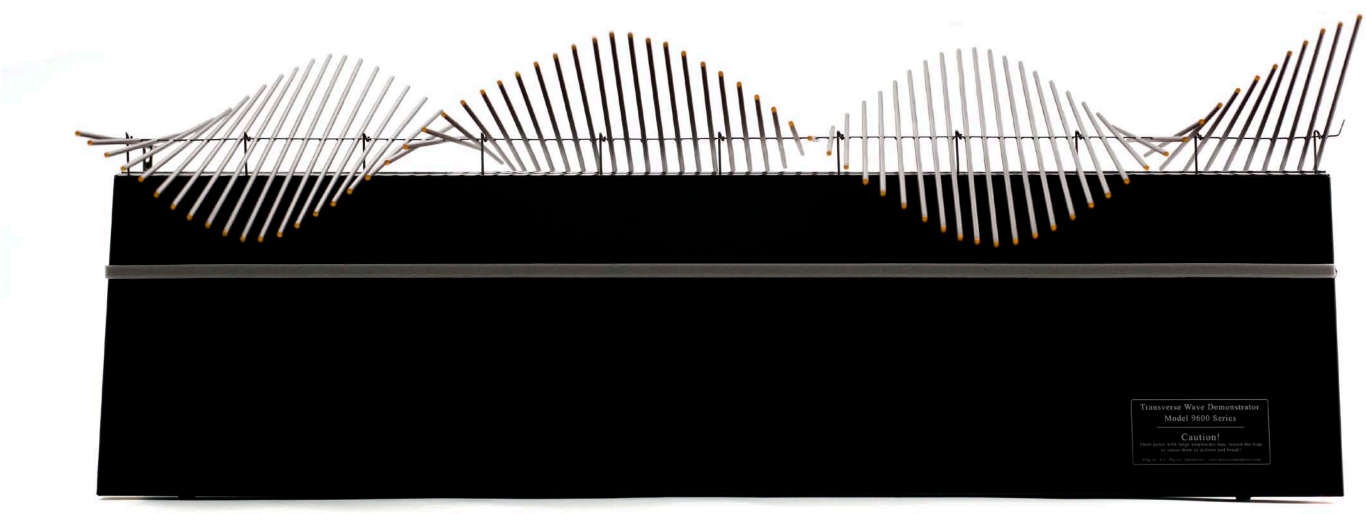
Andrews University
PHYSICS ENTERPRISES

Wave Motion Demonstrator

Instruction Manual



CONTENTS

4 INTRODUCTION

6 THEORY

7 DEMONSTRATIONS

16 APPENDIX

18 GENERAL INFORMATION

INTRODUCTION

The Wave Motion Demonstrator (WMD) uses mechanical waves to illustrate many of the properties and behaviors common to various types of waves. For example, the user can explore how the velocity v, frequency f, and the wavelength λ interact in the mathematical modeling relationship $v = f\lambda$. Also, the user may readily visualize the superposition of waves, easily study resonance conditions that cause standing waves, and demonstrate the reflections produced by various terminations.

The WMD as implemented by professor Bruce Lee at Andrews University is modeled after the wave demonstrator developed by Dr. John N. Shive of Bell Telephone Laboratories in the early 1960's. This manual acquaints the operator with the proper operation of the WMD and describes how many wave properties can be demonstrated on the apparatus. These demonstrations have been found to interest students from the elementary school level through the college level.

HOW IT WORKS

A series of steel rods is attached at their centers to a torsion wire. When a rod is displaced and released, a wave propagates along the wire. The velocity of propagation depends on the torsion spring constant of the wire and the moment of inertia of the rods.

PRODUCT INCLUDES

The complete Wave Motion Demonstrator pictured in Figure 1 consists of three sections:*

- One "wide" section approximately 0.92 m (3 ft) long consisting of seventy-two 0.46-m (18-in) long inertia bars.
- One "narrow" section approximately 0.92 m (3 ft) long consisting of seventy-two 0.23-m (9-in) long inertia bars.
- One "transition" section approximately 0.60 m (2 ft) long consisting of forty-seven inertia bars with lengths exponentially tapered from 0.46 m to 0.23 m.

These three sections are constructed using 5/32-inch steel rods with a rectangular slot milled in their center to hold an approximately 0.04-inch square spring steel wire which is soldered in place with 5% silver-95% tin. When not in use the bases lay flat facilitating storage and transportation of the equipment while protecting the bars. The tips of the rods are painted white on one end and with a fluorescent paint on the other end that shows up brightly in ultraviolet light.

Figure 1. The complete Three-Section Wave Motion Demonstrator.

INTRODUCTION

Accessories to the WMD shown in Figure 2 include:*

- One node clamp. When connected to a desired rod it will fix the position of the rod forming a node and cause a reflection.
- One dashpot consisting of a lightweight piston acting in a container of water. When clipped to a desired rod it will dissipate the wave energy minimizing a reflection.
- Two couplers. They are used to connect the three different demonstrator sections together.

* The single Wave Motion Demonstrator includes only the "wide" section on the left in Figure 1 along with the node clamps and dashpot shown in Figure 2.

Figure 2. Accessories for the WMD.

SETUP AND OPERATION

Grasp the ends of either half of the base and tilt it up cautiously, freeing the bars from the foam strip as necessary to prevent excessive twisting of the central wire.

Connect the horizontal slotted metal fasteners to the screws at each end to form the base into its "A" shape as shown in Figure 3. The apparatus is now ready for use.

Multiple sections are connected using the supplied couplers to join the tabs at each end of the central wire as shown in Figure 4.

Waves can now be produced by simply providing a vertical displacement of the bars at one end.

Figure 3. The WMD opened in its "A" shape.

