My wife’s father was an Adventist preacher. While she and I were courting, one of his female parishioners took me aside and admonished me very seriously that no mathematicians would ever get to heaven!

By contrast, much of what initially attracted me to mathematics appeared in Uriah Smith’s *Daniel and the Revelation*, which showed how the events of history fit like jigsaw puzzle pieces into carefully calculated time prophecies. The first great Adventist disappointment happened because early calendar makers miscalculated the end date for the 2300-year period. It took mathematically savvy believers to discover the error.

As teachers of mathematics, it is our God-given responsibility to cajole, encourage, drive, insist, and force (if possible) our students to think. Peter encourages us to be able to give “to everyone who asks you a reason for the hope that is in you.” Reasons that were valid in the last millennium do not necessarily persuade young people today. More than ever before, we need to “train young people to be thinkers, and not mere reflectors of other people’s thought.” Two great ways to train the mind are through the study of the Bible and of mathematics.

As far back as 600 B.C., Thales insisted that results in mathematics not be accepted unless they could be proved. Today, much of what is passed off as math is watered down, diluted, and polluted by cultural, societal, and political pressures. It is our God-given responsibility to see that students master the basics of mathematics that transcend language, culture, race, and national heritage.

“Higher than the highest human thought can reach is God’s ideal for His children” is the great ideal that we must strive for. Attaining the ideal is not easy. Articles in the special section of this issue stress various facets that, we hope, will provide ideas that will improve your techniques and abilities in communicating mathematics to your students. Since nowadays, mathematics is almost universally regarded as difficult, we need to use history, culture, religion, patriotism, sports, literature, romance, and everything else we can muster as motivation and imperatives for excelling in mathematics.

I had a teacher in one of my classes tell me this past spring, “When I started your Concepts of Mathematics class, I feared and dreaded mathematics. You helped me see that I could teach mathematics. Now my 4th graders beg me to let them do math!”

Authors Nugent and Moore give us some practical strategies that teachers can use in the classroom. Sharilyn Horner offers a list of what she wishes students entering college knew on arrival.

Cathleen Duffy reviews the literature on the great strides that can be achieved by peer tutoring in math. Marion Prince offers insights into the Common Core State Standards for mathematics that have been incorporated into the latest North American Division K-12 math curriculum. Murray Cox addresses gender issues and mathematics.

Math has long been regarded as one of the 3 R’s! As Adventist teachers, let’s replace the 3 R’s with the 4 R’s: Reading, ’Riting, ’Rithmetic, and Religion. Many prominent mathematicians of former years were deeply spiritual. This side of them is consistently downplayed by many of their biographers. I discuss a few of their experiences in my article on Mathematical Vignettes.

Continued on page 46
35. Williams, “Tiering and Scaffolding,” op. cit.
36. The teacher will need to watch the progress of the lessons because by this point, grade 5 students may need a short group lesson to review and support their understanding of the idea of how repeated addition is related to multiplication.
43. Goodlad and Hirst, Peer Tutoring, op. cit., p. 141.
46. Sparks, “Researchers Find That Students Learn by Tutoring Virtual Peers,” op. cit.

Guest Editorial Continued from page 3

Each of these articles in this issue is meant to inspire and encourage you as you lead students into the adventure of mathematics.

The Coordinator for the special section on mathematics in this issue, Wil Clarke, Ph.D., is Professor of Mathematics at La Sierra University in Riverside, California. The editorial staff of the JOURNAL express heartfelt appreciation for his assistance throughout every phase of the production of the issue from the selection of topics and authors through the peer review process, revisions, and preparing the final manuscripts.

NOTES AND REFERENCES
1. 1 Peter 3:15 (NKJV), italics supplied. Texts credited to NKJV are from the New King James Version. Copyright © 1979, 1980, 1982, by Thomas Nelson, Inc. Used by permission. All rights reserved.
4. Ibid.
Making a Difference:
Child abuse is a global problem. In 2010, more than 3.3 million reports of child abuse were filed in the United States, involving nearly six million children. It is estimated that 27 children under the age of 15 die from physical abuse or neglect in the U.S. every week! The actual global child-abuse statistics are difficult to determine due to varying reporting measures, laws pertaining to child abuse, and cultural or social norms. However, the World Health Organization estimates that 40 million children below the age of 15 are abused each year.

Recognizing the Forms of Child Abuse

Child abuse is generally committed by a person in a position of trust or authority: a parent, family member, guardian, teacher, or caregiver. Less than 10 percent of incidents in the U.S. involve strangers. Child abuse occurs at every socioeconomic level, across all ethnic and cultural lines, within all religions and at all levels of education, and is seldom accidental. Typically, abuse fits into the following categories:

- **Neglect/maltreatment**—failure to provide appropriately for a child's welfare, nutritional needs, shelter, clothing, medical care, adequate supervision, or educational opportunities.
- **Physical abuse**—corporal punishment or physical injury inflicted due to the willful acts of another person.
- **Sexual abuse**—engaging a child in sexual activities that he or she cannot comprehend and that violate social norms, laws, and moral standards.
- **Psychological maltreatment**—behaviors that convey that a child is worthless, flawed, unloved, unwanted, or only of value in meeting another person's needs.

The U.S. Department of Health and Human Services provides a helpful fact sheet that can help adults recognize key indicators of child abuse:

- **In the child:** sudden changes in behavior or school performance; learning problems or difficulty in concentrating; anxiety that something bad will happen; lack of adult supervision; overly compliant and passive; withdrawal; coming to school early, staying late, and not wanting to go home.
- **In the parent or guardian:** lack of concern for the child; failure to seek help for the child's physical or medical problems that have been brought to the parent or guardian's attention; denial of, or blaming the child for, problems...
in school or home; asking teachers to use harsh physical discipline if the child misbehaves; seeing the child as evil, worthless, or burdensome; or demanding a performance level that the child cannot achieve.

- In the parent and child: rarely touching or looking at each other; viewing their relationship negatively; or a stated dislike for each other.

**Child Sexual Abuse**

While abuse in any form can severely affect the child’s life and welfare, sexual abuse is perhaps the most devastating.

- Of all reported cases of sex abuse in the U.S. in 2010, child sexual abuse accounted for 9.2 percent.
- One in four girls and one in six boys under the age of 17 is sexually abused by an adult.
- The peak age for abuse is between 7 and 13.
- In more than 90 percent of cases, the victim knew the perpetrator in some way, but fewer than 10 percent reported the abuse.
- Only three percent of offenders are apprehended.
- In more than half the cases, adults knew about a crime and failed to report the situation to law-enforcement authorities.
- Girls are more likely to be abused by family members, and boys by friends of the family.

**Adventist Schools and Child Abuse**

The Seventh-day Adventist Church operates the second-largest Christian school system in the world, with more than 7,800 schools and colleges in more than a hundred countries. Adventists are proud of their heritage of providing Christ-centered education, and many denominational employees are graduates of Adventist schools. Because of this, it is easy for us to live in a world of denial.

Adventist educators must be alert to the possibility that our students may be targets for abuse. We must take proactive measures to protect children from abusive situations and report suspected cases of child abuse in accordance with local laws. This is a difficult challenge, but it comes from Jesus Himself: “If you cause one of these little ones who trust in me to fall into sin, it would be better for you to have a large millstone tied around your neck and be drowned in the depths of the sea” (Matthew 18:6, NLT).

In 1997, the Seventh-day Adventist Church issued the following position statement addressing child sexual abuse:

“Child sexual abuse occurs when a person older or stronger than the child uses his or her power, authority, or position of trust to involve a child in sexual behavior or activity. . . . Sexual abusers may be men or women of any age, nationality, or socio-economic background. They are often men who are married with children, have respectable jobs, and may be regular churchgoers. It is common for offenders to strongly deny their abusive behavior, to refuse to see their actions as a problem, and to rationalize their behavior or place blame on something or someone else. While it is true that many abusers exhibit deeply rooted insecurities and low self-esteem, these problems should never be accepted as an excuse for sexually abusing a child. Most authorities agree that the real issue in child sexual abuse is more related to a desire for power and control than for sex.

“Every child, whether male or female, is to be affirmed as a gift from God. The Bible condemns child sexual abuse in the strongest possible terms. It sees any attempt to confuse, blur, or denigrate personal, generational, or gender boundaries through sexually abusive behavior as an act of betrayal and a gross violation of personhood.”
from the church? Warns Ellen White: “Too much importance cannot be placed on the early training of children. The lessons that the child learns during the first seven years of life have more to do with forming his character than all it learns in future years.”

Protecting the Adventist Campus

Operating a large system of education from preschool to university level poses tremendous challenges in the area of preventing and dealing with child abuse and related liability claims. Every Adventist school administrator, therefore, must craft a child-abuse prevention plan. The first step is to identify potential areas where child abuse might occur on campus. Proactive assessment and planning to prevent child abuse requires that the school administration create policies that do the following:

- Ensure appropriate and adequate supervision at all times;
- Prevent situations where a student is alone with a teacher/other school personnel in order to lessen likelihood of inappropriate contact;
- Make it clear that inappropriate jokes, comments, and personal conduct with students will not be condoned or permitted;
- Instruct school employees to carefully avoid improper forms of touch when interacting with students;
- Have glass panels installed in all classroom areas, and keep them uncovered at all times;
- Minimize one-on-one out-of-school interactions with students by school personnel;
- Monitor and control the use of social media communication between school employees and students;
- Do not allow individual students to be transported alone in an employee’s or volunteer’s vehicle;
- Provide adequate supervision for off-campus school-sponsored trips;
- Ensure adequate lighting throughout the campus; and
- Conduct background checks on all employees and volunteers.

The U.S. Department of Health and Human Services has published a very useful guide, Preventing Child Sexual Abuse Within Youth-Serving Organizations, which includes a planning tool for child sexual-abuse prevention. This document provides a comprehensive risk-assessment process that you can implement in your school. Once you complete the assessment, you will be better equipped to begin developing the appropriate risk-mitigation strategies, policies, codes of conduct, training, and safeguards to ensure a safe campus environment.

What About Higher Education Risks?

The 2011 child-abuse scandal that rocked the campus of Pennsylvania State University should serve as a warning that all schools, regardless of size or grade level, are vulnerable to child abuse and its related claims. Many times, administrators of institutions of higher education seem oblivious to the risk that child abuse may affect their institutions. The 267-page special report on the Penn State case points out: “The most saddening finding by the Special Investigative Counsel is the total and consistent disregard by the most senior leaders at Penn State for the safety and welfare of . . . child victims. . . . Four of the most powerful people at the [University] . . . failed to protect against a child sexual predator harming children for over a decade. . . . They exhibited a striking lack of empathy for [the] . . . victims by failing to inquire as to their safety and well-being.”

Could the same type of situation arise at an Adventist college or university?

Higher education campuses are filled with numerous intersections between minors and adults. The 2012 report Managing the Risks of Minors on Campus, published by Arthur J. Gallagher & Company, identified numerous activities that involve children on campus beyond the more obvious exposures in academic areas like education, child development, social work, psychology, and nursing. The report encourages all colleges and universities to assess carefully their interaction points with students: “Many institutions that inventory their youth serving programs find more programs than they expected. . . . One university thought it had two programs that involved minors. When a full risk assessment was done, it discovered it actually had 166.” Here are just a few times when children could be at risk of abuse, and adults could be threatened by false allegations: traveling on school buses or in private vehicles, student teaching, work experience, academy visitation days, music camps, invitational athletic events or clinics, mission trips, research studies, housing for school field trips, community outreach programs, summer education programs or day camps, and daycare for students’ or employees’ children, just to name a few. The school has a responsibility to prevent abuse of children and youth by other students as well as by employees/volunteers and strangers on campus.

Counting the Costs

The estimated annual cost of child abuse and neglect in the United States for 2008 was $124 billion for legal defense and indemnity costs. The impact has been felt across faith-based communities. From 1950 through the mid 2000’s, Catholic institutions reported they had paid out in excess of $2 billion, and the resulting financial disruption has caused the bankruptcy of six dioceses in the United States.

The Adventist community is not immune from abuse-related costs. Adventist Risk Management (ARM) administers the claims for denominational organizations in North America. In the past two decades (1992-2011), more than 400 claims were made in the United States, involving more than 525 child victims. The incurred cost to the church, in the U.S. alone, has exceeded $30 million. ARM routinely handles 15 to 20 reported claims in the U.S. each year.

The number of victims and the dollars spent represent only the “numerical” data. The true cost of child abuse and its impact on individuals involved, as well as the church and its institu-
Each school has a responsibility to protect its employees, volunteers, and students from charges of abuse. Since such allegations may be real or false, all reported incidents must be thoroughly investigated.

