Math 408(85-89,92,93) Homework 9 Name & Number:

1. Use Residue to evaluate the improper integrals below.

(a) $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$

(b) $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$

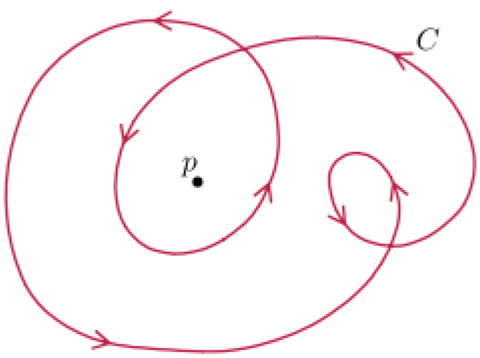
(c) $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$

(d) $\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta}$

2. Let C denote the unit circle $|z|=1$, described in the positive sense. Use the argument principle to determine the value of variation $\Delta_C \arg f(z)$ and integral value of the functions when

(a) $f(z)=z^2$ (b) $f(z)=\frac{(2z-1)^7}{z^3}$ (c) $f(z)=\frac{1}{z^2}$

3. Let f be a function which is analytic inside and on a positively oriented simple closed contour C, and suppose that $f(z)$ is never zero on C. Let the image of under the transformation $w=f(z)$ be the closed contour shown below. Determine the value of $\Delta_C \arg f(z)$ and the number of zeros.



4. Let f be analytic inside and on a simple closed contour C except for a finite number of poles inside C. Denote the zeros by $z_1,\ldots,z_n$ (none of which lies on C) and the poles by $w_1,\ldots,w_m$. If $g(z)$ is analytic inside and on C, prove that

$$\frac{1}{2\pi i}\oint_C g(z)\frac{f'(z)}{f(z)}dz=\sum_{i=1}^{n} g(z_i)-\sum_{j=1}^{m} g(w_j)$$