CAUTION: Do not use larger amplitudes than necessary (not more than 30 to 40 degrees) for demonstrations, since large amplitudes, especially with short pulses or short wavelengths, may permanently damage the central wire or loosen the bars on the wire.

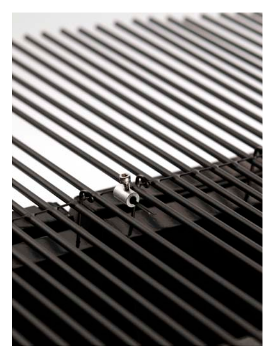


Figure 4. Two WMD sections coupled.

NOTE: During the demonstrations highlighted in this manual an electromechanical actuator was used to generate the waveforms. With this actuator the pulse waveforms could be consistently reproduced and the periodic waveforms could be generated with specified frequencies. In a couple of cases some waveforms were produced by hand, specifically, when waveforms had to be generated at more than one location or when the waveforms must be introduced with minimal contact with the end rod. The actuator connection to the rod on the left end of the demonstrator can sometimes be seen in the figures.

A digital camera was used to take multiple exposures of the WMD during these demonstrations at a rate of 4 exposures/s. The figures in this manual illustrating these demonstrations were selected and edited from these exposures.

THEORY

The propagation velocity of the wave on the demonstrator can be shown to be

$$v = \sqrt{(k/I)} \qquad (1)$$

where k is the torsional spring constant for the central wire and I is the moment of inertia of the bars around the wire. For a thin bar its moment of inertia about its center is

$$I = (mL^2)/12 \qquad (2)$$

where m is the mass of the bar and L is the length of the bar. The narrow demonstrator unit has bars that are exactly half the length (and half the mass) of the wide demonstrator unit. Both units use the same central wire and therefore have the same torsional spring constant k. From this we may deduce that the narrow unit will have 1/8th the moment of inertia and that its propagation velocity will be √8 (or 2.82) times greater than that of the wide unit. Using an experimentally determined value for k, the propagation velocities for the two units were calculated in the Appendix to be 0.47 m/s for the wide unit and 1.32 m/s for the narrow unit.

The wave or characteristic impedance is a concept that is helpful in understanding the behavior of the wave on the demonstrator at a point of discontinuity. The wave impedance for the demonstrator can be shown to be

$$Z = \sqrt{(kI)} \qquad (3)$$

For the wide demonstrator, Z_w is 0.028 N-m-s using the experimentally determined value of k. For the narrow demonstrator, Z_n is 0.010 N-m-s.

At a point of discontinuity in the wave impedance a portion of the wave will be reflected and a portion will be transmitted. The reflected and transmitted portions will be predicted by the reflection coefficient Y_r and transmission coefficient Y_t that can be expressed in terms of the wave impedances. If the wave is propagating on the demonstrator with wave impedance Z_1 and it encounters a new impedance Z_2, the amplitude of the reflected wave A_r compared to the amplitude of the incident wave A_i will be

$$Y_r = A_r/A_i = (Z_1 - Z_2)/(Z_1 + Z_2) \qquad (4)$$

Similarly, the amplitude of the transmitted wave A_t compared to the amplitude of the incident wave A_i is given by

$$Y_t = A_t/A_i = (2Z_1)/(Z_1 + Z_2) \qquad (5)$$

The two limiting cases should be considered. If the wave encounters an open end on the demonstrator, the impedance Z_2 is zero, and the reflection coefficient Y_r is +1 indicating that the wave is totally reflected with the same phase or polarity. If the wave encounters a clamped end on the demonstrator, the impedance Z_2 is infinite and the reflection coefficient is -1 indicating that the wave is totally reflected with a reversal of phase or polarity. On the other hand if the line can be terminated with an impedance equal to its wave impedance, there will be no reflection and all of the wave energy will be transmitted to or dissipated in this termination. The dashpot described in the Introduction is designed to have that matching impedance.

DEMONSTRATIONS

WAVE PROPAGATION

Set up the wide wave demonstrator with the dashpot connected to the far end inertia bar about midway between the central wire and the tip of the rod. The purpose of the dashpot is to absorb most of the energy transmitted down the line and in this way to minimize reflections.

Now give the near end bar of the wave demonstrator a small sharp up and down disturbance.

Then, repeat with a larger pulse followed closely by a smaller pulse. Note that each of the waves travels with the same speed.

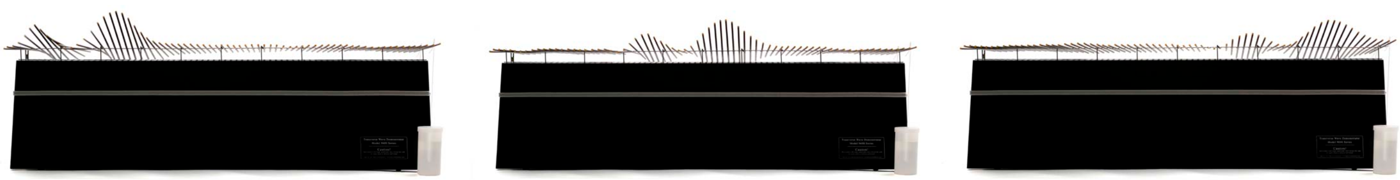


Figure 5. A wave propagating on the wide wave demonstrator (0.5s between pictures). From the change in wave position between these three pictures, the velocity of wave propzagation on this wide wave demonstrator is shown to be about 0.51 m/s. This value compares favorably with the calculated wave propagation velocity of 0.47 m/s presented for the wide unit in the Theory section above. For this 36-inch unit, the time to propagate from one end to the other is then about 1.9 s. See the Appendix.

Set up the narrow wave demonstrator with the dashpot connected to the far end inertia bar near the tip of the rod. Send pulses down the line noting that the speed of propagation is faster than with the wide unit.