Putting Integrity First

Each school has a responsibility to protect its employees, volunteers, and students from charges of abuse. Since such allegations may be real or false, all reported incidents must be thoroughly investigated. Alleged perpetrators should not be allowed to return to the campus until the inquiry is completed. Incidents should be reported promptly to legal authorities, the conference office of education, and the school’s insurance carrier. Engaging legal counsel with experience in handling child-abuse cases may provide valuable assistance in understanding the law as well as guidance with the school’s internal investigation. Remember: The personal life and professional reputation of the accused and the future of the victim are at stake.

When child abuse is alleged, this raises immediate concern about how the school should handle the situation. People tend to hope that if they simply ignore the situation, it will go away or be handled by someone else. However, the allegations concerning child abuse are very serious and must be handled promptly and appropriately, in accordance with the local child-abuse reporting laws. Ignorance about what to do is not an excuse.

In 18 U.S. states, every adult is considered a mandated reporter for suspected incidents of child abuse. Most jurisdictions provide some form of immunity from liability for persons who in good faith report possible child abuse or neglect. Immunity applies as long as the report is not made maliciously or without reasonable grounds. A review of the local child-abuse reporting laws and training on how teachers and school administrators must respond should be included in mandatory in-service seminars each school year for all employees and volunteers. Schools should invite representatives from local law-enforcement agencies to participate in this training.

In most jurisdictions, educators and clergy are considered mandated reporters of suspected cases of child abuse. Failure to report can result in criminal actions against the teacher or pastor. Monsignor William Lynn, the secretary for the clergy in the Philadelphia Archdiocese recommending priest assignments and investigating abuse complaints, was convicted in June 2012 on charges of child endangerment and sentenced to three to six years in prison. “I think this is going to send a very strong signal to every bishop and everybody who worked for a bishop that if they don’t do the right thing, they may go to jail,” said Reverend Thomas Reese, at the Woodstock Theological Center at Georgetown University.

For a variety of reasons, child abuse is often not reported. The abused child may feel threatened that disclosure will cause further physical harm either to himself or herself or to another person. The child may be afraid of what others will think if abuse is revealed, or ashamed and feel he or she is to blame. Many children would rather continue in the abusive situation than go through the trauma of reporting the abuse, which could result in their losing the security of home and love of parents. “For this reason [feeling traumatized], most abused children decide not to tell anyone. Those who do disclose an abusive experience are displaying courage despite their fears and feelings.”

But what about the potential for false allegations? Those do occur, and they can have a devastating impact on
the lives of everyone involved. Nevertheless, it is not the administrator’s responsibility to determine whether a child is lying or embellishing the facts. Instead, he or she has the legal duty to report suspected abuse to authorities and let the child-protection agencies determine the truthfulness of the allegations.

When a child has the courage to confide his or her story to a trusted adult, this becomes a critical intersection in the child’s life—a sign that he or she is reaching out for help. Listed below are key elements for protecting children and for crafting a respectful response:

1. If you suspect that the child is in danger, immediately call the police.
2. Keep control of your emotions. Reaffirm the child and offer reassurance that whatever happened, it was not his or her fault.
3. Stay calm and listen carefully; take notes if possible.
4. Believe the child; expressing disbelief may limit full disclosure.
5. Do not interrogate the child; just get the essential facts and then notify the parents or guardian, if appropriate.
6. Show support and provide encouragement that the child is doing the right thing.
7. Do not promise to keep the information secret. Tell the child what will follow next; provide reassurance that you are there to provide support.
8. Promptly file an official report with law enforcement or a child-protective service in accordance with reporting laws in your jurisdiction. Record the name of the person you spoke with as well as the date and time of the report. Keep a copy of the report on file.
9. Cooperate with the investigation.

Oftentimes, the educators and pastors fear that the mere allegation of child abuse will destroy the school or church. This is never the truth. With thousands of pages written on the Penn State child abuse story, one lesson is clear: “Protecting the institution should always come second to doing what is right. Ultimately, that will be what protects the organization.”

The Use of Corporal Punishment

The use of corporal punishment can create a high liability risk for educators in Adventist schools. At least 106 countries do not prohibit the use of corporal punishment in schools. In the United States, 20 states still allow some form of corporal punishment. Corporal punishment is defined under human-rights laws as “any punishment in which physical force is used and intended to cause some degree of pain or discomfort.” There is no comprehensive definition under U.S. state or federal law. Corporal punishment in schools has been banned in Canada, most of Europe, Japan, South Africa and New Zealand.

Publications

- Choose With Care: Building Child Safe Organisations, © 2001 – ECPAT Australia, South Melbourne, Victoria, Australia: http://www.ecpatal.org
- For Their Sake, © 1992 – American Camp Association, Martinsville, Indiana: http://www.acacamps.org
- Plan to Protect, © 2008 – Winning Kids Inc., Markham, Ontario, Canada: http://www.winningkidsinc.ca

Helpful Websites

- Canadian Center for Child Protection – http://www.protectchildren.ca
- Childhelp Centre – http://www.childhelp.org
- ECPAT Australia – http://www.childwise.net
- FaithTrust Institute – http://www.faithtrustinstitute.org
- International Center for Assault Prevention – http://www.internationalcap.org
- Nonprofit Risk Management Center – http://www.nonprofitrisk.org
- Prevent Child Abuse, America – http://www.preventchildabuse.org
- Reducing the Risk – Christianity Today: http://www.reducingtherisk.com

http://jae.adventist.org
Educators may be held criminally liable for their actions if excessive physical force is used in disciplining students. School administrators should determine the legality of using corporal punishment within their country or local community. Adventist Risk Management supports a prohibition on the use of corporal punishment in church institutions and believes that other non-physical means of student discipline should be used. This will minimize any potential liability against the school employee or volunteer, and protect children from injury and emotional trauma.

Creating a Child Abuse Awareness Culture

Creating a safe, abuse-free campus requires teamwork and proactive effort. School administrators must be willing to develop a “zero tolerance” policy when it comes to child abuse. This policy should be written and clearly communicated to every teacher, volunteer, student, and parent, as well as to school constituents. This message must be clearly communicated: We place the safety of our students first!

One of the emerging child-abuse trends is students physically and sexually abusing other students. Child-on-child abuse incidents can occur when children and teenagers are left unsupervised during school or church activities. These types of incidents often begin as bullying and escalate from name calling to more serious forms of physical or sexual abuse, including cyberbullying and rape.26 Adventist Risk Management received six claims from schools and churches in the United States in 2010 alleging incidents of child-on-child sexual abuse. These types of incidents can result in negligent supervision liability lawsuits being brought against Adventist schools.

One of the key elements of a child-protection policy is a code of conduct for both adults and students. Child abusers, especially sex offenders, use specific strategies in identifying the children they will abuse. First, they win the child’s confidence and trust through a grooming process that may take many months or even years, often even befriending the child’s family. They slowly violate the boundaries by making the child comfortable through playful acts. Corey Jewel Jensen, co-director of the Center for Behavioral Intervention in Beaverton, Oregon, says: “In addition to the tricks they use with children, they also know how to keep other adults from discovering their crimes or reporting them to the police. They say they present an image of ‘morality and respectfulness’ and they make people think ‘that I am not the kind of person who would do something like that’; they act ‘helpful and polite.’”27

A well-developed code of conduct should succinctly and clearly articulate boundaries and behaviors as well as appropriate and inappropriate behaviors. Clear definitions ensure that everyone knows the rules and the consequences for acting outside of the boundaries.28 Having a clear code of conduct and strictly enforcing it protects children from abuse and adults from false allegations.

Background Checks

Most jurisdictions in the United States require teachers and school personnel to be fingerprinted and undergo a criminal background check before being issued licenses or credentials. School administrators should be sure all employees’ personnel files are up-to-date and in compliance with the policies of their conference and local state or county. School volunteers should also be screened. The conference may use a specific service provider or a law-enforcement agency to conduct criminal background checks for volunteers. Schools must follow the proper procedures in order to guard employees’ and volunteers’ privacy and to comply with disclosure laws, which can vary among jurisdictions.

Schools outside North America should consult their legal counsel or local law-enforcement agency to comply with the background screening requirements in their jurisdiction. In areas with no specific regulations, school administrators should implement the following measures before allowing volunteers to interact with students: (1) checking the volunteer’s personal references; (2) requiring a two-adults-with-a-child rule at all times; and (3) requiring a six-month probationary/orientation period to observe the volunteer’s behavior during which he or she receives training regarding appropriate conduct between adults and students.

Shielding the Vulnerable

The 2010 General Conference Session added significant new child-protection language to the Seventh-day Adventist Church Manual. These protections include requirements for volunteer screening, criminal background checks, a two-adults-with-children rule, and a six-month waiting period before adults are placed in positions of leadership involving children.29 In 2011, the North American Division adopted Working Policy FB 20, which requires all conferences and institutions to provide child-abuse prevention training and to conduct criminal background checks on all volunteers who work with children.30 Schools in other parts of the world should consider implementing similar policies.

In order to provide resources to comply with these new policy mandates, the North American Division launched the “Shield the Vulnerable” child-protection program in the summer of 2012. This program provides dynamic interactive resources that can help minimize the risk of child abuse at churches and schools. The program is available to all conferences in the North American Division and provides a criminal background screening service and online child-abuse prevention training for employees, volunteers, and students at a nominal cost.

To support the church’s child-protection initiative, Adventist Risk Management has developed a series of online resources via its Website, http://www.adventistrisk.org, which includes sample policy statements, video
clips, and PowerPoint presentations, which can be customized for use by the local church or school.

The Seven Campaign
Recognizing that child abuse is a global issue, Adventist Risk Management has developed the Seven Campaign, which aims to create a grassroots movement intended to mobilize our 17 million church members to become proactive in child-abuse prevention. Online resources at http://www.7campaign.org include a digital media kit with flyers, posters, talking points, video clips, and other items to help churches and schools develop their own child-abuse prevention program.

You Can Make a Difference
As an Adventist educator, you have the responsibility to create a Christ-centered school environment where students are loved and valued and can develop a sense of self-worth. Keeping students safe from the harm of child abuse is an important step toward that goal.

NOTES AND REFERENCES
29. Secretariat Office, Seventh-day Adventist Church Manual (Silver Spring, Md.: General Conference of Seventh-day Adventists, 2010), pp. 84, 85.

Arthur F. Blinci is the Vice President and Chief Risk Management Officer for Adventist Risk Management, Inc., in Silver Spring, Maryland. He has worked with denominational organizations and traveled throughout the world for the past 35 years presenting numerous risk-management leadership seminars, child-abuse prevention training, and writing safety articles. Blinci earned an Executive Master of Business Administration degree at the Ken Blanchard School of Business—Grand Canyon University; his Bachelor of Science in Business Administration from Andrews University in Berrien Springs, Michigan; and the Associate Risk Management professional designation from the Insurance Institute of America. He resides in Colton, California.
WHAT CAN YOU DO TO STAND UP AGAINST CHILD ABUSE?

<table>
<thead>
<tr>
<th>SCHOOL BULLYING</th>
<th>CHILD LABOR</th>
<th>SEXUAL ABUSE</th>
<th>MEDIAN AGE OF CHILD</th>
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<tr>
<td>Between 20 and 65 percent of school-aged children reported having being bullied verbally or physically</td>
<td>MILLIONS</td>
<td>Up to 36% of girls and 29% of boys have suffered child sex abuse</td>
<td>ONLY 9 YEARS</td>
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THESE STATISTICS ARE SOBERING.
They are also a call to action. We cannot ignore this issue or pretend it only happens to other people.

Because the ministry of risk management makes Adventist Risk Management especially sensitive to the issue of Child Abuse, we have launched The Seven Campaign to Stop Child Abuse. Why “Seven?” Seven simple points can help make a difference and spread the change that is so necessary.

MY PROMISE
1. Tell 7 people how to stop abuse now*
2. Speak out against abuse – don’t remain indifferent
3. Advocate for change with my voice and vote
4. Mobilize my peers
5. Lead by example
6. Express my support creatively through the Virtual Gallery
7. Share The 7 campaign with 7 people

*Information meant how to stop abuse. Suggestions on what you can do in your community, an online gallery where you can express your creativity on this subject, and more is available at www.thersevencampaign.com

The Seven Campaign is a call to action for all Seventh-day Adventists and supporters to stand up and put into practice these principles we hold true. Seventh-day Adventists have historically affirmed the dignity and worth of each human being and decry all forms of physical, sexual and emotional abuse. Let us stand together to protect children.

