Lab 6 Forces in Equilibrium

Objective:

- To test the hypothesis that forces combine by the rules of vector addition and that the net force acting on an object at rest is zero (Newton’s First Law).

Equipment:

- Pasco force table with four pulleys
- Hooked weight set
- Dual Range Force Sensor with force table bracket
- Ruler, protractor, right triangle
- Computer interface with Science Workshop software

Physical principles:

Definitions of Sine, Cosine, and Tangent of an Angle

Consider one of the acute (less than 90°) angles, θ, of the right triangle shown in figure 1. As a result of where they reside, the three sides of the triangle are called the opposite side, adjacent side and hypotenuse. The two sides that make up the right angle (exactly 90°) are always the adjacent side and the opposite side. As a result, the length of the hypotenuse is always greater than the length of each of the other two sides but less than the sum of the lengths of the other two sides. The size of the angle θ can be related to the length of the three sides of the right triangle by the use of the trigonometric functions Sine, Cosine and Tangent, abbreviated sin, cos and tan, respectively. They are defined as shown in Figure 1.

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad (1)
\]
\[
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}
\]
\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

Vector Addition

*Graphical method* - Vectors may be added graphically by repositioning each one so that its tail coincides with the head of the previous one (see Figure. 2). The resultant (sum of the forces) is the vector drawn from the tail of the first vector to the head of the last. The magnitude (length) and angle of the resultant is measured with a ruler and a protractor, respectively. Note: In order
to measure the angle, a set of axes must first be defined.

*Component method* - Vectors may be added by selecting two perpendicular directions called the X and Y axes, and projecting each vector on to these axes. This process is called the resolution of a vector into components in these directions. If the angle $\theta$ that the vector makes from the positive X axis, is used (see Figure 3), these components are given by

\begin{align*}
F_x &= F \cdot \cos \theta \\
F_y &= F \cdot \sin \theta
\end{align*}

(2)

The X component of the resultant is the sum of the X components of the vectors being added, and similarly for the Y component. The angle that the resultant makes with the X axis is given by

\[ \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) \]

(3)

and the magnitude is given by

\[ F = \sqrt{F_x^2 + F_y^2} \]

(4)

**Equilibrium Conditions**

Newton's first law predicts that a body will not accelerate when the net force acting on it is zero. So, for an object to be at rest, the resultant force acting on it is zero. Thus, if three forces act on an object at rest, the following relationship has to be satisfied.

\[ \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \]

(5)

An equivalent statement is

\[ \vec{F}_3 = - (\vec{F}_1 + \vec{F}_2) \]

(6)

so that $F_3$ is equal in magnitude and opposite in direction to the vector sum of the other two forces.
Prediction:

Suppose you have one force, of magnitude 3.0 N, directed in the positive $x$ direction ($\theta_1 = 0^\circ$), and a second force, of magnitude 4.0 N, direction in the positive $y$ direction ($\theta_2 = 90^\circ$).

In your journal, draw a graph that includes these two forces (to scale), the vector sum of these two forces, and the needed force that would be equal in magnitude and opposite in direction to the resultant of these two forces. What is the magnitude and direction of $\mathbf{F}$?

Procedure:

Setup Science Workshop:
1. Connect the din plug from the force sensor into analog channel A of the Science Workshop Interface.
2. Open Science Workshop.
3. Click on Sampling Options to change the sampling rate to 500 Hz. Set a stop time of 2 s.
4. Click and drag the analog plug (right hand side) icon to analog channel A, choose Force Sensor.
5. Double click on the Force Sensor icon below analog channel A, to calibrate.
6. Enter 0 in the Low Value box on the left. With no tension in the force sensor string click on the Low Value Read to enter the voltage for zero force.
7. Support 500 g of hooked weights from the end of one of these strings across a pulley and connect a second one to the force sensor. Position the force sensor opposite the pulley. The tension force equals the weight of the mass, which is $(0.500 \text{ kg})(9.8 \text{ m/s}^2) = 4.90 \text{ N}$. Position this pulley opposite the force sensor. Be sure that the string is perpendicular to the end face of the force sensor. Enter 4.9 in the High Value box on the left. Click on the High Value Read to enter the voltage for the 4.9 N force. Click on OK to accept the calibration.
8. Click and drag the graph icon onto the force sensor icon. Click on the statistics icon $\Sigma$ on the lower left, then click on the $\Sigma$ in the statistics window and select Count. Similarly select Mean and Standard Deviation.