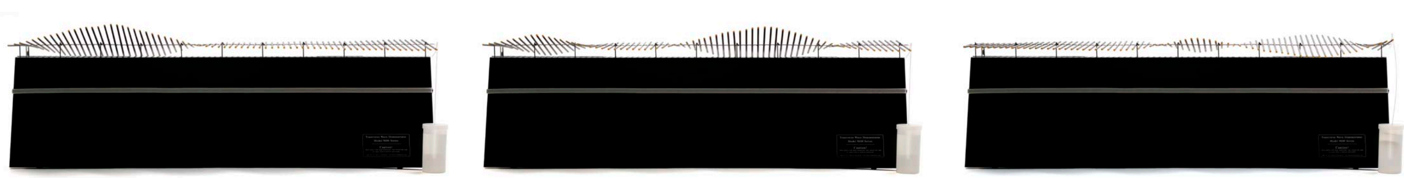


Figure 6. The same wave excitation used in Figure 5 propagating on the narrow wave demonstrator (0.25 s between pictures). From the change in wave position in these pictures, the velocity of wave propagation on the narrow wave demonstrator is shown to be about 1.42 m/s. This again compares favorably with the calculated value of 1.32 m/s in the Theory Section. So for this 36-inch unit, the time to propagate from one end to the other is about 0.7 s. Note that the number of rods between peaks of the pulse in pictures taken at a known time interval can be compared between the wide and narrow units to obtain the ratio of the propagation speeds (approximately 2.8 or √8 as expected).

DEMONSTRATIONS

PERIODIC WAVES

Using the wide wave demonstrator with the dashpot attached as before, generate periodic disturbances at several frequencies. Since the propagation velocity ***v*** of the waves is fixed and related to the frequency ***f*** and spatial wavelength ***λ*** according to $v = f\lambda$, a higher frequency will exhibit a shorter wavelength, and vice versa.

Figure 7. The wide wave demonstrator excited by a higher and a lower frequency periodic waveform (2 Hz and 1 Hz, respectively). For the 2-Hz excitation, the wavelength is about 20 bars, and for the 1-Hz excitation, the wavelength is about 40 bars.

Using the narrow wave demonstrator with the dashpot attached as before, apply the same higher and lower frequency waveforms used before.

Figure 8. The narrow unit when excited by these same periodic waveforms. For the 2-Hz excitation, the wavelength is about 57 bars. For this narrow unit the velocity of propagation is $\sqrt{8}$ times faster and the wavelength is $20\sqrt{(8)} = 57$ bars as expected. For the 1-Hz excitation the wavelength is seen to be longer than the unit.

It should be noted that the waves we have been observing propagating from one end of the demonstrators to the other are transmitting energy. At the receiving end of the unit this energy could be harnessed to do work – in our case it is adding heat to the water in the dashpot.

DEMONSTRATIONS

REFLECTION OF WAVES

With the far end of the wide wave demonstrator free (no dashpot), start a sharp positive pulse followed closely by a smaller positive pulse traveling down the demonstrator. From the Theory Section we expect that the open end will totally reflect the wave with the same polarity.

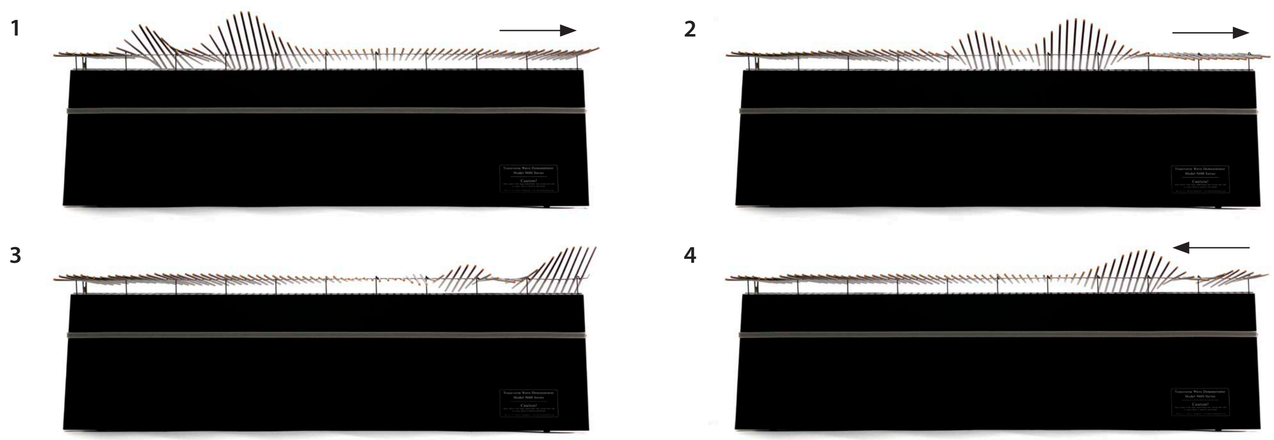


Figure 9. A sequence of pictures (0.5 s between pictures) illustrating how a positive pulse reflects from the open end of the unit. Note that the positive pulse waveforms are reflected with the same polarity as expected.

With the far end of the wide wave demonstrator clamped (using the node clamp on the last inertia bar), again start the same large and smaller positive pulses traveling down the demonstrator. From the Theory section we expect that these pulses will be totally reflected with an inversion of polarity.

