Lab 3 Momentum Change and Impulse

Objectives:

- To measure the change in momentum of a cart in a collision and the impulse acting on it during the collision and to compare these values as a test of the impulse-momentum theorem.
- To compare the action and reaction forces that two carts exert on each other in a collision.
- To compare the total momentum of two cars before and after a collision.

Equipment:

- Track with two carts, accessory weights
- Two motion sensors
- Two force sensors mounted on carts
- Electronic balance

Physical principles:

Momentum

A body with mass \( m \) and velocity \( \vec{v} \) has by virtue of its motion an important property called momentum. This momentum is the product of the object’s mass and velocity and is labeled by the symbol \( \vec{p} \).

\[ \vec{p} = m \vec{v} \]  \hspace{1cm} (1)

A simple relationship between force, momentum and time can be found from Newton's second law of motion.

\[ \vec{F} = m \vec{a} = m \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta m \vec{v}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} \]  \hspace{1cm} (2)

so that the force has the meaning of the change in momentum over elapsed time.

Impulse and the Impulse-Momentum Theorem

Multiplying equation (2) by \( \Delta t \) gives

\[ I = \vec{F} \Delta t = \Delta \vec{p} \]  \hspace{1cm} (3)

where \( I \) is called the vector impulse, the product of the force and the time that the force acts on the system. When the force is varying in time the impulse is the area under the force versus time graph. This statement that the impulse that acts on an object equals the change in its momentum is called the impulse-momentum theorem.
Lab 3 Momentum Change and Impulse

Objectives:

- To measure the change in momentum of a cart in a collision and the impulse acting on it during the collision and to compare these values as a test of the impulse-momentum theorem.
- To compare the action and reaction forces that two carts exert on each other in a collision.
- To compare the total momentum of two cars before and after a collision.

Equipment:

- Track with two carts, accessory weights
- Two motion sensors
- Two force sensors mounted on carts
- Electronic balance

Physical principles:

Momentum

A body with mass $m$ and velocity $v$ has by virtue of its motion an important property called momentum. This momentum is the product of the object’s mass and velocity and is labeled by the symbol $p$.

$$\vec{p} = m\vec{v} \quad (1)$$

A simple relationship between force, momentum and time can be found from Newton's second law of motion.

$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta m\vec{v}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t} \quad (2)$$

so that the force has the meaning of the change in momentum over elapsed time.

Impulse and the Impulse-Momentum Theorem

Multiplying equation (2) by $\Delta t$ gives

$$I = \vec{F}\Delta t = \Delta\vec{p} \quad (3)$$

where $I$ is called the vector impulse, the product of the force and the time that the force acts on the system. When the force is varying in time the impulse is the area under the force versus time graph. This statement that the impulse that acts on an object equals the change in its momentum is called the impulse-momentum theorem.
Momentum Conservation

When the system consists of several parts, the force in equation (2) is the vector sum of the individual forces and the momentum is the vector sum of the momenta of all parts of the system. If in a collision the only forces that act are the collision forces between the colliding objects then the total impulse acting on the system is zero. This is because of Newton’s third law, the law of action and reaction. In our experiment the collision force of cart A on cart B is in one direction and is equal and opposite to the reaction force, the force that cart B exerts on cart A. Since both carts are part of our system the net force acting on the system is the sum of these forces which is zero. The resulting impulse is zero so that the total momentum of the system does not change, the total momentum before the collision is the same as the total momentum after the collision.

\[
\vec{p}_i = \vec{p}_f \quad \text{where} \quad \vec{p}_i = m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} \quad \text{and} \quad \vec{p}_f = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}
\]  

(4)

This principle of conservation of momentum is one of the most important ideas in physics.

Procedure:

In this experiment cart A collides with a second stationary cart B. The velocities of each cart is measured with a motion sensor, that of cart A is measured with a motion sensor in digital channels 1 and 2, that of cart B with a second motion sensor that is plugged into digital channels 3 and 4. An additional mass of 1 kg is placed on cart A. We take the positive direction to be that indicated by motion sensor A so that the velocities of cart A are correctly indicated. Since motion sensor B measures positive to be in the opposite direction we need to reverse the sign of the velocity of cart B measured by motion sensor B. The collision forces that each cart exerts on the other are measured by the force sensors mounted on the carts. The force sensor on cart A measures the force that cart B exerts on cart A and the force sensor on cart B measures the force that cart A exerts on cart B.