The goal of this campaign is to raise the level of awareness about child abuse and create a culture of vigilance and best practices. ARM hopes that the Seven Campaign will spur churches and schools into action; that they will implement a Child Protection Plan in their church or school. Raising the visibility of this issue will allow for more open discussion and increased awareness.
THE 7 CAMPAIGN IS A GLOBAL CAMPAIGN
TO RAISE AWARENESS AND ADVOCATE FOR AN END TO
THE ABUSE OF CHILDREN AROUND THE WORLD

WE NEED YOUR HELP

HOW CAN YOU HELP STOP CHILD ABUSE?
USE your voice your influence your life
TO END CHILD ABUSE

Make a PROMISE to BE the DIFFERENCE VISIT www.thesevencampaign.com
If you teach math, maybe you love mathematics for its own sake; I do. Or maybe you’ve been assigned to teach the subject. Have you felt that no matter how you teach math, it comes across as dry as the proverbial hills of Gilboa? Do you find yourself discouraged at hammering basic skills into reluctant minds class period after class period? Been there! Done that!

The good news is that we math teachers can enliven our classes by sprinkling our math instruction with bits of humor or biography to break the monotony. A joke or pun here and there, no matter how big a groan it elicits, at least recaptures students’ attention. If we can throw in a (short) story now and then, our students will love us for it. After all, mathematics is much more connected with the humanities than with the sciences, in spite of popular opinion to the contrary. If we can bring out the human side once in a while, some of our students may warm ever so slightly to the subject!

Allow me the liberty of suggesting some ways of doing this.

In algebra, I often ask my students to factor something like $x^2 - 5x - 6$. Distressingly frequently, I get the answer $-1$, $6$. What they have done is create the virtual equation $x^2 - 5x - 6 = 0$ and, after factoring the expression, solved it for $x$. They have done more work than I requested. Furthermore, I have to deduct points for their failure to precisely follow instructions.

When my students solve a virtual equation instead of merely factoring it as requested, I remind them of Christ’s injuration: “If any man will sue thee at the law, and take away thy coat, let him have thy cloak also. And whosoever shall compel thee to go a mile, go with him twain.” Then I say, “In our Christian walk, we are to do more than required by the law. But in mathematics, you should never do more than you are asked to do!”

As teachers, we often hear statements like, “I don’t understand it!” or “It’s just not true.” I remember as a child trying to come to terms with my mother’s admonition that “two wrongs don’t make a right,” with the fact that a negative times a negative is a positive. This was an irreconcilable philosophical paradox to me.

A physicist working at the Los Alamos National Laboratories consulted the great 20th-century mathematician, John von Neumann, about a problem. Von Neumann stated the problem was a simple application of the mathematical method of characteristics. The physicist immediately confessed that he didn’t understand this method.

Von Neumann replied, “Young man, in mathematics you don’t understand things, you just get used to them.”

I confess that I have never reconciled philosophically that a negative times a negative is a positive. But I have long since gotten used to the concept.
Mathematicians

We can also find a few tales to tell about mathematicians. When I find such a story, I write it on a sticky note and place it in an appropriate place in my teacher’s edition. These stories stimulate students’ interest. When you run across a mathematician’s name in the textbook, it’s easy to go to the Internet and search for him or her. Beware—since the Internet is not refereed, there is always a danger that erroneous material can be and has been inserted. Here are some examples of how I use mathematician stories.

A Young Mathematical Genius—Carl Friedrich Gauss

When I talk about adding a series of numbers, such as \(1 + 2 + 3 + 4 + \ldots\), I tell my students about Carl Gauss who, after misbehaving in class, was told to add the numbers between 1 and 100.4 His teacher, hoping for some peace and quiet for a while, turned back to his work. However, almost immediately, young Gauss inquired, “Is it 5,050?” (This was the correct answer.) I then show his reasoning.

This same precocious Gauss at the tender age of 3 corrected his merchant father’s payroll!5 Including this anecdote not only allows me to introduce the techniques for calculating series, but also to show that a mathematician could have had behavior problems!

Religion and Mathematicians

Many mathematicians were deeply religious. This may come as a surprise to many students. Here are a few examples.

Blaise Pascal

When we teach the binomial theorem or combinatorics, we encounter the Pascal Triangle. (See Figure 1 on page 16.)

Each entry in the table is obtained by adding the two numbers immediately above and to the right and left of the position. As I teach about these connections, I often give my students a brief account of Pascal’s spiritual struggle with mathematics.

Blaise Pascal (1623-1662) grew up in a God-fearing home. Very early, he showed an interest in mathematics. His disapproving parents removed all math books from their home, which only whetted his appetite. By age 14, Pascal had invented a digital instrument to help his father calculate the taxes he had to collect. By the age of 16, he had presented his first paper on mathematics, including elements of projective geometry.6 However, he had an uneasy relationship with mathematics. His early training led him to believe that doing mathematics was wrong. And so, several times in his life, he confessed this interest in math as a sin and abandoned it—only to be attracted to it again.

Finally, during an electrical storm, the horses pulling Pascal’s carriage bolted and left the carriage hanging precariously over the edge of a bridge. Pascal was rescued, unharmed, which he
regarded as a direct communication from God. This time, he
gave up his dalliance with mathematics forever and started
writing religious works. His work *Pensées*, which describes his
thoughts about God and His dealings with humanity, has be-
come a classic that is well known throughout the world.7

**Augustin-Louis Cauchy**

Another religious mathematician was Augustin-Louis
Cauchy.

For more than 100 years after Newton invented calculus,
mathematicians tried unsuccessfully to provide a rigorous
foundation for it. When young Augustin-Louis Cauchy first
started teaching, he expressed an interest to apply the rigor of
Euclidean Geometry to the calculus. He wrote his own calculus
textbook, *Cours d'Analyse*, to do exactly that.8

We trace the branch of mathematics
called analysis to Cauchy’s
ground-breaking work. Mathematicians regard Cauchy with great es-
tem today, although this was not al-
ways so.

A devout Catholic, Cauchy used
this devotion as a standard by which
he criticized his colleagues regarding
their alleged laxness in religious mat-
ers. Other mathematicians hated
him for this, and some even called
him mad.9 It appears that this critical
religious trait kept Cauchy from get-
ing at least one prestigious appoint-
ment he had sought.10

I use this story when teaching cal-
culus or analysis to encourage stu-
dents to stand for the right even if it
is unpopular.

**Maria Gaetana Agnesi**

Maria Gaetana Agnesi is another mathematician whose story I introduce to my students. Every calculus student has en-
countered the bell-shaped curve called the Witch of Agnesi,
which looks something like Figure 2.

The infamous name for this curve comes from John Col-
son’s mistranslation of the Italian word *versiera*, a term used
for a rope on a sailing vessel. The name *witch* has stuck, despite
its inaccuracy.11

Maria Agnesi became very interested in mathematics as a
child. She read Newton eagerly and eventually published a two-
volume text *Analytical Institutions* (*Instituzioni Analitiche ad
uso Della Gioventu Italiana*) in which she presented and ex-
tended much of Leonhard Euler’s work. The witch is a curve
she discussed in an unpublished commentary on Guillaume
L’Hospital’s work.12

Far from being a witch, Agnesi was deeply religious. At the
age of 20, she desired to enter a convent. Her father forbad it.
Later, she was appointed to the chair of mathematics at the
University of Bologna. There is no record of her ever lecturing
there, however. This is undoubtedly due to the fact that women
did not lecture at universities in those days. An example of her
brilliance: As a child, Agnesi mastered at least seven languages.13

After publishing her mathematical works and after her father’s
death, she turned her intellect to studying theology. In her later
years, Agnesi devoted her attention to ministering to the poor,
homeless, and sick. She worked very closely with the Christian
ministry of the Blue Nuns. There are conflicting reports as to whether she
actually entered their order later in
life.

**Sir John Napier**

When teaching trigonometry, we
eventually get to the product-to-sum
identity:

\[
\cos x \cos y = \frac{1}{2} [\cos(x + y) + 
\cos(x - y)]
\]

This formula led John Napier to an
"Aha" experience. He asked himself,
"Why can’t I use this formula to create
a table of numbers that will allow me
to turn multiplication into addition?"

After over 20 years of hard work, he
created a table of logarithms.14 How-
ever, this phenomenal effort possibly
contributed to his losing his mind to-
ward the end of his life.

The table of logarithms he created allowed anyone to turn
multiplication into addition and division into subtraction.
When we think about addition versus multiplication of four-
digit numbers without a calculator, we suddenly realize what a
marvelous invention Napier had made, which enabled even the
sea captain unskilled in mathematics to calculate his position
in the vast ocean much more accurately. If the captain of the
*Mayflower* had had access to this table, he probably would have
landed the pilgrims in Virginia—their goal—instead of Mas-
sachusetts, and the history of America would have been very
different!

In fact, the use of logarithms enabled the British navy to
land their ships and army accurately anywhere on earth.

If your students enjoy trivia, you can share the fact that
Napier spelled his name in several different ways. However,
never once did he spell it “Napier” as we do today. In his formal writing, which was always in Latin, he always spelled it “Neperus.”

Sir John Napier was born in 1550 at the beginning of the Scottish reformation. After becoming an evangelistic Protestant, Napier wrote A Plaine Discovery of the Whole Revelation of Saint John, which he regarded as his greatest work. He thus can be placed as a direct intellectual ancestor of the Millerite movement and Adventism.

Sir Isaac Newton

When doing math, we often use the calculator’s square-root button, but have you ever wondered how the calculator finds the square root so quickly? The algorithm it uses comes from one invented by Sir Isaac Newton to solve difficult equations numerically.

Newton is, of course, better known for his discoveries relating to calculus and gravity. During the winter of 1665/1666, England was struck by a vicious strain of plague. The University of Cambridge closed, and Newton, soon to be one of its most famous students, went home to the family farm. The year 1666 has come to be known by the Latin term *annus mirabilis* or the “miracle year.” During that brief period, Newton developed calculus, the theory of gravitation, the laws of motion, and his theory of light.

We have a delightful story from Newton’s youth. After the death of his stepfather, who hated and banished him, Newton went home. His mother asked him to cut a hole in the barn door so the cat could get in and out. After cutting the hole, Newton noticed that the cat had a bunch of kittens, so out of concern for them, he cut a smaller hole next to the larger one so they could get in and out, too!

An avid student of the prophecies of Daniel and Revelation, Newton wrote *The Prophecies of Daniel and the Apocalypse*, which was published posthumously. So he, too, is a direct ancestor of Adventism. LeRoy Edwin Froom quotes frequently from both Napier and Newton in his definitive four-volume work *The Prophetic Faith of Our Fathers*.

Augustus De Morgan

Augustus De Morgan was the first mathematician to give a rigorous formulation of the principle of mathematical induction. This is an extremely powerful tool that is often used in proving theorems in all branches of mathematics. His name is also attached to a set of laws in logic and set theory.

De Morgan seemed to appear out of obscurity. Although he was regarded as a brilliant student of mathematics at Cambridge University, in order to receive his M.A. there, he had to sign a document that he believed certain theological doctrines. He did not accept the wording of one or more of these
doctrines, so he refused to sign. For this reason, he never received his M.A. He published several mathematical books but was barred from the upper levels of mathematical research by his decision. He stands as an example of someone who lived by his conscience.

He stated in his will: “I commend my future with hope and confidence to Almighty God; to God the Father of our Lord Jesus Christ, whom I believe in my heart to be the Son of God but whom I have not confessed with my lips because in my time such confession has always been the way up in the world.”

Niels Henrik Abel

In algebra classes, students learn to solve polynomial equations. About 200 years after Christ, Diophantus developed the procedure for solving quadratic equations. He was the first mathematician to use variables to represent numbers. In the middle 1500s, Giralomo Cardan (or Cardano) published the methods for solving cubic equations in his book *Ars Magna*. Shortly thereafter, mathematicians discovered how to solve quadratic (fourth-degree) equations. This started a grand search for the general solution of a fifth-degree equation.

The young Norwegian mathematician Niels Abel concluded this search once and for all by proving that it is impossible to solve fifth or higher degree equations by the method of radicals. Abel and another young mathematician, Evariste Galois, attacked the problem independently in the novel fashion of showing that it couldn’t be done! The set of abstract algebraic commutative groups are called Abelian groups in his honor. He was so brilliant that he was able to graduate from the University of Oslo after attending for only one year. Abel lived honorably though in great poverty, dying of tuberculosis when he was only 26.

Interestingly, the Abel family had a unique heraldic crest. It consists of a snake in the Tree of Knowledge of Good and Evil. Adam reaches out with his right hand to receive a fruit from Eve’s left hand. Meanwhile, Eve is plucking another fruit with her right hand. Niels Abel used this crest, explaining that his surname came from the second son of the Edenic pair.

These anecdotes show that using stories and biographical sketches can lend interest to your instruction and help motivate students. Recently, at a funeral, I met a former student who reminisced about how much she enjoyed the calculus class she took from me “because of all the stories you told.”