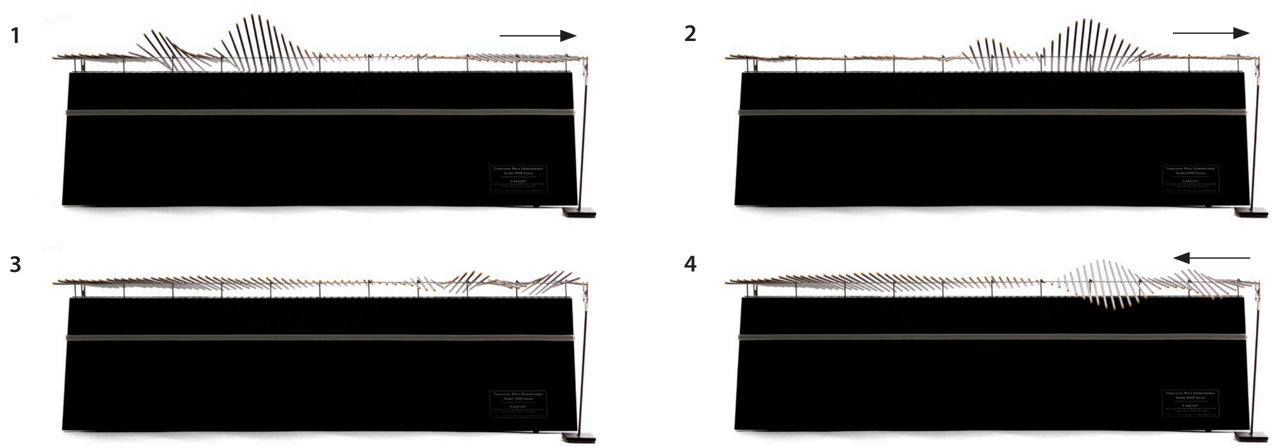


Figure 10. A sequence of pictures (0.5 s between pictures) illustrating how a positive pulse reflects from a clamped end of the unit. Note that the pulse waveforms are totally reflected with the opposite polarity as expected.

Similar behaviors are observed for reflections of sound waves, other forms of mechanical waves, guided electrical waves, and electromagnetic waves at points of discontinuity in the mediums through which they are propagating. Open and clamped terminations result in total reflections, whereas partial reflections will be produced by more modest discontinuities of the medium.

CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE OF WAVES, SUPERPOSITION

Simultaneously start short positive pulses from both ends of the wide wave demonstrator. At the point where the pulses meet their amplitudes add (constructive interference). Note that the shapes of the waves after passing through each other are preserved.

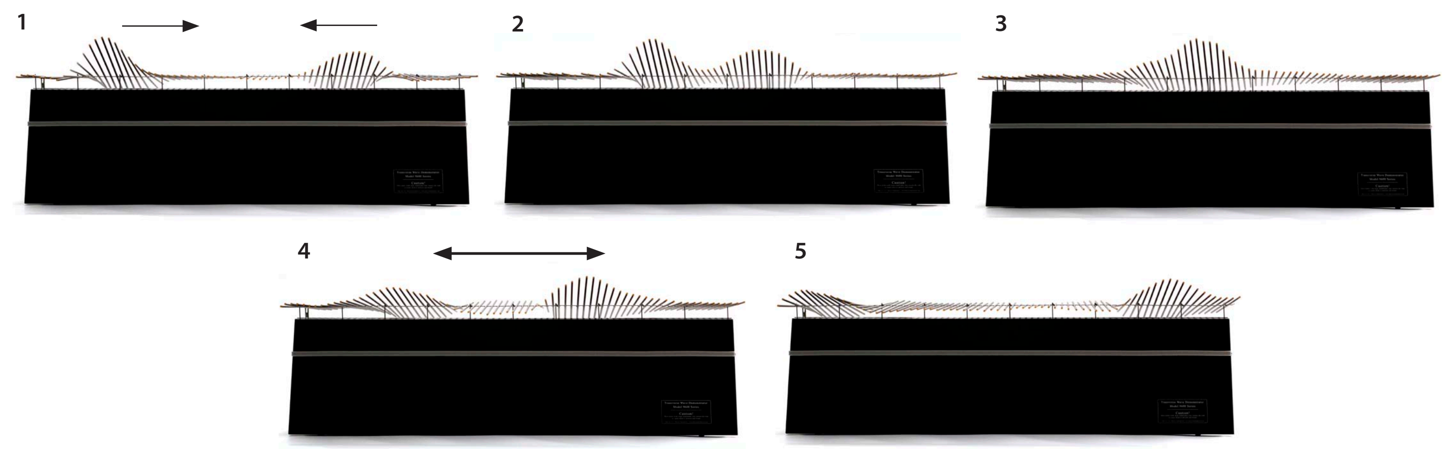


Figure 11. A sequence of pictures (0.25 s between pictures) illustrating this constructive interference. This summing of the amplitudes as the pulses meet is known as superposition.

This time, simultaneously start a short positive pulse at one end and a short negative pulse at the other end of the wide wave demonstrator. As the pulses meet their amplitudes add to nearly zero (destructive interference). Again note that the shapes of the waves are preserved after passing through each other.

