The Effective Mathematics Classroom

What does the research say about teaching and learning mathematics?

- Structure teaching of mathematical concepts and skills around **problems to be solved** (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
- Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
- Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
- Students interaction in ways that both support and challenge one another's strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
- Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999).
- There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):
    1. Teaching for conceptual understanding
    2. Developing children's procedural literacy
    3. Promoting strategic competence through meaningful problem-solving investigations
- Students in the middle grades are experiencing important crossroads in their mathematical education. They are "forming conclusions about their mathematical abilities, interest, and motivation that will influence how they approach mathematics in later years" (Protheroe, 2007, p. 52).
- Instruction at the middle grades should build on students' emerging capabilities for increasingly abstract reasoning, including:
    - ✓ Thinking hypothetically
    - ✓ Comprehending cause and effect
    - ✓ Reasoning in both concrete and abstract terms (Protheroe, 2007)

Classroom Observations

Classroom observations are most effective when following a clinical supervision approach (Cogan, 1973; Holland, 1998). During a classroom observation cycle, the classroom observer and the teacher meet for a **preconference**, during which the terms of the classroom observation are established. A focusing question is selected, and the classroom observer negotiates entry into the teacher's classroom. Focusing questions provide a focus for classroom observation and data collection, and could emerge from "big idea" questions such as:

- What instructional strategy are you looking to expand?
- What are the expected outcomes of the classroom observation?

During the **observation**, data is collected by the classroom observer while the teacher teaches the lesson. The observer collects data regarding only the focusing question that was agreed upon during the preconference. The tool for data collection must match the purpose of the observation.

After the observation, the classroom observer and teacher meet for a **postconference**. During that time, the teacher looks at the data that is collected, and the observer asks the teacher what he/she notices from the data. Based on the teacher's responses, a conversation focusing on the questions addressed during the preconference. It is entirely possible (and, indeed, likely) that the focusing question is not answered, but the postconference conversation results in an additional list of questions that can guide continuing classroom observations and post-observation discussions.

Classroom observations: What should the teacher be doing?

In an effective mathematics classroom, an observer should find that the teacher is (Protheroe, 2007):

- **Demonstrating acceptance of students' divergent ideas.** The teacher challenges students to think deeply about the problems they are solving, reaching beyond the solutions and algorithms required to solve the problem. This ensures that students are explaining both how they found their solution and why they chose a particular method of solution.

- **Influencing learning by posing challenging and interesting questions.** The teacher poses questions that not only stimulate students' innate curiosity, but also encourages them to investigate further.

- **Projecting a positive attitude about mathematics and about students' ability to "do" mathematics.** The teacher constantly builds students' sense of efficacy and instills in her students a belief that not only is the goal of "doing mathematics" attainable, but also they are personally capable of reaching that goal. Mathematics is not presented as something magical or mysterious.

Classroom observations: What should the students be doing?

In an effective mathematics classroom, an observer should find that students are (Protheroe, 2007):

- **Actively engaged in doing mathematics.** Students should be metaphorically rolling up their sleeves and "doing mathematics" themselves, not watching others do the mathematics for them or in front of them.

- **Solving challenging problems.** Students should be investigating meaningful real-world problems whenever possible. Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating new knowledge and understanding about the real world every day.

- **Making interdisciplinary connections.** Mathematics is not a field that exists in isolation. Students learn best when they connect mathematics to other disciplines, including art, architecture, science, health, and literature. Using literature as a springboard for mathematical investigation is a useful tool that teachers can use to introduce problem solving situations that could have “messy” results. Such connections help students develop an understanding of the academic vocabulary required to “do mathematics” and connect the language of mathematical ideas with numerical representations.

- **Sharing mathematical ideas.** It is essential that students have the opportunity to discuss mathematics with one another, refining and critiquing each other’s ideas and understandings. Communication can occur through paired work, small group work, or class presentations.

- **Using multiple representations to communicate mathematical ideas.** Students should have multiple opportunities to use a variety of representations to communicate their mathematical ideas, including drawing a picture, writing in a journal, or engaging in meaningful whole-class discussions.