I encourage you to start creating a list of anecdotes about people who have influenced whatever subjects you teach. A website that will help you get started is http://en.wikipedia.org/wiki/List_of_Christian_thinkers_in_science.

As you add to your list of anecdotes, please think of compiling them for an article in *The Journal of Adventist Education*. I’m sure others will enjoy reading about them, too.

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NOTES AND REFERENCES

1. Matthew 5:40, 41, KJV.
3. See http://en.wikipedia.org/wiki/Wikipedia, where concerns are expressed about the “quality of writing and the amount of vandalism” that can be found there.
10. Ibid., p. 290.
Train a child in the way he should go, and when he is old he will not turn from it."¹

This text can be applied both to a lifelong relationship with God and to other worthwhile commitments. How do teachers and parents raise up children to enter into science, technology, engineering, and mathematics (STEM) careers? There is an unwritten rule that 100 percent of students need to be prepared to pursue collegiate-level mathematics. However, the National Science Foundation has estimated that only 18 percent of U.S. female college freshmen plan to enter STEM careers, and a 2009 poll indicated that as few as five percent of U.S. girls age 8 to 17 were interested in engineering.²

Further problems continue at the workplace level, where even though women are more likely than men to be hired, Cathy Trower of Harvard University found a higher turnover rate for women in STEM careers due to lower job satisfaction, as compared to their male counterparts.³ How can we guide more male and female students to study math and science, and to find happiness in these fields?

For Christians, career choice has both a temporal and eternal component. Parents and teachers guide young people to make informed choices based on their gifts, the world’s needs, and God’s calling. Hence, it is important to properly interpret this comment by Ellen G. White in 1905: “Many of the branches of study that consume the student’s time are not essential to usefulness or happiness. . . . If need be, a young woman can dispense with a knowledge of . . . algebra . . . but it is indispensable that she learn to make good bread, to fashion neatly-fitting garments, and to perform efficiently the many duties that pertain to homemaking.”⁴ In today’s world, studying mathematics has more practical applications than a hundred years ago before the era of technology, when almost no women attended college. The statement’s principle is still relevant, however—women (and men) need practical training that will be useful in their
daily lives and careers.

So what type of education will provide a solid and happy future for students? This article reviews modern educational literature concerning student preference and ability in mathematics classes and offers suggestions for teaching based on this literature, recent student surveys, and the author’s 20 years of teaching experience.

**Student Interest**

Students’ needs, desires, ability, and access do not necessarily converge or even intersect. More frequently, interest supersedes ability, regardless of need or access. For instance, recent studies agree with Heller and Ziegler who showed that females from around the world express less interest than males in physics and mathematics, and the magnitude of the difference increases at upper-grade levels. Currently, worldwide, more women than men enroll in almost every area of graduate education—with the exception of mathematics, engineering, computer sciences, and the physical sciences.

Gender differences are also evident in a number of countries other than the U.S. Consider the following male/female ratios in academic areas for 12th and 13th graders in Germany: two to one in math and chemistry, eight to one in physics, but one to three in biology. Around the world, significantly fewer women than men work in mathematical fields.

The male/female ratio for the SAT achievement test, mathematics portion (SAT-M), performing at the average level is 2:1. This ratio jumps to 4:1 for the top 15 percent and 132:1 for the highest 2 percent. Interest in STEM courses can be measured by student success on Advanced Placement (AP) courses. A recent post by The National Math and Science Initiative (NMSI) revealed that 2011 data indicate persistent achievement disparities in STEM courses at the K-12 level, as revealed in Advanced Placement scores.

**Are All Inequalities the Result of Discrimination?**

At the undergraduate level in the average U.S. university, more women than men receive passing grades. This does not result from discrimination against men, but simply reflects the fact that more females than males enroll in college. Likewise, more women than men receive Bachelor of Arts degrees (133 to 100), according to the National Science Foundation, simply because they outnumber male candidates. (The situation might indicate a biased climate if one were able to demonstrate favoritism toward female students or discrimination against male students.)

What about research showing that women make only 77 percent as much as men? This currently occurs because women, on average, work fewer hours in less highly remunerated occupations.

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Figure 1. Advanced Placement (AP) Average Scores by Gender, 2009

However, this is rapidly changing as the number of women increases in fields such as medicine, dentistry, and law, which were once dominated by men. In America, the number of enrolling and graduating women continues to surpass the number of men in many of the fields traditionally dominated by men.11

Now, what about the statistics previously mentioned concerning student interest levels? Boys score higher than girls on the SAT-M as well as the ACT.12 There are groups that allege that high-stakes standardized test questions are biased toward males. Conversely, there are groups that claim that because the male and female brains are formed differently in the womb, boys and girls develop different interests and hobbies. Should the SAT-M testing difference be treated as a simple inequality or an egregious form of discrimination? The way a teacher chooses to regard—and address—this dichotomy will shape the type of pedagogy he or she employs in the classroom and flavor his or her interaction with students.

Educators’ Views of Gender Testing Differences

Teachers who consider differential math-test scoring as simple disparity may point out that even though males score significantly higher on the SAT-M than females do, the SAT poorly predicts college readiness or college GPA. But the fact that women score lower on the SAT-M disadvantages them, since 92 percent of universities require SAT scores, and 75 percent of them use these scores as part of the admission process.13 Therefore, even if testing differences are considered to be the product of genetic differences, the situation quickly becomes unfair if test scores are used to disadvantage one group. The use of SAT-M scores effectively reduces a girl’s chance for a college education and scholarships.

The claim of unfairness is further bolstered when one considers that more selective schools require a higher score on the SAT-M than do open-admissions institutions. For example, Texas A&M requires a minimum of 600 on the SAT-M for acceptance. This means “one in every four college-bound males in Texas is eligible, [but] only one in every seven females qualify.”14 And the more selective the school, the larger the gender difference in mathematical performance.15 Mathematics thereby becomes the doorway, or the blockade, that leads into, or prevents, access to higher-paid, more-prestigious jobs. Each of the top 15 highest-paying careers requiring a college degree has a heavy dose of mathematics as a common denominator.16 With midcareer salaries between $90,000 and $100,000, the most profitable majors include aerospace, electrical, chemical, and industrial engineering, physics, statistics, biochemistry, and mathematics.17 Herein lies the root of the assertion that lack of access to college opportunities produces an unfair situation due to the SAT-score differences between boys and girls. Though there are surely biologically related gender differences, the continual low performance in mathematics among women is not one of them.18 Instead, candidate interest levels and opportunity play a foundational role.

Gender Differences in Math

Teachers who consider math score differences as simply indicating gender-based phenomena may instead point to the fact that the non-gendered human fetus forms differently during the first trimester when sex is determined, depending on whether it is flooded with estrogen or testosterone. For example, among other differences, the brain develops a higher neuron density if the fetus becomes female, and the eyeball acquires a higher propensity for tracking fast-moving objects in the male fetus. These natural variations, which develop before birth, result in differences in the senses (sight, smell, hearing) as well as in personality traits (risk taking, empathy, aggression), according to Evans and Chancellor.19

In general, females are more verbal than males. They tend to know more words at an earlier age, have more acute hearing, are better able to read faces and body language, have a higher perceptual speed, are more content to observe, and in stressful situations will ask for help sooner or put up with the situation longer. Boys, on the other hand, are better at spatial reasoning, have more acute vision, learn best using kinesthetic activities, have a greater need for activity, recall better with visual cues, and in stressful situations tend toward “fight-or-flight” responses.20 Input from the senses, as interpreted by the brain, can be claimed to create differences, which result in variations in test scores.

Further widening the achievement gap is the fact that boys tend to score higher on complex problem-solving due to the personality trait of risk-taking and their willingness to consider multiple problem-solving procedures, whereas girls are more timid about trying new approaches and adhere more closely to prescribed formulas or algorithms.21 Even though females receive higher grades in both high school and college, males outperform them on tests that cover material not specifically taught in class.22 On beginning-level problems, boys and girls score about the same, but boys score higher on complex and multi-step procedures.23 In other words: Girls are better at following the teacher’s instruction, while boys are better at independent thinking.

Teaching Implications

As a rule, the art of teaching presupposes that there are differences among
students. Certainly in God’s eyes everyone is equally valued, but in the classroom not everyone is intellectually equal, is equally educable, and, for a variety of reasons, not everyone is equally motivated. For educators, worrying about why differences exist between the genders is probably less important than focusing on a plan of action for the students seated in their classroom. At that moment, they need an education rather than an a priori explanation as to why they are the way they are. It is prudent not to pit nature against nurture but instead concentrate on research-based methodologies to enhance students’ collaborative and separate contributions. An appropriate plan of action should include four important areas: motivation, spatial-skills practice, representative teaching, and student involvement.

**Motivation**

First and foremost, the student has to want to learn. Or as Belcheir expressed it succinctly: “ultimately nothing an instructor can say or do will make a difference if the student is unmotivated to implement it.” This reflects the wisdom of the old cliché “you can lead a horse to water, but you can’t make him drink.” Of course, you can always salt the oats!

Motivation is the teacher’s primary objective in educating both boys and girls—and each student requires a different type and level of motivation. For example, in a comparison of girls and boys with similar grades, girls report feeling more hopelessness and shame and less enjoyment and pride in their accomplishments than boys do. Though recent research is finding declining differences between the genders in mathematical achievement, there does not seem to be a comparable decline in the affective differences. In other words, progress is being made in cognitive performance, but the gap remains large in terms of attitudes about mathematical learning. Any student who wants to succeed must first acquire the appropriate attitude toward learning.

Studies in the U.S. have shown that motivational differences start to take a toll as girls progress toward high school. There are almost no gender differences in mathematics performance at age 9. Small differences are seen by age 13, and larger differences become clear (in favor of males) around age 17. In the U.S., the most damaging period for a young girl occurs when she transitions to middle school. Girls “are characterized by a debilitating pattern of mathematics-related emotions, and of underlying competence beliefs and value beliefs which can be observed as early as at the age of eleven.”

The best approach for teaching girls is to focus on the influence of the environment on affective factors such as self-confidence, attitude, and perception. Teachers need to find ways to convince girls that they can learn mathematics. Providing encouragement is especially important since for girls, confidence is significantly related to their mathematics achievement because studies show that both confidence and achievement decline in the middle school years. Thus, it is critical that teachers help girls form correct (and positive) estimations of their capacity to learn mathematics.

Most students find regular continual feedback helpful for motivational purposes. Student success usually results from a combination of high-quality instruction and motivation. When pedagogy is solid, expectations are clear, and consequences consistent, students are more likely to achieve to the best of their ability.

**Representative Teaching**

While teachers should be aware of archetypal gender strengths and preferences, they should not embrace harmful stereotypes. Believing that every student can achieve and using a variety of teaching methods will help to ensure success for every student. Girls benefit from hearing more than seeing; therefore, descriptive explanations are helpful. Boys are attracted to moving objects, so teachers should move about during lectures. Girls will ask an overabundance of questions, so teachers need to use the Socratic method to provide answers. Boys tend not to ask during a struggle; thus it can be helpful to pair them up in
order to encourage communication. Girls will read faces, so a teacher must be wary of his or her body-language clues during the answering process. Although boys tend to look down or away when answering questions, this lack of eye contact is not an indication of ignorance or disrespect. Often, boys are simply processing information (and in some cultures it is considered impolite to make eye contact), so providing ample time to answer will help elicit thoughtful responses.33

Teaching to attitude is similar. The mindset of students can be gauged by their reaction to puzzles and brain-teasers. Do they persist or give up easily? Does failure induce in them a sense of challenge or helplessness? Some students believe that puzzles come easy to smart people, so if an answer is not clear to them, then they classify themselves as not being smart. Students with this perspective believe that intelligence is a static, inherited trait, and do not comprehend that it can be grown. To the contrary, intelligence and spatial skills are most certainly developed rather than innate. The learning environment created by the teacher can change the mindset—and the achievement—of his or her students.34

Spatial Skills
One of the more significant contrasts revealed by the majority of modern research on gender differences is large inequalities between boys and girls in the area of spatial skills. Of particular interest is the disparity in the ability to mentally rotate three-dimensional objects. The disparity is important because girls’ SAT-M performance is significantly predicted by this skill.35 Strategies that will have the greatest impact for girls combine both affective factors and spatial skill practice. For instance, between 1989 and 2009, boys won 17 of 20 National Geographic Bees, and even at the local level, girls tend to score significantly lower on social-studies tests where spatial and geographic abilities are required. Spatial skills, however, can be learned, and girls who participate in activities that improve their spatial awareness show improvement in related skills and achievement in STEM classes. Early-childhood education should include shape manipulation, visualization, measurement, and estimation, all of which develop skills that play a prominent role in higher-level mathematics. Throughout the grades, teachers should incorporate mental-rotation skills and other spatial-skill training into the curricula.36

Including puzzles and/or games that teach concepts such as proportionality and symmetry are beneficial, as well. Blocks and other building toys like LEGO sets and drawing will help students develop spatial skills. Be sure to consider the color and theme of the toys in order to appeal to both genders. Games such as chess can be introduced, especially to girls, since chess strengthens spatial skills as well as logic and trains participants to think ahead.37

Sports also improve spatial skills through movement—throwing and catching, aiming, and judging reaction time. All students, but girls in particular, will reap many benefits from greater participation in athletics and physical education.38 Dr. J. J. Edwards found that, besides spatial skill improvement, exercise provides the added benefit of motivation and camaraderie.39

Spatial skills can also be improved by the use of video games that involve fast-paced hand-eye coordination. Students who spend more structured time on the computer develop their capacity to solve technological problems and understanding of software.40 As students become aware of applications, software, and connectivity, this builds interest and appreciation for technological applications.