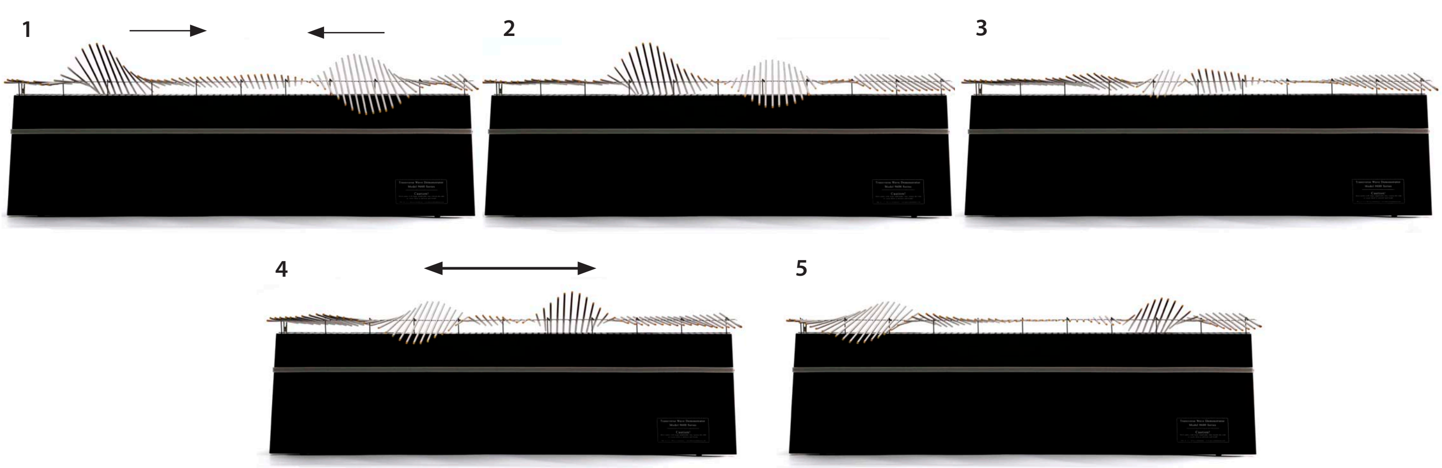


Figure 12. A sequence of pictures (0.25 s between pictures) illustrating this destructive interference as the two waves superimpose.

STANDING WAVES AND RESONANCE (1)

Interference of periodic waves is observed by sending periodic waves from one end and allowing them to reflect either from a free or a clamped end.

Initially, clamp the far end. The end held in the hand and driven is "almost" clamped so there is a node at both ends and there will be phase reversal upon reflection at both ends. By sending periodic waves of the proper frequency from one end, the reflections can build up and produce a resonance. To produce this resonance the length of the wave demonstrator is always an integral multiple of a half wavelength. If the wave demonstrator length is L, then $L = n\lambda/2$ where n is the mode number.

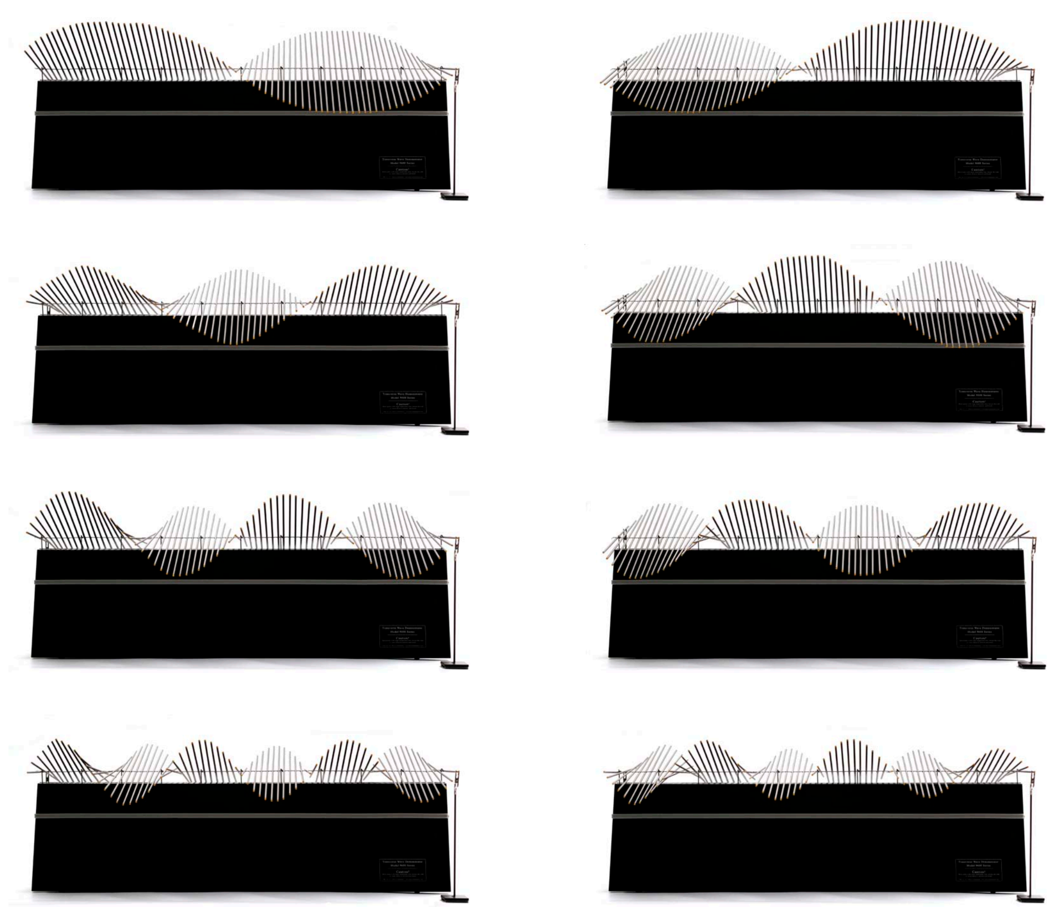


Figure 13. Standing waves with nodes at each end. Modes 2, 3, 4, and 6 are shown. These standing waves build up where the wave reflected from the far end returns to the driven end with the proper phase to add to the driven wave. Some practice is required to get the "feel" for the proper frequencies. Sensing feedback from the apparatus helps greatly in establishing the resonant frequencies.

DEMONSTRATIONS

STANDING WAVES AND RESONANCE (2)

If the far end is left free, resonance will again occur at many frequencies with an antinode at the free end and a node at the driven end. Note that the length of the wave demonstrator is now an odd multiple of a quarter wavelength. So in this case the length ***L= (2n-1) λ/4*** where again ***n*** is the mode number.