Setup Science Workshop: (Done by Instructor)

1. Open Science Workshop.
2. Plug in the motion sensor A into digital channels 1 and 2.
3. Drag the digital plug icon over digital channel 1 and select motion sensor. Double click on the motion sensor icon and set the sampling rate to 25.
4. Plug the force sensor into analog channel A.
5. Drag the analog plug icon over analog channel A and select force sensor. Double click on the force sensor to calibrate it. Set the low value to zero, and click on low value read with no mass hanging from the force sensor. Set the high value to 9.8 N and click on the high value read with a 1000 g mass (weight = 9.8 N) hanging from the force sensor.
6. Repeat steps 2 - 5 for cart B plugging the motion sensor into digital channels 3 and 4 and the force sensor into analogue channel B.
7. Click on Sampling Options to change the sampling rate to 500 Hz and the stop time to 2 sec.
8. Make a graph of velocity versus time by clicking on the graph icon, dragging it over the
Momentum Conservation

When the system consists of several parts, the force in equation (2) is the vector sum of the individual forces and the momentum is the vector sum of the momenta of all parts of the system. If in a collision the only forces that act are the collision forces between the colliding objects then the total impulse acting on the system is zero. This is because of Newton’s third law, the law of action and reaction. In our experiment the collision force of cart A on cart B is in one direction and is equal and opposite to the reaction force, the force that cart B exerts on cart A. Since both carts are part of our system the net force acting on the system is the sum of these forces which is zero. The resulting impulse is zero so that the total momentum of the system does not change, the total momentum before the collision is the same as the total momentum after the collision.

\[ \vec{p}_i = \vec{p}_f \quad \text{where} \]
\[ \vec{p}_i = m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} \quad \text{and} \quad \vec{p}_f = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf} \]  \hspace{1cm} (4)

This principle of conservation of momentum is one of the most important ideas in physics.

Procedure:

In this experiment cart A collides with a second stationary cart B. The velocities of each cart is measured with a motion sensor, that of cart A is measured with a motion sensor in digital channels 1 and 2, that of cart B with a second motion sensor that is plugged into digital channels 3 and 4. An additional mass of 1 kg is placed on cart A. We take the positive direction to be that indicated by motion sensor A so that the velocities of cart A are correctly indicated. Since motion sensor B measures positive to be in the opposite direction we need to reverse the sign of the velocity of cart B measured by motion sensor B. The collision forces that each cart exerts on the other are measured by the force sensors mounted on the carts. The force sensor on cart A measures the force that cart B exerts on cart A and the force sensor on cart B measures the force that cart A exerts on cart B.

Setup Science Workshop: (Done by Instructor)
1. Open Science Workshop.
2. Plug in the motion sensor A into digital channels 1 and 2.
3. Drag the digital plug icon over digital channel 1 and select motion sensor. Double click on the motion sensor icon and set the sampling rate to 25.
4. Plug the force sensor into analog channel A.
5. Drag the analog plug icon over analog channel A and select force sensor. Double click on the force sensor to calibrate it. Set the low value to zero, and click on low value read with no mass hanging from the force sensor. Set the high value to 9.8 N and click on the high value read with a 1000 g mass (weight = 9.8 N) hanging from the force sensor.
6. Repeat steps 2 - 5 for cart B plugging the motion sensor into digital channels 3 and 4 and the force sensor into analogue channel B.
7. Click on Sampling Options to change the sampling rate to 500 Hz and the stop time to 2 sec.
8. Make a graph of velocity versus time by clicking on the graph icon, dragging it over the
motion sensor icon, selecting digital channel 1, velocity, and clicking on OK. Add the force sensor graph to the graph window by clicking on the multiple graph icon (lower right icon in the lower left corner of the graph) and selecting analog A and force sensor, OK.

9. Add the velocity of cart B and force sensor B graphs to the graph window by clicking on the multiple graph icon (lower right icon in the lower left corner of the graph) and selecting digital channel 3, velocity, OK and then analog B and force sensor, OK.

10. Start statistics by clicking on the Σ button in the graph window, then click on the Σ button beside the cart A velocity graph and select mean and standard deviation. Click on the Σ button beside the force sensor graph and select integration. Repeat this for the cart B velocity and force graphs.