Student Involvement
Teachers must confront students’ stereotypes concerning mathematics and their ability to do math. All students should be urged to participate in mathematics-related activities, and girls especially should be encouraged to apply to math and science programs, regardless of the schools’ cut-off scores. Students can be admitted on worthiness based on alternate sources of ability/effect.41 Other suggestions: Provide hands-on opportunities, encourage all students to be creative, and teach students to take calculated risks and make educated guesses. Unless there are scoring penalties, students should attempt to answer all test questions. Collaboration time on mathematics is also important for confidence building.

Conclusion
Whether one considers mathematical score differences between boys and girls to be innate or the result of flawed policies and prejudicial treatment, the use of SAT-M scores for admission purposes runs the risk of creating barriers for girls attempting access to a college education and especially to STEM majors. Compounding this problem is the likelihood American higher education will continue to require high SAT and ACT scores for entrance. These types of tests present the appearance of supporting high standards, so dropping the requirement would give the impression of de-emphasizing academic rigor.42

To shape young people’s attitudes about STEM careers, teachers, administrators, and parents should emphasize that mathematics, formulas, and equations are the source of today’s technology—including cell phones, medical advances, new cosmetics, and advances in digital and sound technology, and
that mathematics and science are the common denominator in high-paying careers. Career counseling, peer representation in STEM classes, and role models are of paramount importance in encouraging both genders to succeed at math and science and to choose STEM-related careers. Teachers should teach to students' strengths while simultaneously avoiding harmful stereotypes. For further information, I highly recommend the report Why So Few? Women in Science, Technology, Engineering, and Mathematics by Hill, Corbett, and Rose listed in the references. And finally, teachers need to keep learning themselves and seek input from professionals to develop a pedagogical style that enhances every student’s academic achievement.

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NOTES AND REFERENCES
3. Ibid.
8. Ibid.
14. Ibid.
16. Ibid.
35. Casey, Nuttall, and Pezaris, “Mediators of Gender Differences in Mathematics College Entrance Scores,” op. cit.
37. Ibid.
38. Ibid.
40. Eliot, Pink Brain, Blue Brain, op. cit.
42. Nankervis, “Gender Inequities in University Admission,” op. cit.
43. Eliot, Pink Brain, Blue Brain, op. cit.
44. See Endnote 2.
A nationwide state-sponsored curriculum for mathematics and language arts will soon be implemented in public schools across the United States. This will also include a new system for assessing students’ mastery of the curriculum. The main focus of this article is what mathematics teachers will need to do to adapt their instruction because of the coming changes. Resources are available to help teachers to prepare for the implementation of the new standards known as the Common Core State Standards (CCSS).

An examination of the new Elementary Mathematics Standards in North American Division (NAD) Seventh-day Adventist Schools (2012) reveals that the mathematics curriculum for grades K-8 has been aligned with the Common Core State Standards of Mathematics (CCSS-M). On the first page of the recently released elementary standards, the mathematical practices that are part of the Common Core are listed. The NAD Mathematics Curriculum Guide (2003) for the secondary level, based on the 2000 National Council of Teachers of Mathematics (NCTM) content and process standards, is undergoing a similar update. Since the elementary NAD standards include all of the Common Core without neglecting the integration of faith, which is included in the essential questions and big ideas, the secondary NAD standards should also align well with the CCSS while maintaining a faith-based focus.

What makes the Common Core standards different from previous standards? Will it be worth the effort to incorporate them into the NAD standards? What benefits are anticipated in the area

BY MARIAN PRINCE
of student learning and comprehension?

But first, an overview of the history of the CCSS. With the decline in world ranking of U.S. students’ scores on the National Assessment of Education Progress (NAEP), the Trends in Mathematics and Science Study (TIMSS), and other international measures of student academic achievement,\(^1\) educators and politicians have been looking for a way to coordinate the standards across the states without implementing a national curriculum (which is frequently done in other countries), since curriculum is locally determined in the U.S. In 2010, the Council of Chief State and School Officials (CCSSO) and the National Governors Association (NGA) released the Common Core State Standards in Mathematics (CCSS-M) and English Language Arts, which most of the states are adopting.\(^2\) The standards (http://www.corestandards.org/) are designed to ensure that students will develop the skills necessary for college and careers in a global economy.\(^3\)

After the NCTM developed curriculum and evaluation standards in 1989 regarding the mathematical content that should be taught, it became evident that teachers needed guidance regarding how to train students to become mathematically proficient. So in 2000, the NCTM published Process Standards.\(^4\) A year later, the National Research Council recommended that teachers develop five strands of Mathematical Proficiency in their students.\(^5\)

For a number of years, these guidelines did not have a large-scale impact on the way mathematics was taught in U.S. classrooms.\(^6\) But, given the broad base of participation and collaboration by so many states, the authors of the CCSS-M resolved that with the full implementation of the Common Core State Standards, the way mathematics is taught and learned would change. When the CCSS were released in 2010, they contained eight Mathematical Practices that are to be applied at each grade level. These Mathematical Practices are designed to complement the content standards.

### The CCSS Mathematical Practices

Figure 1 shows the relationship among the practices. The CCSS Mathematical Practices can be briefly summarized as follows:

1. **Make sense of problems and persevere in solving them.** Students are able to state what the problem asks them to do. They can identify solution strategies that fit within the conditions and limiting factors of the problem. They continually check their solutions, adapt their approach if necessary, and are willing to think “outside the box.”

2. **Reason abstractly and quantitatively.** Students are able to understand what the numbers in the problem represent and how the numbers affect the problem. By transforming the problem into symbols and using the properties of mathematics, they are able to find the solution and to understand the meaning of their answer in practical terms.

3. **Construct viable arguments and critique the reasoning of others.** Students are able to craft logical arguments, and after listening to the arguments of others, are able to respectfully ask questions to identify strengths and weaknesses. They are capable of clearly communicating their own arguments and suggesting improvement(s) in the logic of other students’ arguments.

4. **Model with mathematics.** Students are able to apply mathematics to everyday life and the workplace. Using a variety of tools (diagrams, tables, graphs, flowcharts, and formulas), students are able to decide if their model makes sense and to interpret the results of their solutions based on the situation.

5. **Use appropriate tools strategically.** When solving a mathematical problem, students are able to make sound decisions about when to use technology, manipulatives, and other handheld tools and be able to identify resources that inform the problem-solving process. They are also capable of recognizing the limitations of certain tools.

6. **Calculate and communicate precisely.** In their attempt to communicate precisely to others, students will use clear defi-
nitions and appropriate units, explain any symbols used, and label graphs and diagrams with the necessary information. They will calculate accurately and express numerical answers with an appropriate degree of precision.

7. Look for and make use of structure. Students are able to identify, generalize, and extend patterns appropriately. Examples: Recognizing that 3 plus 7 is the same as 7 plus 3; and that the new fact, 7 x 8, is the same as 7 x 5 + 7 x 3 that are more familiar; students will be able to conclude that 9 = 2 + 7 and 14 = 2 x 7 in the expression $x^2 + 9x + 14$.

8. Look for and express regularity in repeated reasoning. Students are able to notice repeated calculations or patterns and to find shortcuts to use while solving problems. They are able to judge whether the results are reasonable and accurate.

Why Are These Mathematical Practices Important?

Adventist teachers in the North American Division need to be knowledgeable about the Common Core Standards, since the church’s mathematics curriculum corresponds to these standards. Students transferring from your school will need to be prepared for the CCSS mathematical practices in their new classrooms. Likewise, you will need to know how the CCSS-M have shaped the mathematical background of students entering your classroom.

Moreover, learning more about the CCSS mathematical practices and incorporating them into your instructional repertoire should improve your teaching of mathematics—whether or not you have a strong math background or a degree in mathematics or mathematics education.

As of the writing of this article, for the 2014-2015 school year, all but five of the 50 states have agreed to implement a new system of assessment. This is an unprecedented state-level cooperation on academic standards. Two consortia, Smarter Balanced Assessment Consortium (SBAC) and the Partnership for Assessment of Readiness for College and Careers (PARCC), have received Race to the Top funds to develop an assessment system to measure the full breadth of the CCSS in order to provide instructionally relevant information, fair accountability measures, and valid data to inform policy decisions. Through the use of Computer Adaptive Testing, electronic grading of constructed response items, and performance tasks (some through computer simulation), the assessment system incorporates three main approaches: summative, interim/benchmark, and formative in a secure online-testing environment. This testing system, if implemented as designed, should serve as a driving force to bring about change in the way mathematics is taught in the American classrooms.

Review the sample 6th-grade task in Figure 2 that the SBAC released for feedback and review. This item is designed to measure whether students can apply mathematics to make a decision based on understanding of proportional reasoning, including application of unit rates.

The students are to calculate how much each student will pay for each trip and write a letter to the teacher recommending which field trip to take based on students’ first- and second-place votes, costs, and distance. Because this problem requires using Mathematical Practices 1, 2, 3, 4, 6, 7, and 8, students who have not developed the habit of applying the Mathematical Practices will likely not do well on this item.

When this assessment system is put into place, teachers at all grade levels will have to use instructional practices that encourage their students to internalize and use these mathematical practices as they learn the required content.

What Do the Mathematical Practices Look Like in a Classroom?

There are videos of exemplary lessons that show students participating in lessons that require them to use one or more Mathematical Practices. One of the Public Video Lessons featured on http://www.insidemathematics.org is a 5th- and 6th-grade lesson on multiple representations of numeric patterning called “The Button Task.” (See Figure 3.) After predicting the number of but-
tons needed to make Pattern 11, students discuss in their groups the responses given by Learners A and B. To aid students in discussing the principles involved, the teacher asks them to share questions they would like to ask Learners A and B.

When the teacher asked each group to share what they had heard at their table, some students asked, “Where did Learner B get the 4?” while other students said that they understood Learner A’s strategy better than Learner B’s. The teacher then distributed manipulatives for students to use in investigating each solution.

The students critiqued the two solutions to the mathematical problem while looking for patterns, persevering in solving a problem, reasoning abstractly and quantitatively, using structure, and modeling with mathematics. Thus, “The Button Task” helped facilitate the development of mathematical practices in these students. Many other activities are given to demonstrate the other CCSS mathematical practices, but because of space limitations, only one example can be included here.

The SBAC has made a goal to provide an online repository by the 2014–2015 school year where teachers can find resources for implementing formative assessment as well as model CCSS lesson plans and student work. This goal is well on the way to being realized. The Inside Mathematics Website (http://www.insidemathematics.org/) has a number of videos showing teachers planning, teaching, and reflecting on model lessons that involve students in using the various mathematical practices.

Also included are samples of student work that demonstrate the effectiveness of the lessons, as well as model CCSS lessons (http://www.gomaisa.org/) that are in the process of being created by the Michigan Association of Intermediate School Administrators (MAISA). A Website where teachers can find all these Common Core resources was created by Danielle Seabold, mathematics consultant at the Kalamazoo Regional Educational Service Agency (KRESA), which you can explore here: http://bit.ly/MI-CCSS-M. Kent County (Michigan) Intermediate School District has built an online system (Curriculum Crafter) that allows paid subscribers to keep up with the changes in the standards. There is limited free access to this tool at http://www.curriculum crafter.org/. Each Regional Educational Service Agency in Michigan and the other participating states have accumulated resources to help schools align their instruction with the Common Core Content and Practice Standards.

**What Do Mathematics Teachers Need to Be Doing?**

Math teachers need to be including the instructional practices that research has shown to enhance student achievement. Marzano and his colleagues supplied the research results for Identifying Similarities and Differences; Summarizing and Note Taking; Reinforcing Effort and Providing Recognition; Homework and Practice; Nonlinguistic Representations; Cooperative Learning; Setting Objectives and Providing Feedback; Generating and Testing Hypotheses; and Cues, Questions, and Advance Organizers. Teachers will find it helpful to review these practices periodically, since they are foundational classroom strategies in developing the CCSS Mathematical Practices.