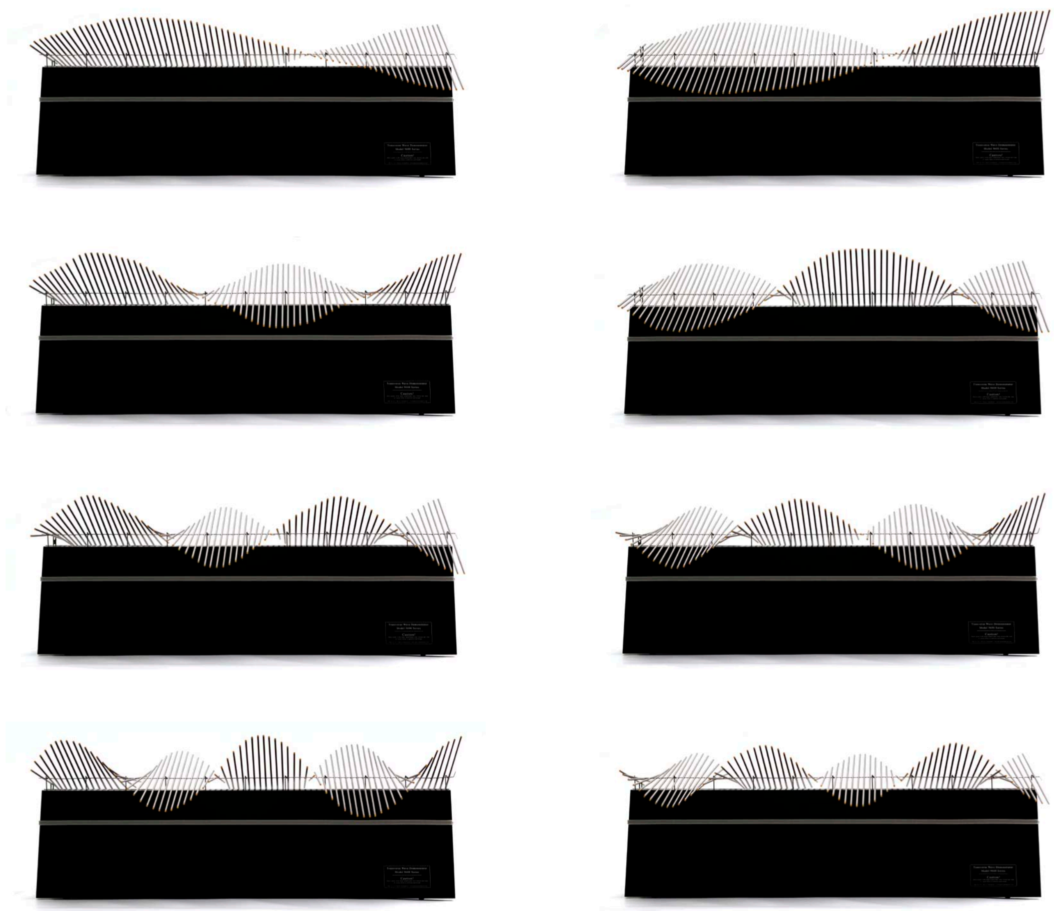


Figure 14. Standing waves with a node at the near end and an anti-node at the far end. Modes 2, 3, 4, and 5 are shown.

STANDING WAVES AND RESONANCE (3)

A third way to produce standing waves is with antinodes at both ends. This is somewhat more difficult and requires a little more practice.

By holding the end of the wire or the end bar near the wire very loosely, standing waves with antinodes on each end can be produced. In this case again $L = n\lambda/2$.