Setup system:

1. Measure and record in your journal the masses of the carts, \( M_A \) and \( M_B \) including the masses of the force sensors.
2. Place the cart on the track and level the track so that the cart does not accelerate toward motion sensor B. Cardboard reflectors are mounted on the carts to enhance the ultrasound reflections.

Data Collection: Cart A with 1 kg Mass

1. Place a 1 kg mass on cart A and click on the REC button as your lab partner sends cart A toward the stationary cart B. Take care to tend the force sensor cables so as not to exert forces on the carts. An initial speed of about .3 m/s for cart A works well.
2. Click and drag the curser to select the best data from the velocity vs. time graph of cart A just before the collision. Click and drag the curser to select the best velocity data just after the collision. Record in Table 1 the mean values of the velocities \( v_{Ai} \) and \( v_{Af} \). Repeat this for the velocity of cart B just after the collision. Remember to make \( v_{Bf} \) positive.
3. Click and drag the curser to select the area under the force vs. time graph of cart A during the collision and record in Table 1 this area as the impulse \( I_A \). Repeat this for cart B.

Analysis of Data:

Complete Table 2. Compute the momenta as the products of the mass times the velocity.

1. The first column is the initial momentum of cart A. This is also the initial momentum of the system since cart B is initially at rest.
2. The second column is the momentum of cart A after the collision.
3. The third column is the change in the momentum of cart A, the second column minus the first column.
4. The fourth column is the final momentum of cart B. This is also the change in the momentum of cart B since it is initially at rest.
5. The last column is the final momentum of the system and it is the sum of the second and fourth columns.
motion sensor icon, selecting digital channel 1, velocity, and clicking on OK. Add the force sensor graph to the graph window by clicking on the multiple graph icon (lower right icon in the lower left corner of the graph) and selecting analog A and force sensor, OK.

9. Add the velocity of cart B and force sensor B graphs to the graph window by clicking on the multiple graph icon (lower right icon in the lower left corner of the graph) and selecting digital channel 3, velocity, OK and then analog B and force sensor, OK.

10. Start statistics by clicking on the Σ button in the graph window, then click on the Σ button beside the cart A velocity graph and select mean and standard deviation. Click on the Σ button beside the force sensor graph and select integration. Repeat this for the cart B velocity and force graphs.

Setup system:

1. Measure and record in your journal the masses of the carts, $M_A$ and $M_B$ including the masses of the force sensors.
2. Place the cart on the track and level the track so that the cart does not accelerate toward motion sensor B. Cardboard reflectors are mounted on the carts to enhance the ultrasound reflections.

Data Collection: Cart A with 1 kg Mass

1. Place a 1 kg mass on cart A and click on the REC button as your lab partner sends cart A toward the stationary cart B. Take care to tend the force sensor cables so as not to exert forces on the carts. An initial speed of about $0.3 \text{ m/s}$ for cart A works well.
2. Click and drag the cursor to select the best data from the velocity vs. time graph of cart A just before the collision. Click and drag the cursor to select the best velocity data just after the collision. Record in Table 1 the mean values of the velocities $v_{A_i}$ and $v_{A_f}$. Repeat this for the velocity of cart B just after the collision. Remember to make $v_{B_f}$ positive.
3. Click and drag the cursor to select the area under the force vs. time graph of cart A during the collision and record in Table 1 this area as the impulse $I_A$. Repeat this for cart B.

Analysis of Data:

Complete Table 2. Compute the momenta as the products of the mass times the velocity.

1. The first column is the initial momentum of cart A. This is also the initial momentum of the system since cart B is initially at rest.
2. The second column is the momentum of cart A after the collision.
3. The third column is the change in the momentum of cart A, the second column minus the first column.
4. The fourth column is the final momentum of cart B. This is also the change in the momentum of cart B since it is initially at rest.
5. The last column is the final momentum of the system and it is the sum of the second and fourth columns.
6. Calculate the percent error of the difference between impulse in Table 1 and momentum change in Table 2 for both carts A and B by using

\[
\% \text{Err}_{Ip} = 100 \frac{|I - \Delta p|}{I}
\]  

\hspace{1cm} (5)

7. Calculate the percent error of the difference between the initial and final momentum in Table 2 by using

\[
\% \text{Err} = 100 \frac{|p_f - p_i|}{p_i}
\]  

\hspace{1cm} (6)

8. Graph the difference between the force of cart A on cart N and the force of cart B on cart A. To do this click on the calculator, click on input, analogue A, force, then press the - to subtract, and click on input, analogue B, force. Enter Net Force as the calculation name and NF as the short name and press enter. Click on the add graph icon, click on calculations, Net Force to see a graph of the difference of the force - reaction pair. This should be zero. Calculate the percent error of the greatest deviation from zero to the maximum force exerted.