During each marking period, teachers can also incorporate several activities into their lessons that engage students in mathematical discussions, during which they use some or all of the CCSS Mathematical Practices and thereby deepen the students’ understanding of the mathematics embedded in various tasks. A number of resources online provide assistance. On the Inside Mathematics site, the Problem of the Month link is particularly valuable. Grouped by mathematical strand, each task is presented at multiple levels so that teachers of any grade can select the age- and skill-appropriate version of the task for their students. Look at the Math Tasks tab in the KRESA Live Binder (http://bit.ly/MI-CCSS-M), and follow the links to lessons from the National Council of Supervisors of Mathematics (NCSM) under the tab, Great Tasks: NCSM, and the Mathematics Assessment Project (MAP) under the tab, Grades 6-12 MAP tasks. Check out each tab, where you will find an assortment of resources to help you start implementing the Common Core. The common-core standards are found in the tab, CCSS-M Info and CCSS-M Unpacked. You can download the CCSS to your iPhone with a link in the tab, eResources (Click on CCSS Apps).

If it seems as if the CCSS-M is just piling on more work to the many things that you already have to teach your students, consider flipping your classroom. The math department at Byron High School in Minnesota decided to prepare its students for the problem-solving skills required on the SBAC assessment by using
this approach for homework. Instead of lecturing about a topic and then assigning practice problems to be done outside of class, the instructors videotaped their lectures so that students could spend time during class working on their homework problems with the benefit of their teachers’ help.

Teachers can adapt and transform textbook problems that do not require much mathematical thinking into a task that engages students in the Mathematical Practices. Dan Meyer explains how he changes dull mathematics lessons into engaging problems that start students reasoning about mathematics. His presentation can be seen at http://www.youtube.com/watch?v= NWUFj8w9Ps. Caulfield, Harkness, and Riley reported how they transformed a traditional textbook question on finding the probabilities of a spinner from a one-right-answer problem to a two-day investigation, during which students used proportional reasoning, geometric properties, and probabilistic thinking while using what today are called the CCSS Mathematical Practices. (This article was written before the Common Core State Standards Initiative came into existence.)

Teachers need to practice the art of questioning every day because this technique is vital for developing students’ understanding of the Mathematical Practices and their use of mathematical reasoning. To build up a storehouse of effective questions, insert key questions into your daily lesson plans until they automatically come to mind during instruction. NCTM has a useful two-part resource on “Asking Good Questions and Promoting Discourse” at http://www.nctm.org/resources/content.aspx?id=25149.

Conclusion
The Common Core Standards, implemented with the Mathematical Practices, are designed to improve the way instruction is managed for all students. The NAD math standards align well with the CCSS-M, and, as the elementary 2012 NAD mathematical standards have demonstrated, make it possible to integrate faith with learning mathematics a central goal of the NAD curriculum guide. The new curriculum will equip Adventist teachers to better prepare all their students for their future as members of society and of the heavenly kingdom.

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2. With the combined efforts of the CCSSO and the NGA, the CCSS has become a state-based initiative that has the participation of 45 states, as of the writing of this article (http://www.corestandards.org/in-the-states). This wide acceptance of the Common Core across the U.S. lends credence to the claims made by the CCSS authors that instruction in the classroom must change.
3. CCSS’s “Career and College Readiness” includes accommodations made for students who go directly from secondary school into the workplace. Whether or not students attend college, the Common Core Standards are designed to equip them with the skills they need after high school.
4. The five Process Standards that appeared throughout NCTM’s Principles and Standards for School Mathematics (Reston, Va.: NCTM, 2000) were Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.
11. In Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement (2001), Robert Marzano, Debra Pickering, and Jane Pollock did a meta-analysis of educational research in instructional practices that do increase student achievement.
14. Philip Daro, Senior Fellow of Mathematics, Pearson Foundation/Gates Foundation Partnership, Senior Fellow-Director, Strategic Education Research Partnership. He was also a CCSS-M author.
The Principles and Standards for School Mathematics of the National Council of Teachers of Mathematics, the Common Core State Standards for Mathematical Practice, and the NAD Curriculum Guides for mathematics are consistent in calling for instructional activities that allow students to experience mathematical processes, like exploring, solving, justifying, measuring, estimating, representing, applying, and explaining. An important objective of engaging students in these processes is the development of conceptual understanding along with factual knowledge and procedural skills.

This article is designed to promote teaching methods that engage students in active learning and result in deep conceptual understanding along with factual knowledge and procedural skills.

This article is designed to promote teaching methods that engage students in active learning and result in deep conceptual understanding along with factual knowledge and procedural skills.

Lawrence and Hennessey’s book, Sizing Up Measurement: Activities for Grades 6–8 Classrooms, has activities similar to those in this article, as well as a rich collection of other measuring activities. For measurement activities for grades K-5, see Bachman and Confer. For other activities, lesson plans, and resources, I recommend the Illuminations Website of the National Council of Teachers of Mathematics.

Getting Started
Give your students a circle cut from heavy paper. Ask:
“Do you know how to measure its circumference and area? Do you know more than one way to do it?”
“Do you know the difference between the two formulas, \(2\pi r\) and \(\pi r^2\)?”
“Do you know the correct units to use for circumference and area?”
“What is the definition of \(\pi\), and why is this symbol used in the formulas for both circumference and area?”

Circumference
According to Dunham, the ancient Greeks knew that the ratio of the circumference \(C\) of a circle to its diameter \(d\) is constant. Today we call this ratio \(\pi\), that is, \(\pi = C/d\), or \(C = \pi d\). Because the diameter is twice the radius \(r\), we also have the formula \(C = 2\pi r\).

Activity 1: Circumference (Estimated time: 10 minutes)
This activity will help students understand the important concept of circumference and its relationship with \(\pi\).
Give the students several circles of different diameters; for example, plastic lids from storage containers. With each circle, ask them to follow these steps:
1. Wrap a narrow strip of paper tape around the circle and tear it off to show the circumference, \(C\).
2. Lay another strip of tape along a diameter and tear it off to show the length of the diameter, \(d\).
3. Use the shorter piece of tape, \(d\), to measure the longer one, \(C\).
4. Measure the circumference and diameter with a ruler, then use a calculator to divide the circumference by the diameter. [Make sure students use the correct units.]

5. Write a conclusion about the investigation.

The students should find, of course, that the circumference is always a bit longer than three diameters, that is, $\pi$ is slightly larger than 3. Students should also realize that circumference is the same as length and thus should be measured using units like centimeters or inches.

Another way for students to arrive at an estimate of $\pi$ is to use a flexible measuring tape to measure the circumference and diameter and then divide the two quantities.

Area

Dunham describes how the ancient Greek mathematician Archimedes proved the number $\pi$ that is used in the formula for circumference is also used in the formula for area. In his proof, he considered a right triangle whose base was the circumference $C$ of the circle and whose height was the radius $r$ of the circle, as shown in the figure above. Then, using an ingenious argument that is too complex to repeat here, he showed that the area of the circle can be neither greater than nor less than the area of the triangle. He concluded, therefore, that the area of the circle was identical to the area of the triangle.

Using the familiar formula for the area of a triangle, $A = \frac{1}{2} \times \text{base} \times \text{height}$, and the formula for the circumference of a circle, $C = 2\pi r$, we find the formula for the area of a circle as follows: Area of circle = Area of right triangle

$$A = \frac{1}{2} \times C \times r = \frac{1}{2} \times 2\pi r \times r = \pi r^2.$$

When we simply give students this area formula, $A = \pi r^2$, and ask them to calculate the areas of circles drawn in textbooks, how much do they really understand about circular area? I have found that they understand very little, which is why I developed the following laboratory activity.

Activity 2: Circular Area (Estimated time: 60 minutes or longer)

The activity sheet on page 33 shows a sequence of five methods for measuring the area of a circle. The goal of the activity is for the students to develop a deeper understanding of the concept of circular area, as well as area in general.

Each group will need a circle cut from heavy paper, like card stock or a manila file folder. To draw the circles, use an embroidery hoop about 15 cm in diameter. (The hoop will also be used for Method 5.) Lay the hoop on the heavy paper and draw around the inside of the hoop. Then cut out the circles. (If you choose to omit Method 5, you do not need an embroidery hoop. Instead, you can use a compass to draw the circles.)

It will be helpful for you to mark the center of each circle in advance. One way to locate the center is to use the embroidery hoop to draw a circle on plain paper or tracing paper. Carefully fold the circle in half so that it coincides with itself, and then fold again in a perpendicular direction. The two fold lines should meet at the center. Use a ruler to check. Now, with a sharp pencil or other object, punch a small hole through the center of the circle.
Activity 2: Basic Materials Needed for Each Group

Paper circle
Metric ruler (centimeters)

Additional materials will be needed for each method, as listed below along with comments.

Method 1: Covering with squares
Square centimeter grid paper (the grid should be larger than the circle)

Comments: This first method highlights the concept of area—covering with squares. Emphasize the area unit, square centimeters (cm²).

Method 2: Using a formula
Calculator

Method 3: Weighing
Balance or scale for weighing a paper circle
Rectangular pieces of paper cut from the same paper that was used to make the circles

Comments: Do this method with the whole class. Otherwise, you will need several scales or will have to develop a plan whereby the students circulate from station to station. For best results, cut the strips as accurately as possible, and use a sensitive balance.

Method 4: Cutting and rearranging the pieces
Scissors, preferably one pair for each student
Protractor or other means of dividing the circle into 12 sectors of the same size

Comments: (a) You can save much lab time by marking six diameters on the circles in advance, or by providing each group with a copy of the diagram below. They can lay it over the center of their circle, mark 12 (or just six) points on their circle, and then draw the diameters.

(b) Notice that after the circle is cut, the pieces can be rearranged to form something that looks like a parallelogram. The area of a parallelogram is given by $A = \text{base} \times \text{height}$, and the base of the “parallelogram” above is...
half the circumference, \( \frac{1}{2} \times 2\pi r \), while
the height of the “parallelogram” is \( r \). Thus the area is approximately \( \frac{1}{2} \times 2\pi r \times r = \pi r^2 \). Nice!

Method 5: Transforming circular area
to rectangular area

Enough marbles about 1.5 cm in di-
diameter to fill the embroidery hoop one
layer deep

Small cardboard box (i.e., shoe box)

Stiff piece of cardboard or book to
push the marbles into one end of the
box

Comment: As with Method 3, I sug-
gest you do this activity with the whole
class. Otherwise, it requires many mar-
bles, boxes, and embroidery hoops or a
circulation plan.

The author would like to thank
Richard Wright for helpful com-
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these activities with her students.

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tivities for Grades 3-5 Classrooms (Sausalito,

7. Illuminations: Resources for Teaching Math:
8, 2013. This Website has many activities, lessons,
links to online resources, and mathematics stan-
dards espoused by the National Council of
Teachers of Mathematics. The materials are cate-
gorized by grade level and mathematics content.
Many of the lessons have built-in applets that
students can use for exploration and discovery.

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Anna (not her real name) had struggled with mathematical concepts since kindergarten. Each year, number sense, adding, subtracting, and multiplication seemed farther from Anna's grasp. But in the 5th grade, Anna's math teacher introduced her to the use of manipulatives. By using this new approach, Anna was able to solve equations such as $3x + 4 = 2x + 10$ with ease by the end of the third quarter.

The purpose of a classroom is to engage all learners in the process of education—even those who seem slow to comprehend certain concepts. Learning is a progressive experience, so teachers must firmly establish its foundation and provide opportunities for investigation, active involvement, and contemplation by each learner, which will facilitate the learning process. Educators Harry and Rosemary Wong estimate that in the typical classroom, students spend only 35 percent of the school day actively engaged in learning.¹ Teachers must move students beyond listening to a lecture, answering textbook questions, and completing worksheets to become more actively engaged in the classroom.

The abstract nature of mathematics necessitates active participation by students in order to achieve mastery, especially when studying algebraic concepts. Since it is widely accepted that algebra is the gatekeeper course for college preparatory courses,² each math teacher needs to identify methods that will actively engage his or her students and create an environment where students can achieve success.

This article will provide a brief history of math manipulatives, review some of the research supporting their use in mathematics classrooms, and describe a few specific manipulatives that are available on the market today.