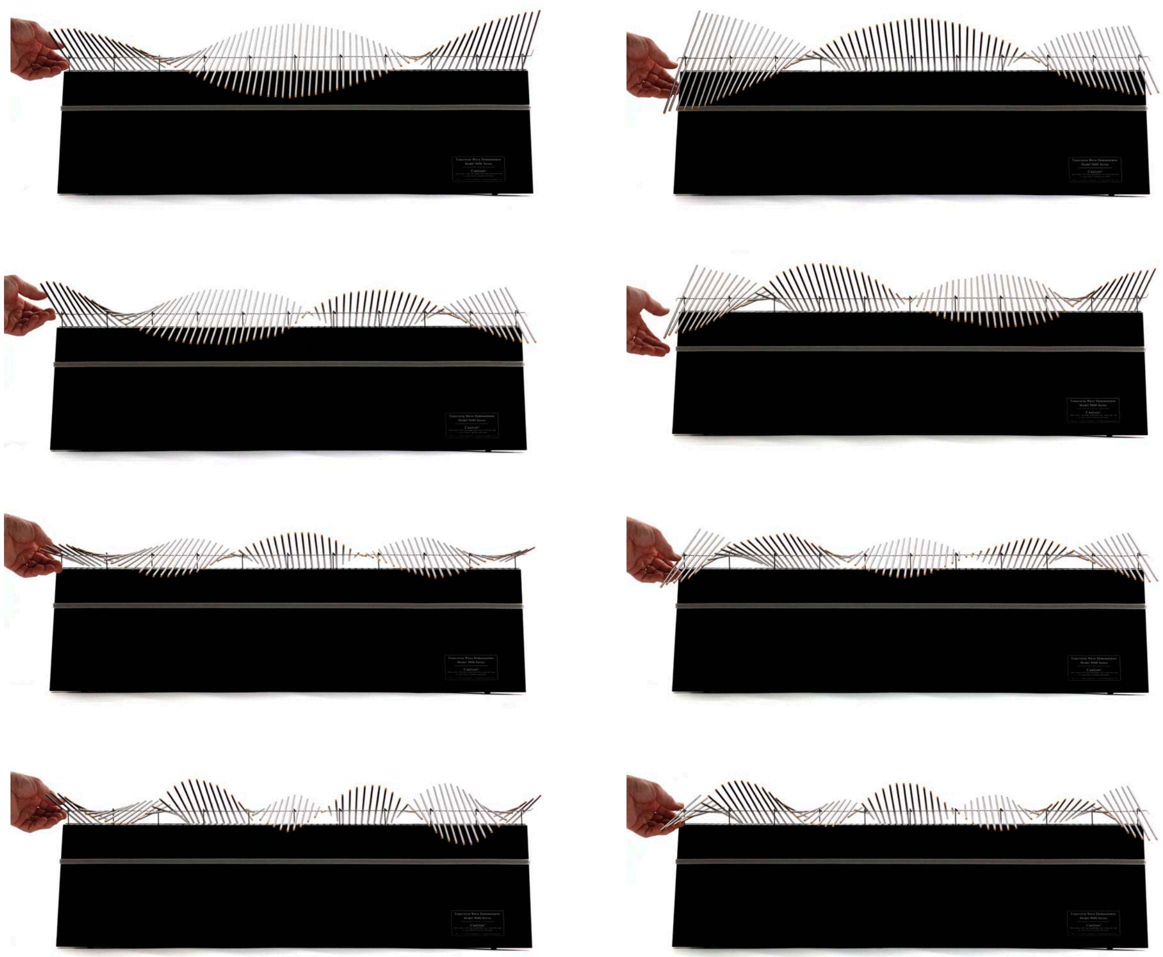


Figure 15. Standing waves with antinodes at both ends. Modes 2, 3, 4, and 6 are shown. These modes are similar to those occurring in resonant open pipes with length always an integral multiple of a half wavelength.

DEMONSTRATIONS

IMPEDANCE MATCHING (1)

If the narrow unit is coupled directly to the wide unit, there will be a significant reflection produced at the transition since the wave impedances are different. The wave impedances for the two units are calculated in the Appendix. The narrow unit has lower wave impedance than the wide unit, and the calculated reflection coefficient is plus 0.48. So we will expect to see a reflection generated at the transition between the units with an amplitude about half of the incident wave amplitude and with the same polarity. See Figure 16.

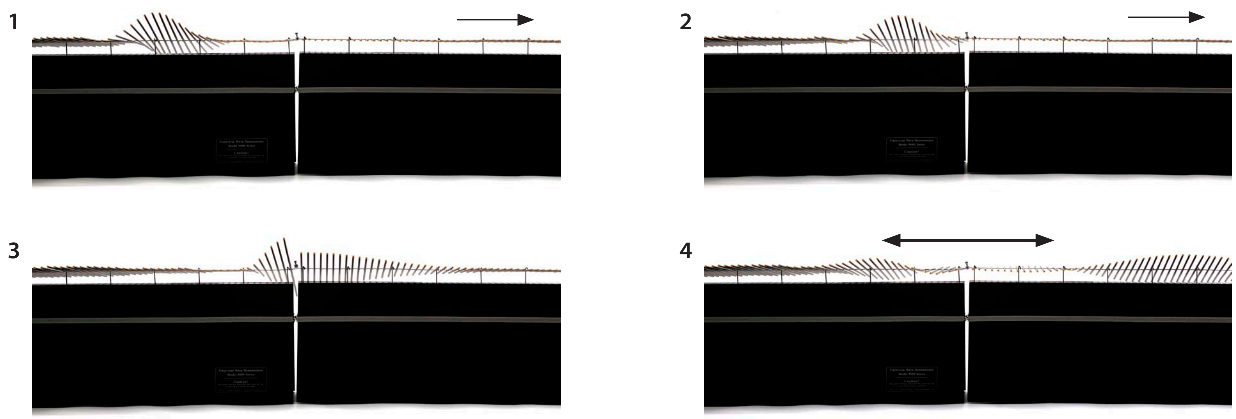


Figure 16. Reflection at the transition between the wide and narrow units when they are connected directly together. Note the positive reflection from the positive incident pulse. Pictures were taken at 0.25s intervals.

If the tapered transition section is now inserted between the wide and the narrow units, it will provide a smooth transition between the wave impedance of the wide unit and the lower wave impedance of the narrow unit. The pulse will travel smoothly to the right with little reflection.

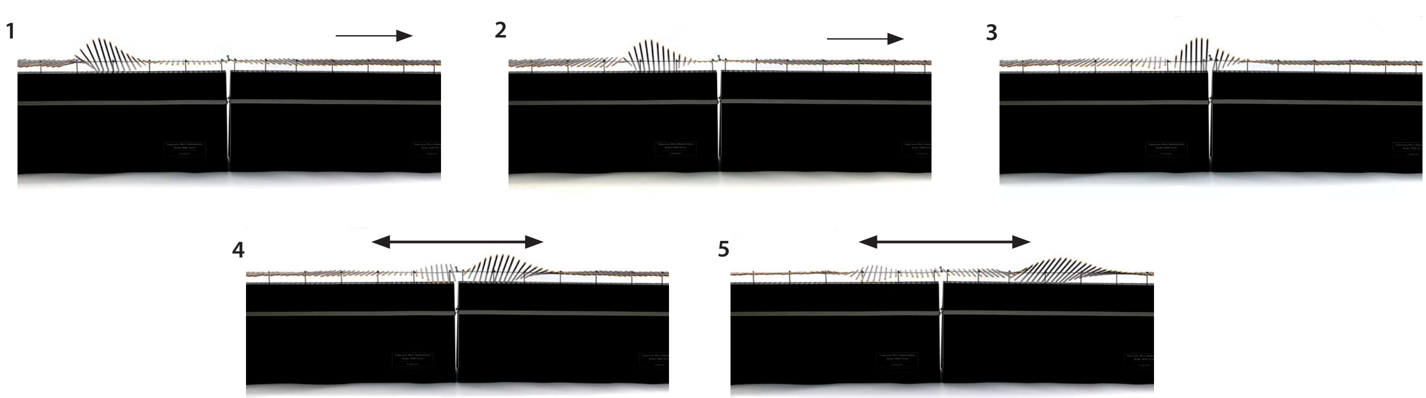


Figure 17. The wide demonstrator connected to the tapered section showing the smooth transition of the positive pulse as it travels to the right. Note the minimal reflections at the transition between the two units. Pictures were taken at 0.25s intervals.