**In your conclusions you should:**

1. Compare the impulse acting on each cart to its change in momentum. To within what percent did you find these to be the same?
2. Compare the initial and final total momenta. To within what percent did you find these to be the same?
3. Compare the action-reaction forces. To within what percent did you find these to be the same?
4. Speculate on the origin(s) of error.
5. Describe what you learned in this experiment.

\[M_A = \text{__________} \quad M_{A+1 \text{ kg}} = \text{__________} \quad M_B = \text{__________}\]

**Table 1 Velocity and Impulse Data**

<table>
<thead>
<tr>
<th>(v_{Ai} \text{ (m/s)})</th>
<th>(v_{Af} \text{ (m/s)})</th>
<th>(v_{Bf} \text{ (m/s)})</th>
<th>(I_A \text{ (N s)})</th>
<th>(I_B \text{ (N s)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{______})</td>
<td>(\text{______})</td>
<td>(\text{______})</td>
<td>(\text{______})</td>
<td>(\text{______})</td>
</tr>
</tbody>
</table>

**Table 2 Momentum Calculations**

<table>
<thead>
<tr>
<th>(p_{Ai} = p_i \text{ (N s)})</th>
<th>(p_{Af} \text{ (N s)})</th>
<th>(\Delta p_A \text{ (N s)})</th>
<th>(p_{Bf} = \Delta p_B \text{ (N s)})</th>
<th>(p_f \text{ (N s)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{______})</td>
<td>(\text{______})</td>
<td>(\text{______})</td>
<td>(\text{______})</td>
<td>(\text{______})</td>
</tr>
</tbody>
</table>
6. Calculate the percent error of the difference between impulse in Table 1 and momentum change in Table 2 for both carts A and B by using

\[
\% \text{Err} = 100 \frac{|I - \Delta p|}{I}
\]  
(5)

7. Calculate the percent error of the difference between the initial and final momentum in Table 2 by using

\[
\% \text{Err} = 100 \frac{|p_f - p_i|}{p_i}
\]  
(6)

8. Graph the difference between the force of cart A on cart N and the force of cart B on cart A. To do this click on the calculator, click on input, analogue A, force, then press the - to subtract, and click on input, analogue B, force. Enter Net Force as the calculation name and NF as the short name and press enter. Click on the add graph icon, click on calculations, Net Force to see a graph of the difference of the force - reaction pair. This should be zero. Calculate the percent error of the greatest deviation from zero to the maximum force exerted.

**In your conclusions you should:**
1. Compare the impulse acting on each cart to its change in momentum. To within what percent did you find these to be the same?
2. Compare the initial and final total momenta. To within what percent did you find these to be the same?
3. Compare the action - reaction forces. To within what percent did you find these to be the same?
4. Speculate on the origin(s) of error.
5. Describe what you learned in this experiment.

\[M_A = \text{__________} \quad M_{A+1 \text{ kg}} = \text{__________} \quad M_B = \text{__________}\]

**Table 1 Velocity and Impulse Data**

<table>
<thead>
<tr>
<th>(v_{Ai} \text{ (m/s)})</th>
<th>(v_{Af} \text{ (m/s)})</th>
<th>(v_{bf} \text{ (m/s)})</th>
<th>(I_A \text{ (N s)})</th>
<th>(I_B \text{ (N s)})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2 Momentum Calculations**

<table>
<thead>
<tr>
<th>(p_{Ai} = p_i \text{ (N s)})</th>
<th>(p_{Af} \text{ (N s)})</th>
<th>(\Delta p_A \text{ (N s)})</th>
<th>(p_{Bf} = \Delta p_B \text{ (N s)})</th>
<th>(p_f \text{ (N s)})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>