Burns and Humphreys suggest that gaps in the teaching of algebra prevent students from understanding why the subject is necessary for life. The typical algebra classroom focuses on “procedures, problems, rules, and rituals of algebra while failing to reveal its place in the world of mathematics.”³

**Definition and History of Manipulatives**

Manipulatives can be any hands-on activities, interactive objects, or technology that teachers use to help students understand the objectives of the curriculum.⁴ In a historical study of manipulatives, Picciotto found that Zoltan Dienes was the first teacher on record to use manipulatives to help students comprehend algebraic concepts.⁵ In 1963, Dienes assembled a team of educators, psychologists, and students at Harvard University in what became known as the Mathematics Learning Project. (At this point, the word *manipulatives* had not come into common usage.) Dienes studied the use of balance beams, blocks, hooks, and other items to represent math concepts. He found that these materials were bene-
ficial in demonstrating a variety of mathematics principles such as the Distributive Law and Commutative Property but did not produce consistent results when used in factoring trinomials. Mary Laycock enhanced the blocks used by Dienes by including multi-base blocks. Modifying both ideas, Peter Rasmussen worked with base ten tiles, which became the precursor to the popular Algebra Tiles and many other manipulatives.

The International Center for Leadership in Education sponsors an annual Model Schools Conference to help schools develop academic programs that foster rigor and ensure relevance for all students. At these conferences, teachers and administrators from around North America present activities and programs that have helped their students experience success in every academic area. In the Keynote Address at the 2006 Model Schools Conference, Bill Daggett implored educators to present relevant assignments that move beyond the mere recall of facts to solving multifaceted real-world problems. Many times, students who seem to know the facts of math fail to perform as expected on standardized tests. These students know that $12 \times 12 = 144$, but they don’t know why. Until they do, math will simply be numbers in the textbook that have no connection to their lives.

Although some real-world problems may be included in the typical math textbook, Marzano and Pickering observed that application of new material outside of the textbook reinforces the significance of the information. Moreover, math students in all grades need bridges to link the textbook with the real world. Research suggests that manipulatives can provide that link, helping students recognize math applications outside of the textbook and classroom. (See Robert Moore’s article on the use of manipulatives to measure a circle on page 30 of this issue.)

Using Manipulatives Successfully

As stated above, manipulatives have been used to illustrate algebraic concepts for more than 40 years. Despite this fact, most pre-algebra and algebra students do not have regular contact with them. Perhaps their teachers don’t see the need, haven’t had training in how to align manipulatives with the textbook units, or aren’t convinced that manipulatives can make a difference. In a two-year study of manipulatives in the algebra classroom, Leinenbach and Raymond compared the term averages before and after using manipulatives for one teacher’s five algebra classes. When the students learned algebra only from a textbook, without using manipulatives, the average grade in the five classes was 78.89 percent. During and just after manipulatives use in teaching algebraic concepts, the mean score increased to 83.77 percent.

In 1994, Ernest evaluated the training of 40 middle school and high school teachers on the use of manipulatives for algebra and geometry. The Evaluation of Eisenhower Workshop qualitative checklist was used to measure the teachers’ success in using manipulatives after receiving training. She found that in most of the classrooms, 100 percent of the students participated when manipulatives were used, and the quality of instruction ranged from very good to excellent. Leinenbach and Raymond discovered that students’ performance on assessment instruments improved when they actively participated in the learning process. When evaluating the combined use of textbooks and manipulatives in mastering math objectives (contrasted with instruction using only the textbook), these researchers observed a 23 percent rise in students’ test and quiz scores. Table 1 displays the difference in class averages using the textbook alone, and in combination with manipulatives.
In a study of 7th and 8th graders in a multigrade mathematics classroom, the author of this article found that the students’ attitudes about mathematics improved when manipulatives were combined with textbook use. In this study, the Mathematics Attitudes Survey (MAS) was given before and after students used manipulatives. The MAS measured student attitudes toward math in several areas. The results of the MAS revealed that when students used manipulatives, their attitudes toward math success, confidence while doing math, and usefulness of math were more positive than when manipulatives were not used. The study also found that students completed math assignments faster and with more accuracy when using manipulatives.13

These positive outcomes of combining manipulatives use with textbook instruction offer a stark contrast to what happens to students who experience constant failure in the math classroom. When Ewing interviewed 43 students regarding their high school math experience, he discovered that many had given up on math and consequently on school because they had become convinced that success was unattainable. The research subjects told Ewing that their teachers expected them to learn math from the textbook and seldom engaged them in active learning strategies. As a consequence, many of these students became bored and frustrated with assignments, failed exams, and eventually quit school. None of the students interviewed was in a classroom that used creative learning strategies such as manipulatives.14

**Manipulatives Available**

Moyer noticed that students in classrooms using manipulatives without stated objectives had more fun, but not higher math grades.15 Unfortunately, teachers who are interested in using manipulatives that are aligned with objectives, textbooks, and state standards have few commercial alternatives. However, solutions can be found. Administrators should provide seminars and guidance in best practices for the use of manipulatives. After receiving training, math teachers can collaborate to harmonize manipulative use with curriculum objectives and experiment to determine which strategies work best with their students.

Many commercially produced manipulatives are suitable for pre-algebra, algebra, and of course, geometry classrooms. Because of space limitations, only a small sampling is listed in Table 2 on page 37. Some can be obtained as boxed sets; others must be ordered individually. Boxed sets tend to be more economical but may not contain sufficient pieces for an entire class. All of the manipulatives listed can be obtained from the Internet or ordered from mathematics catalogs.

**The Challenge**

In conclusion, research shows that when students use manipulatives,

- they are more successful in math classes;
- they participate more in the math classroom;
- their algebra quiz and test scores improve;
- their algebra term grades improve; and
- they have a more positive attitude toward mathematics.

Our students deserve every available opportunity to succeed. Math teachers need to create learning environments that will motivate students and prepare them for a global community. Manipulatives can help students to both *do* and *understand* mathematics.

**Betty F. Nugent, Ed.D.,** has taught mathematics at the elementary, middle school, and high school level for more than 20 years. She has completed extensive research on the use of manipulatives with middle school students, with special emphasis on multigrade schools. Currently, she is the Principal of Mt. Sinai Junior Academy in Orlando, Florida.

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**Table 2. Selected Manipulatives for Algebra**

<table>
<thead>
<tr>
<th>MANIPULATIVE</th>
<th>DESCRIPTION/SUGGESTED USE</th>
<th>MARKET DESIGN</th>
</tr>
</thead>
</table>
| ALGEBRA TILES       | These squares and rectangles help students visualize the distributive law and are benefit-
                      |     ial in factoring polynomials.                                                         |               |
| BASE TEN BLOCKS     | An assortment of rods, cubes, and blocks that can be used to develop number sense, solve
                      |     equations, and everything in between.                                                |               |
| GEOBOARDS           | These square, usually 11- by 11-inch boards hold elastic in place on pegs as students
                      |     experiment with perimeter, area, symmetry, fractions, angles, and various spatial
                      |     activities.                                                                           |               |
| COLOR TILES         | These versatile, 1-inch color tiles help develop basic arithmetic skills through the
                      |     modeling of different math concepts.                                                 |               |
| TANGRAMS            | This set of seven geometric shapes can be used to support standards related to spatial
                      |     sense by using the properties of parallelism, perpendicularity, and symmetry in
                      |     solving real-world problems.                                                         |               |
| HANDS-ON EQUATIONS  | The fun and easy way to learn algebra for students in grades 3-8. Makes 4 x + 3 =
                      |     3 x + 9 child’s play!                                                                 |               |
| PATTERN BLOCKS      | Multicolored, multi-shaped blocks that can be used to form amazing patterns.             |               |
| WRAP-UPS            | Teachers can help students gain a strong foundation for upper-level math using these
                      |     self-correcting Wrap-ups.                                                             |               |
Educators everywhere can recount times when their students have been exemplary teachers. In one of my multigrade classes, while grade 2 students were exploring how multiplication works, it was a keen kindergarten student who helped the 2nd graders see that we were adding same-sized numbers over and over. As the 2nd graders struggled to master the concept, Breann observed what they were doing and said, “All of the numbers are the same, Mrs. Duffy.”

Years later, in another grade 2 math class, Lizzi’s cheery request opened the lesson: “Mrs. Duffy, may I show you what I learned about addition?” Lizzi randomly chose numerals and wrote them in two 20-digit rows on the whiteboard. Then, adding from left to right, she exclaimed, “See! You can use regrouping in any size of addition question.” As a result of Lizzi’s explanation, even the struggling students readily mastered regrouping in both addition and subtraction. When students can make contributions of such caliber, a love of learning is kindled, which enables students to see past the classroom walls to embrace the challenge of preparing “for the joy of service in this world and for the higher joy of wider service in the world to come.”

Memories of the effectiveness of these student-led teaching experiences inspired me to investigate approaches that multigrade teachers can use to nurture their students’ ability to teach and encourage one another. Peer tutoring, where one student acts as tutor and the other as tutee, emerged as a most helpful and positive instructional tool.

However, it soon became obvious that training would be needed to ensure that peers became effective tutors, and that developing question-asking skills would be the key to implementing an effective peer-tutoring program.

This article will first review the research on peer tutoring and how it can benefit multigrade students. Then some designs will be suggested for questioning strategies that facilitate the use of mathematical language during both whole-class and peer-tutoring sessions. After exploring ways to incorporate peer-tutoring sessions into the multigrade class, some easy-to-apply guidelines for peer tutoring and mathematical questioning techniques will...
follow. The last three sections will include examples of how to apply this information in math lessons.

**Research Reveals the Benefits of Peer Tutoring**

Research suggests that peer tutoring is a reliable and effective way to vary class instruction. Researchers, reviewers and meta-analysts have shown that peers can be reliable teachers. Depending on its design, peer tutoring can produce both short-term and long-term gains for tutors and tutees in all subjects with the greatest gains seen in mathematics. In algebra classes, peer tutoring has been shown to be as effective as class instruction. Analysts therefore encourage the use of peer tutoring as a supplement for teaching, rather than solely to add variety to drill-and-practice activities. Peer tutoring fits well in the multigrade class because it saves time and makes it easier to organize and assign tutoring pairs, since varied combinations of students are available in a single classroom: Cross-age groups, with the best age range of two to four years, are available for work that reviews concepts already introduced to older students. Same-age groups are also available in the multigrade classroom. They can benefit from either reciprocal peer tutoring or general tutoring techniques, which enable peers of any ability to tutor.

**Relationship Building a Bonus**

Tutoring also has positive affective benefits. Multigrade teachers can readily organize same-gender pairs and student-chosen pairs, which increases students’ enjoyment of activities because it enhances their sense of belonging and rapport as they relate with friends and children similar to themselves.

Peers relate well to one another better because they have much in common. Thus, peer tutoring can positively affect class dynamics and encourage cooperation as students gain insights into the teaching process and engage in meaningful practice. This helps class members to become productive citizens who show concern for others and use their time more productively. They develop the interpersonal skills needed for Christian witness, and their caring actions reveal that God is growing them in His kingdom.

**Time Savings and Flexibility**

The activities used by peer tutors can also be incorporated into unstructured periods, which provides further time savings and flexibility for the multigrade teacher. The use of math games during tutoring sessions can make the subject more meaningful and interesting to students. As students learn these games and find the activities to be rewarding, they will enjoy playing them at a learning center, which further reinforces math concepts and facilitates mastery of other curriculum subjects.

Multigrade teachers need instructional techniques that result in optimal learning within limited time frames because their time is frequently interrupted or restructured. Peer tutoring helps teachers and students make better use of time and allows teachers the flexibility to effectively implement short on-task sessions (i.e., 15 three-minute sessions during a three-week unit). Amazingly, peer tutoring actually can make a multigrade class easier to manage. Class size is cut in half when the teacher uses a reciprocal peer-tutoring model in which partners take turns tutoring each other.

Children with special needs receive more individualized instruction and companionship because of increased...
“engaged academic time”24 and because the teacher is able to deal promptly with incorrect responses. Most important, as students’ needs are met, the peer-tutoring sessions allow the teacher more time to assess and gain new perspectives on students’ strengths and to analyze how best to meet their needs.25 Throughout the tutoring sessions, the teacher can encourage the use of mathematical terms, support tutors and tutees in their use of prompts, facilitate and reinforce positive interactions among peers, troubleshoot by finding skilled tutors and asking them to model a difficult step for their classmates, and make adjustments for the next tutoring sessions.

Getting Started
So how do you get started? First, be sure to plan the specific, mathematically related objective(s) that you want your students to meet as you develop a script, or series of prompts, for the peer tutors to use. Research has shown that structuring the tutoring sessions is beneficial26 because structure promotes high-level mathematical thinking.27 Scripts, also called prompts, scaffolds, or temporary supports, have been used effectively by tutors to teach their peers algebra, rounding numbers, and bar graphs.28 Further, scripts help tutors facilitate high-level thinking because this tool helps them model questions appropriately, independently scaffold, or adapt a task to meet a peer’s needs. Scripts provide tutors with vocabulary that enables them to encourage their peers to give more accurate mathematical explanations.29 Mathematics readily lends itself to structuring, as it has its own vocabulary. Structuring lessons also establish meaningful goals, giving students a sense of purpose while allowing the strengths of individual students to blossom.30

Zeroing in on the Language of Mathematics
Choose key questions to encourage the use of appropriate terminology and to provide scaffolds for tutors to use during their work with the tutee. If you
have students brainstorm relevant mathematical language in small groups, question starters from both mathematics and language arts will provide insights on how to craft lesson-specific scaffolds. First, consider the five basic teaching scenarios that encapsulate mathematics instruction: patterning, sorting, following examples, correcting non-examples, and modeling. Figure 1 provides examples of questions developed from these ideas for a lesson on addition with regrouping.