IMPEDANCE MATCHING (2)

Figure 18. The same positive pulse as it travels to the right beginning on the wide demonstrator, traversing the tapered unit, and moving on to the narrow unit. The pictures were taken at 0.5s second intervals. Note how the pulse waveform speeds up and spreads out as it moves onto the narrow demonstrator as expected.

The tapered section acts as an impedance matching “transformer”. Many electrical circuits use impedance matching devices. An antenna that matches the feed line impedance to the impedance of free space is one example. An interesting mechanical example is the function of the three bones in the middle ear. Sound energy entering the ear must be transferred to the fluid of the inner ear (or cochlea). The middle ear serves as an impedance matching device between the air and the cochlea.

APPENDIX

WAVE MOTION DEMONSTRATOR CALCULATIONS (1)

Physical Data for the Wave Motion Demonstrator

- **Wide Unit**

Bar length	L	= 45.6 cm
Bar mass	m	= 43.2 g/bar
Bar separation	d	= 1.27 cm/bar

- **Narrow Unit**

Bar length	L	= 22.8 cm
Bar mass	m	= 21.9 g/bar
Bar separation	d	= 1.27 cm/bar

The Equation of Motion for a Torsional Wave Traveling in the z Direction

$$I\, \partial^2\Theta(z,t)/\partial t^2 = k\, \partial^2\Theta(z,t)/\partial z^2$$

where ***I*** is the moment of inertia of the rods and ***k*** is the torsion spring constant of the central wire.

If we assume a sinusoidal wave of frequency ω traveling in the positive z direction with velocity v, then

$$\Theta(z,t) = \Theta o\, \exp(j\omega(t-z/v))$$

and substituting in the above wave equation, we find that

$$v = \mathrm{sqrt}(k/I)$$

Moment of Inertia I about the Center of the Rods

- **Wide Unit**
  $I = mL^2/12 = 7.49E\text{-}4$ kg-m^2/bar

- **Narrow Unit**
  $I = mL^2/12 = 9.49E\text{-}5$ kg-m^2/bar

Spring Constant k for the Central Wire

To find the torsional spring constant ***k*** experimentally, a 50 g weight is hung from the end of a bar of the wide unit while an adjacent bar is clamped so as to keep the bar with the weight horizontal. The deflection distance between the two bars ***c*** was found to be 2.5 cm.

Deflection	c	= 2.5 cm	where *c* is the chord length
Deflect angle	α	= 0.11 rad/bar	where α = 2 arcsin(c/L)
Weight	w	= 50 g	
Force	F	= 0.49 N	
Force act at	L/2	= 22.8 cm	
Spring const	k	= 1.02 N-m/rad/bar	where k = (F*L/2)/α

WAVE MOTION DEMONSTRATOR CALCULATIONS (2)

Propagation Constant β = sqrt(k/I)

- **Wide Unit**
  Prop constant β = 37 bars/s

- **Narrow Unit**
  Prop constant β = 104 bars/s

  Ratio Nar U/WideU = 2.81 ~ sqrt(8)

Refering to Figure 5 in the text and counting bars between the large peak positions, the peak is seen to move 20 bars in 0.5s for the wide unit giving an experimental value for the propagation constant of 40 bars/s.

In Figure 6 for the narrow unit, the peak is seen to move 28 bars in 0.25s giving an experimental value for the propagation constant of 112 bars/s.

These experimental values are about 8% higher than predicted.

Corresponding Velocity in m/s

- **Wide Unit**
  Velocity v = 0.47 m/s where bar separation = d

- **Narrow Unit**
  Velocity v = 1.32 m/s

Time to Traverse the Length of the Unit

- **Wide Unit**
  Traverse time = 1.96 s where unit length = 0.92 m

- **Narrow Unit**
  Traverse time = 0.70 s

Characteristic or Wave Impedance of the Wave Motion Demonstrator

The wave impedance Z= force/velocity.

$$Z = k\,\Theta(z,t)/v\,\Theta(z,t)$$
$$Z = k/\mathrm{sqrt}(k/I) = \mathrm{sqrt}(kI)$$

- **Wide Unit**
  Wave imped. Z_w = 0.028 N-m-s

- **Narrow Unit**
  Wave imped. Z_n = 0.010 N-m-s

GENERAL INFORMATION

REFERENCES

Shive, J. N. (1961). Similarities in Wave Behavior, Teacher's Ed. Bell Telephone Laboratories.

LIABILITY

This product has been designed for educational demonstration purposes only! Use in research, medical, commercial, or industrial applications is prohibited. Any use of this product outside of its intended purpose is done so at the risk of the end user, who shall assume full liability, and fully indemnify A.U. Physics Enterprises and its agents, for any and all damages resulting from such prohibited use.

WARRANTY

The Wave Motion Demonstrator is warranted by A.U. Physics Enterprises for a period of one year from the date of purchase. This warranty covers any defects in workmanship or materials. It does not cover accidental damage, damage as a result of operator error, negligence, or abuse. Should a problem be found, DO NOT attempt to disassemble the apparatus, as this will void its warranty. The entire unit should be returned for repair.

If you purchased this product through our distributor please contact them for further information on warranty policy.

CONTACT