Data collection:
Set up the following situations so that in each case the magnitudes of the forces are unequal. In each case, adjust the position of the force sensor in both the angular and radial direction so that the knot in the strings is exactly over the cross-hairs in the center of the force table. The pulleys should be adjusted so that the strings are exactly horizontal and as close to the force table as possible without actually touching the table.
1. Support hooked masses of 200 g \( (F_1 = 0.200 \text{ kg} \times 9.8 \text{ m/s}^2 = 1.96 \text{ N}) \) and 300 g \( (F_2 = 0.300 \text{ kg} \times 9.8 \text{ m/s}^2 = 2.94 \text{ N}) \) from strings over the pulleys so that the angle between forces \( F_1 \) and \( F_2 \) is 90° as shown in Figure 5. The force \( F_{FS} \) is the value displayed by the force sensor. Enter in Table 1 the two masses and weights \( F_1 \) and \( F_2 \). Click on REC and click and drag to select the force data. Record the value of the force \( F_{FS} \) and its standard deviation. From the force table record the direction \( \theta_{FS} \) that the force sensor string is pulling. Compute the magnitude and direction of the sum of the forces \( F_1 \) and \( F_2 \) using equations (3) and (4) and compare your result with \( \theta_{FS} - 180^\circ \) and \( F_{FS} \).

Select your x axis to be along the line of force \( F_i \) at the angle of \( 0^\circ \). Make a sketch in your journal showing these forces as arrows and write the values of each force alongside its arrow. Draw to scale two vectors for \( F_1 \) and \( F_2 \) and add the vectors graphically. Use at least 1/2 of a page for the graphical solution in order to improve the accuracy of your measurement. Then compare the results from the force sensor with the graphical measurements.

2. Support three different masses in an arrangement approximately as shown in Figure 6. Record the masses in Table 2. Calculate and record in table 2 the weight forces and force components. Add the components of \( F_1, F_2, \) and \( F_3 \). Compute the magnitude and direction of the sum of these forces using equations (3) and (4) and compare your result with \( \theta_{FS} - 180^\circ \) and \( F_{FS} \).

Make a sketch in your journal showing these forces as arrows and write the values of each force alongside its arrow. Draw a rough sketch of the sum of these vectors. Label each vector and the sum.

**Conclusions:**

In your conclusions, discuss whether your measurements satisfy the requirements of Newton’s First Law. What can you say about the proposition that forces combine according to the rules of vector addition?

For extra credit repeat the measurements and calculations you did in step 2 using four different weight forces. Compare the measured force sensor result with the calculated component method solution for the four weight forces. Do these results satisfy the requirements of Newton’s First Law?
Table 1 Adding two forces that are at 90°

<table>
<thead>
<tr>
<th>Mass m (kg)</th>
<th>Weight m* g (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F₁ (along x, θ₁ = 0°)</td>
<td></td>
</tr>
<tr>
<td>F₂ (along y, θ₂ = 90°)</td>
<td></td>
</tr>
</tbody>
</table>

| F₉S = | θ₉S = | θ₉S - 180° = |

\[ F_{calc} = \sqrt{F_{x}^2 + F_{y}^2} = \text{_______}, \quad \theta_{calc} = \tan^{-1}\left(\frac{F_{y}}{F_{x}}\right) = \text{_______} \]

\[ \% \text{ error of } F = \left| \frac{F_{FS} - F_{calc}}{F_{calc}} \right| = \text{_______} \quad \text{angle error} = \theta_{FS} - 180° - \theta_{calc} = \text{_______} \]

Table 2 Adding three forces by the component method.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>m*g (N)</th>
<th>Angle θ</th>
<th>Fₓ (N)</th>
<th>Fᵧ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F₁ (along x, θ₁ is zero)</td>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F₂ (θ₂ is in the first quadrant.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F₃ (θ₁ is in the second quadrant)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Components of the vector sum of F₁ and F₂

\[ F_{FS} = \text{_______} \quad \theta_{FS} = \text{_______} \quad \theta_{FS} - 180° = \text{_______} \]

\[ F_{calc} = \sqrt{F_{x}^2 + F_{y}^2} = \text{_______}, \quad \theta_{calc} = \tan^{-1}\left(\frac{F_{y}}{F_{x}}\right) = \text{_______} \]

\[ \% \text{ error of } F = 100\cdot \left| \frac{F_{FS} - F_{calc}}{F_{calc}} \right| = \text{_______} \quad \text{angle error} = \theta_{FS} - 180° - \theta_{calc} = \text{_______} \]