Curriculum guides and reference books provide additional sample questions and vocabulary. Collaborative group designs also offer guidelines for teachers as they design scaffolds for their student tutors. A collaborative group design can be especially valuable because the assigned roles allow students to work from perspectives that emphasize their individual learning strengths, and can make them better tutors as well. Any type of collaborative role can be emphasized during each tutoring session or questions can be taken from each modality. Teachers also have the option of assigning tutoring pairs to share their work collaboratively with other partners. Math-related descriptions for prompts used from the perspective of each role are shown in Figure 2.

## Deciding How and Where to Use Peer Tutors

After developing your prompts, the next step is to determine where and how you want to use your peer tutors. Warm-up lessons, quick reviews, drills, and even peer-tutoring training sessions are three- to five-minute activities that work well with peer tutors. Perhaps you wish to begin a new math unit with a cross-age project or a cooperative problem-solving activity—peer tutoring can help you assess your stu-
students’ current application of mathematical language.

Peer tutoring can also be incorporated into daily teaching as an instructional strategy for entire lessons as well as for practice, reinforcement, and re-teaching. For example, you could use a specific questioning series to teach one to three students (same ability or cross-age), and then have these students use the same questions to tutor partners, who then tutor others. When using peers as instructors in this way, teachers have the option to tier the level of practice exercises either within the same grade or across grades. See Figure 3. Or perhaps after a whole-class lesson with the teacher, grade 8 students could review multiplication of decimals and whole numbers. They could then teach grade 6 multiplication for only decimals, grade 6 tutors could teach 3-digit by 2-digit multiplication to grade 5 students, who in turn review the concept of repeated addition with students in grade 2.

Using Reciprocal Peer Tutoring

Another idea for practice time, reinforcement, and re-teaching: Include reciprocal peer tutoring, where the student has a prompt card that shows each step for a problem. As Figure 5 shows, the tutee makes an initial attempt at completing the question. If the response is correct, he or she tries another question or switches roles with the tutor. If the answer is incorrect, the tutor indicates the step from which the tutee should begin work and provides a second chance. If the answer is still incorrect, the tutor models the solution, after which the tutee repeats it and then goes onto another problem. The tutor and tutee may switch roles after a certain number of questions or after a stated time period.

<table>
<thead>
<tr>
<th>Figure 3. Example of Tiered Problems for a Fraction Unit*</th>
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<tbody>
<tr>
<td><strong>Level A:</strong> How can 2 people share 3 brownies? Strategy: Repeated halving</td>
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<tr>
<td>How can 2 people share 5 brownies?</td>
</tr>
<tr>
<td>How can 4 people share 3 brownies?</td>
</tr>
<tr>
<td>How can 3 people share 4 brownies?</td>
</tr>
<tr>
<td><strong>Level B:</strong> How can 4 people share 3 brownies? Strategy: Dividing into 1/4, 1/3, 1/6</td>
</tr>
<tr>
<td>How can 3 people share 4 brownies?</td>
</tr>
<tr>
<td>How can 3 people share 5 brownies?</td>
</tr>
<tr>
<td>How can 6 people share 4 brownies?</td>
</tr>
<tr>
<td><strong>Level C:</strong> How can 3 people share 5 brownies? Strategy: Halving plus Level B strategies</td>
</tr>
<tr>
<td>How can 3 people share 2 brownies?</td>
</tr>
<tr>
<td>How can 6 people share 4 brownies?</td>
</tr>
<tr>
<td>How can 5 people share 4 brownies?</td>
</tr>
</tbody>
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*Adapted from Lori Williams, Teaching Children Mathematics (February 2008), p. 326.

<table>
<thead>
<tr>
<th>Figure 4. Sample Reciprocal Peer Tutoring Worksheet</th>
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<tbody>
<tr>
<td><strong>First Try</strong></td>
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<tr>
<td><strong>Second Try</strong></td>
</tr>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td><strong>Third Try/Comments</strong></td>
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<tr>
<th>Figure 5. Sample Prompt Card—The Steps of an Effective Tutoring Session</th>
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<tbody>
<tr>
<td><strong>Greeting:</strong> “Hi. How are you doing?”</td>
</tr>
<tr>
<td><strong>Set the stage:</strong> “Let’s look at our assignment. What are we working on today?”</td>
</tr>
<tr>
<td><strong>Work Time:</strong> “Please, show me…”</td>
</tr>
<tr>
<td><strong>Closing:</strong> “Thank you for _______. Let’s record our work.”</td>
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<th>Figure 6. Ideas for Reinforcing a Classmate’s Correct Answer</th>
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<tr>
<td>“Good work. Thoughtful answer.”</td>
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<tr>
<td>“Good, you used regrouping [or any relevant mathematical term] in this question.”</td>
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<tr>
<td>“You put a lot of effort into ______.”</td>
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<table>
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<tr>
<th>Figure 7. Ideas for Helping a Classmate Correct an Answer</th>
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<tbody>
<tr>
<td>“That’s close. Try again from here.”</td>
</tr>
<tr>
<td>“What math fact do you need? How does that change things?”</td>
</tr>
<tr>
<td>“What math words would help us?”</td>
</tr>
<tr>
<td>“Your regrouping is clear to here.”</td>
</tr>
<tr>
<td>“How can you follow through from here?”</td>
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</table>
Motivation

Once you have chosen a tutoring model, consider the option of incorporating a rewards system. Rewards may be contingent upon individual academic performance or used to validate goals for cooperation within groups. For example, Hawkins et al. asked students to generate their own ideas for reinforcers. Then, during each session, the teacher randomly chose whether the rewards would be contingent upon appropriate tutoring behavior or perfect test scores.

Previous to setting peer tutors to work, the teacher must incorporate tutor training into everyday lessons. Coaching in both math language and interpersonal support strategies helps tutors work more comfortably and successfully. When students are prepared, this helps them to take ownership of their tutoring sessions.

Some simple, easily planned interpersonal support strategies can be implemented to make peer tutoring more meaningful and to encourage accountability and responsibility for all students. A tutoring session can be role-played. As Figure 6 shows, effective tutoring sessions should begin with a greeting, followed by a question to establish the purpose for the session. Next comes the body of the session, then its conclusion. Within the main section of the tutoring session, tutors need to know how to respond to both correct and incorrect answers. The teacher can make specific suggestions or take five minutes to brainstorm ideas with the class.

Teachers can scaffold communication development for tutors and tutees by making reference cards for “The Steps of an Effective Tutoring Session” (Figure 5), “Ideas for Helping a Classmate Correct an Answer” (Figure 7), “Today’s Math Words,” and “Ideas for
Reinforcing a Classmate’s Correct Answer” (Figure 6).

Record-Keeping Strategies

The teacher must plan a strategy for pairs to record their work independently. Charts that students can use to track their progress give sessions a sense of purpose, help students develop a feeling of accomplishment, and provide accountability.42 The teacher will also want to create a record chart on which to record his or her observations during tutoring sessions. On the students’ record charts, there should be a place for comments so that tutors can reflect on the lesson that they have tutored.

Debriefing can be helpful because students, like teachers, need feedback on their teaching skills,43 and students may make helpful suggestions for future lessons.44 Tutor involvement in assessment and planning has been evaluated positively in the literature.45

As you consider how tutors can help the teacher structure subsequent sessions, the positive impact of peer-tutoring techniques on the tutor needs to be re-emphasized. Peer tutoring has been used purposefully and effectively to both teach and tutor new math skills. Student tutors especially advance in proficiency as they teach mathematics to their peers.46

The reciprocal peer-tutoring system uses prompt cards that enable tutors to evaluate their peers even though the tutors are still learning or consolidating their own skills.47 Teachers may also compose mathematically focused scripts that enable tutors to practice higher-level mathematical thinking skills.48 Tutoring helps students develop both interpersonal and intra-personal skills, which enhance their experience in mathematics and make the subject a favorite class.49

Conclusion

To summarize, research and practice show that peer tutoring works. The multigrade classroom is an ideal fit for peer-tutoring programs. Structured lessons, flexibility of design, scaffolds such as prompt cards and role plays, students’ use of mathematical language, and the development of positive interdependence all make peer tutoring in mathematics class practical and user-friendly. As your tutoring program grows, you will find that lesson development becomes a partnership with students. Positive interdependence grows among students and between students and their teacher. This benefit far outweighs any extra initial preparation time.50

NOTES AND REFERENCES

1. Students’ names in this article have been changed to protect their privacy.


14. Cross-gender pairs sometimes show limited results, but appropriate peer training could readily overcome these challenges.


17. Ibid., p. 62.


19. Topping and Bamford, “The Paired Maths Handbook,” op. cit.; and Topping, et al., “Cross-Age Peer Tutoring in Mathematics With Seven- and 11-Year-Olds: Influence on Mathematical Vocabulary, Strategic Dialogue and Self-Concept,” have developed some math games. As I reviewed the games and the information accompanying them, I saw the need to be sure that activities match the curriculum and the abilities of students.


31. Juanita V. Copley, “Questions Specific to Patterns, Functions, and Algebra,” in Spotlight on Young Children and Math, Derry Korkek, ed. (National Association for the Education of Young Children, 2003), p. 24. This is algebra for Pre-K to grade 1, but the general questioning strategy applies to older children.


34. Meneses and Gresham, “Relative Efficacy of Reciprocal and Nonreciprocal Peer Tutoring for Students at Risk for Academic Failure,” op. cit.
35. Williams, “Tiering and Scaffolding,” op. cit.
36. The teacher will need to watch the progress of the lessons because by this point, grade 5 students may need a short group lesson to review and support their understanding of the idea of how repeated addition is related to multiplication.
46. Sparks, “Researchers Find That Students Learn by Tutoring Virtual Peers,” op. cit.

**Guest Editorial** Continued from page 3

Each of these articles in this issue is meant to inspire and encourage you as you lead students into the adventure of mathematics.

The Coordinator for the special section on mathematics in this issue, Wil Clarke, Ph.D., is Professor of Mathematics at La Sierra University in Riverside, California. The editorial staff of the *Journal* express heartfelt appreciation for his assistance throughout every phase of the production of the issue from the selection of topics and authors through the peer review process, revisions, and preparing the final manuscripts.

**NOTES AND REFERENCES**

1. 1 Peter 3:15 (NKJV), italics supplied. Texts credited to NKJV are from the *New King James Version*. Copyright © 1979, 1980, 1982, by Thomas Nelson, Inc. Used by permission. All rights reserved.
4. Ibid.
I’ve yet to meet a college student who is incapable of successfully completing college-level math. Yet, the complaint I hear whenever a student is frustrated with a grade or a concept is “I just can’t do math.” This can truly be a self-fulfilling prophecy, but when students push past their own self-imposed limitations on learning, they often find themselves quite successful.

. . . that they really do need math.

Very few of us, even math teachers, use the quadratic formula on a daily basis. But the “mental muscles” developed in math class are used throughout problem-solving situations, not just the ones with numbers.

. . . that memorization isn’t enough.

A student will be able to accomplish a lot more math in less time if he or she has memorized certain mathematical facts, such as the multiplication tables. But memorization should never replace a real understanding of the concept. What would a student do if he or she somehow forgot the product of three and eight? Would he reach for a calculator? Or would she remember that “three times eight” means having three eights, reducing the problem to simple addition?

. . . that basic math skills are vital to success in college math.

In a perfect world, students would wring every cent out of their high school education (and render my job obsolete) by taking every possible math course offered, entering college with Advanced Placement credit. Short of this ideal, every student should leave high school with, at minimum, a strong foundation of arithmetic, Algebra I, and basic geometry, including:

An excellent working knowledge of the basic mathematical operations, including exponents and roots, as they apply to rational and irrational numbers. Most of these can, and should, be done without the aid of calculators.

An understanding of how variables work, and the ability to solve linear, polynomial, and rational equations.

Familiarity with graphs of one and two variables.

Mastery of the basic vocabulary of geometry and related formulas for angles, length, perimeter, and area.

Some exposure to systems of equations and methods for solving them.

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I W I S H M Y  F R E S H M A N

S T U D E N T S

Knew . . .

that they really can learn math.

BY SHARILYN HORNER