- **Using manipulatives and other tools.** Students, at the middle grades in particular, are just beginning to develop their sense of abstract reasoning. Concrete models, such as manipulatives, can provide students with a way to bridge from the concrete understandings of mathematics that they bring from elementary school to the abstract understandings that will be required of them as they study algebra in high school. Teachers teach their students how to use manipulatives, and support the use of manipulatives to solve meaningful problems that are aligned with the lesson’s objectives.

Classroom observations: What kinds of questions to ask?

Teachers should ask questions that promote higher-level thinking. That does not mean that a teacher should not be asking questions at the lower end of Bloom’s Taxonomy of cognitive rigor. In fact, it is important that a teacher begins a lesson with questions at the Recall and Understand levels of Bloom’s Taxonomy. However, in order to solve meaningful problems, students must be challenged with higher-level questions that follow the lower-level questions. Students will find difficulty applying their mathematical ideas or analyzing a mathematical situation if they are not asked higher-level questions in classroom activities and discussions.

What are some best practices for mathematics instruction?

In general, a *best practice* is a way of doing something that is shown to generate the desired results. In terms of mathematics instruction, we typically think of a *best practice* as a teaching strategy or lesson structure that promotes a deep student understanding of mathematics.

The Education Alliance (2006) looked at a variety of research studies, and identified a list of instructional strategies that could be considered to be best practices in mathematics education:

- Focus lessons on specific concept/skills that are standards-based
- Differentiate instruction through flexible grouping, individualizing lessons, compacting, using tiered assignments, and varying question levels
- Ensure that instructional activities are learner-centered and emphasize inquiry/problem-solving
- Use experience and prior knowledge as a basis for building new knowledge
- Use cooperative learning strategies and make real-life connections
- Use scaffolding to make connections to concepts, procedures, and understanding
- Ask probing questions which require students to justify their responses
- Emphasize the development of basic computational skills (p. 17)

The National Center for Educational Achievement (NCEA, 2009) examined higher performing schools in five states (California, Florida, Massachusetts, Michigan, and Texas) and determined that in terms of instructional strategies, higher performing middle and high schools use mathematical instructional strategies that include classroom activities which:

- Have a high level of student engagement
- Demand higher-order thinking
- Follow an inquiry-based model of instruction – including a combination of cooperative learning, direct instruction, labs or hands-on investigations, and manipulatives
- Connect to students' prior knowledge to make meaningful real-world applications
- Integrate literacy activities into the courses – including content-based reading strategies and academic vocabulary development

Additionally, NCEA researchers found that it was important for teachers to create classrooms that foster an environment where students "feel safe trying to answer questions, make presentations, and do experiments, even if they make a mistake" (p. 24).

Comparing Effective Mathematics Instruction with Less Effective Mathematics Instruction

In general, there are two prevalent approaches to mathematics instruction. In skills-based instruction, which is a more traditional approach to teaching mathematics, teachers focus exclusively on developing computational skills and quick recall of facts. In concepts-based instruction, teachers encourage students to solve a problem in a way that is meaningful to them and to explain *how* they solved the problem, resulting in an increased awareness that there is more than one way to solve most problems. Most researchers (e.g., Grouws, 2004) agree that both approaches are important – that teachers should strive for procedural fluency that is grounded in conceptual understanding. In fact, the notion of *numerical fluency*, or the ability to work flexibly with numbers and operations on those numbers (Texas Education Agency, 2006), lies at the heart of an effective algebra readiness program.

Teachers make an abundance of instructional decisions that can either discourage or promote an effective learning environment for mathematics. Consider the following examples of instructional decisions made by some teachers:

Less Effective Instructional Decisions	More Effective Instructional Decisions
✕ Mr. Ashley shows his students step by step how to solve problems and expects them to do the problems exactly they way he does. ✕ Ms. Lopez ensures that her students do not get lost by requiring them to stop when they finish an assignment and wait for others to finish. ✕ To keep them interested in math, Mr. Flanagan works problems for his students and "magically" comes up with answers. ✕ Two students are working problems on the board while the rest of the class watches. ✕ Students have been given 30 ordered pairs of numbers and are graphing them. ✕ Students find the mean, median and mode of a set of numbers. ✕ The students in Mr. Jones class are sitting in rows and are all quietly working on their assignment. ✕ At the end of class Ms. Stark collects	✓ Ms. Hernandez asks Tim to explain how he arrived at the answer to his problem. ✓ Mr. Roberts stimulates students' curiosity and encourages them to investigate further by asking them questions that begin with, "What would happen if..?" ✓ Ms. Perkins shows her students how "cool" math is and assures them that they all can learn algebra. ✓ The students in Mr. McCollum's class are talking to each other about math problems. ✓ Students are working on creating a graph that shows the path of an approaching hurricane. ✓ Students are conducting an experiment, collecting the data and making predictions. ✓ Students are sharing ideas while working in pairs or small groups. ✓ Students have done their work on chart paper and are holding the chart paper while explaining to the class how they reached their

