Math 408(66,68,74-80) Homework 8 Name & Number:

1. Find the Laurent series of the following in the given region.

(a) $f(z)=\frac{e^z}{z^2}$ in the power of z for $|z|>0$

(b) $g(z)=\frac{1}{z+1}$ in power of z in $|z|>1$

(c) $h(z)=\frac{z}{(z-1)(z-3)}$ in power of z-1 for two regions

(i) $0<|z-1|<2$

(ii) $|z-1|>2$

2. Use Cauchy's Residue theorem to evaluate the integral of each of the function below around $|z|=3.5$ in the positive direction.

(a) $\frac{e^{-z}}{(z-1)^2}$ (b) $z^2e^{\frac{1}{z}}$ (c) $\frac{z+1}{z^2-2z}$

3. Show that any singular point of the function is a pole. Determine the order m of each pole, and find the corresponding residues.

(a) $\frac{z^2+2}{z-1}$

(b) $\left(\frac{z}{2z+1}\right)^3$

4. Find the value of the integral $\oint_C \frac{dz}{z^3(z+4)}$, taken counterclockwise around the circle

(a) $|z|=2$

(b) $|z+2|=3$

5. Suppose that a function f is analytic at z_0, and write $g(z)=f(z)/(z-z_0)$. Show that

(a) if $f(z_0)\neq 0$, then z_0 is a simple pole of g, with residue $f(z_0)$.

(b) if $f(z_0)=0$, then z_0 is a removable singularity point of g.