everyone's worksheet and grades them. ✗ Students are in groups. One student in the group works out the problem while the others closely observe. ✗ Mr. Johnson will only allow calculators in his classroom during the second half of the year. He believes that students need to learn all their facts before they use calculators. ✗ Ms. Brown is showing her students how they can use a formula to easily find the value of any term in a sequence. ✗ During the first week of school Ms. Fitzwater holds up the text book and says, "I hope you are all ready to work very hard this year. This is a very thick book and we will be covering every single thing in it." ✗ Mr. Swanson believes that all students should get the same instruction at the same time. To accomplish this he only uses whole group instruction. ✗ In Mr. McBride's class he spends 99% of class time on skills and computation because his students have difficulty understanding word problems.	conclusions. ✓ Students are acting out a problem in front of the class. Others in the class participate in a discussion of the problem. ✓ Students are using calculators to determine patterns when multiplying integers. ✓ Mr. Osborne tells his students that their text book is only one resource that he uses in his classroom. Tonight their homework is out of that resource. ✓ Students are using color tiles to build the terms in a sequence. ✓ Some students are working in groups, some in pairs and some individually. Not all students are working on exactly the same thing. ✓ Students read about the history of the Pythagorean Theorem. After reading, they solve problems using the theorem. Students then write about what they did compared to the original uses of the Pythagorean Theorem.

References

Artzt, A. F., Armour-Thomas, E., & Curcio, F. R. (2008). *Becoming a reflective mathematics teacher*. New York: Lawrence Erlbaum Associates.

Cogan, M. L. (1973). *Clinical supervision*. Boston: Hougton Mifflin.

Davidson, N. (Ed.). (1990). *Cooperative learning in mathematics: A handbook for teachers*. Menlo Park, CA: Addison-Wesley.

The Education Alliance. (2006). *Closing the Achievement Gap: Best Practices in Teaching Mathematics*. Charleston, WV: The Education Alliance.

Grouws, D. (2004). "Chapter 7: Mathematics." In Cawelti, G, ed., *Handbook of Research on Improving Student Achievement*. Arlington, VA: Educational Research Service.

Holland, P. E. (1988). Keeping faith with Cogan: Current theorizing in a maturing practice of clinical supervision. *Journal of Curriculum and Supervision*, 3(2), 97 – 108.

Johnson, D., & Johnson, R. (1975). *Learning together and alone: Cooperation, competition, and individualization*. Englewood Cliffs, NJ: Prentice Hall.

National Center for Educational Achievement. (2009). *Core Practices in Math and Science: An Investigation of Consistently Higher Performing Schools in Five States.* Austin, TX: National Center for Educational Achievement.

National Center for Educational Achievement. (2010). *Best Practice Framework*. Accessed online at http://www.just4kids.org/en/texas/best_practices/framework.cfm , January 24, 2010.

National Research Council (NRC). (1999). *How people learn: Brain, mind, experience, and school. J. D. Bransford, A. L. Brown and R. R. Cocking (Eds). Washington, DC:* National Academy Press.

Posamentier, A. S., Hartman, H. J., & Kaiser, C. (1998). *Tips for the mathematics teacher*. Thousand Oaks, CA: Corwin Press.
Note: Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997; Artzt & Armour-Thomas, 1992 are summarized in Posamentier, Hartman, & Kaiser, 1998

Protheroe, N. (2007). "What Does Good Math Instruction Look Like?" *Principal* 7(1), pp. 51 – 54.

Shellard, E., & Moyer, P. S. (2002). *What Principals Need to Know about Teaching Math*. Alexandria, VA: National Association of Elementary School Principals and Education Research Service.

Texas Education Agency. (2006). *Mathematics TEKS Refinements, K-5*. Austin, TX: Texas Education Agency.