## Discover Density Set Experiment Guide and Teacher's Notes



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## Introduction

## Overview

The Discover Density Set provides materials and activities to guide students through some basic graphical analysis techniques.

In each case, an example is worked through and explained. The student then performs a similar analysis using the materials in the set.

In the first analysis, students discover the concept as a mathematical constant relating measurements of a particular material.

Second, they are asked to experimentally discover an equation that predicts the mass of spheres of unknown but uniform composition, based on their diameters.

Finally, they are led to develop an equation in three variables that predicts the mass of cylinders of unknown but uniform composition, based on measurements of their diameters and lengths.

The only mathematical formula students are expected to know is that for slope. If they recall special volume formulas for spheres and cylinders, they are asked to not use this information in the development of their equations. After graphical analysis yields the desired equations, students can use volume formulas and tabulated density data to verify the equations they have discovered experimentally.

## Other Uses

Because the items in the set are machined to close tolerances, and average dimensions and masses are given in the teacher's guide, the set may be used for other purposes, such as a traditional density set, or to test students' abilities to make accurate measurements, etc.

## Equipment

- 4 Metal Rectangular Solids (A-D)
- 4 Gray Plastic Rectangular Solids (E-H)
- 1 Transparent Plastic Rectangular Solid (I)
- 1 Black Plastic Rectangular Solid (J)
- 4 Black Plastic Cylinders of Constant Length (K-N)
- 4 Black Plastic Cylinders of Constant Diameter (O-R)
- 4 Transparent Plastic Spheres (S-V)



## Prelab 1 Introduction

## The Speed of Sound

Figure 1
Graph of distance vs time

This sample problem presents experimental data and leads you through a process to obtain an equation relating the data. You will follow the same process in Experiment 1 using the materials in this set.

## Experimental Data

A lightning bolt struck the earth, and upon seeing it, a number of observers started timing, using stopwatches. The observers each stopped their watches when they heard the thunder. The times recorded and the distances from the observers to the lightning strike are recorded in the table. The variables are labeled " t " for time and " d " for distance:

| Time, t (s) | Distance, d (km) |
| :---: | :---: |
| 3.7 | 1.2 |
| 5.2 | 1.8 |
| 8.3 | 2.8 |
| 12.1 | 4.1 |
| 14.9 | 5.1 |

The data is then graphed with distance on the y -axis and time on the x -axis. A straight line can be drawn through the points to closely approximate the pattern of the data. We may assume there are random errors caused by factors such as differences in human reaction time and errors in measuring distance. Such errors may be the reason the points do not line up perfectly.


## Data Analysis

In algebra, the formula for a line on a graph is often given by:

$$
y=m x+b,
$$

where $y$ is the variable on the vertical axis,
$x$ is the variable on the horizontal axis,
$m$ is the slope of the line, and
b is the point on the vertical axis where the line intersects.

The slope is found by marking two points on the line, and dividing the difference in $y$-coordinates (called the rise) by the difference in $x$-coordinates (the run).

Since all parts of a straight line have the same slope, the slope is a constant for this experiment.

For this data, $b$ is zero, and $m$ is $2.0 \mathrm{~km} / 6.0 \mathrm{~s}=0.33 \mathrm{~km} / \mathrm{s}$.

Notice that dimensional units are part of the rise and the run. The slope is found in this case by dividing a distance by a time. You might recognize this as the formula for speed.

The algebraic equation $y=m x+b$ may be translated into an equation appropriate to this situation by replacing the algebra symbols with the variables from this problem.

Thus,
distance $=$ speed $\boldsymbol{*}$ time, a very familiar equation!

The speed in this case is the speed of sound, and agrees with published data ( $0.343 \mathrm{~km} / \mathrm{s}$ ), considering uncertainty.

This example is a simple illustration of how numerical data from an experiment can be transformed into a meaningful equation.

## Experiment 1 Relating Mass and Volume of Rectangular Solids

## Introduction

In this activity you are given four rectangular solid metal pieces and four similar plastic pieces. You are asked to take measurements, organize and graph the data, and write equations relating the mass and volume for each material. A minimum of instructions are given. You should study and follow the example in Prelab 1, titled "The Speed of Sound."

## Equipment

- Rectangular Solids A-J
- Ruler or Caliper
- Balance or Scale


## Procedure

1. Create a table to record the length, width, height, volume, and mass of the four metal pieces A-D and a similar table for the four gray plastic pieces E-H.
2. Record the length, width, and height in centimeters. If you are using a metric ruler, estimate to the nearest 0.01 cm when finding these dimensions. Use the rules regarding significant figures or another appropriate expression of uncertainty.
3. Consider the volume to be the independent variable (x-axis) and the mass to be the dependent variable ( y -axis) when graphing the data. Prepare a graph that shows the data for all eight objects, labeling your data points for the metal pieces with circles and the plastic with squares.
4. Draw a best-fit line for the data from the metal pieces and another for the plastic. If straight lines passing through the origin do not represent the data well, recheck your measurements and calculations for any data points that do not fit the pattern.
5. Calculate the slope of each line, and include dimensional units as part of your calculations. Show your calculations, and use significant figures or another appropriate expression of uncertainty.
6. Although each item had its own mass and volume, the slope of the line for the metal pieces is constant. The metal pieces are all aluminum, and the slope is termed the density of aluminum. Find a published value for the density of aluminum, and compare to your value. Does your value agree within the limits of uncertainty?
7. The gray plastic is polyvinyl chloride, or PVC. Its published density is 1.33 to $1.41 \mathrm{~g} / \mathrm{cm}^{3}$. Does your value agree within the limits of uncertainty?
8. Write equations for each of the two lines obtained. Use meaningful symbols, such as "m" and " $v$ " for the mass and volume. Include dimensional units in the constant.
9. Find the mass, volume, and density of the transparent rectangular solid (I) and the black rectangular solid (J). Plot them on the same graph as the aluminum and PVC. Ignoring the color, can you say with confidence that they are or are not the same type of plastic as PVC, or as each other?

## Prelab 2

## The Mass of Fluorite Octahedra

## Introduction

Developing a mathematical equation from a set of experimental data is an extremely useful skill. The examples that follow show a method that works for a great many physics phenomena. You will then be asked to apply this method in Experiment 2.

The mineral fluorite is often found in geometric shapes having eight faces that are equilateral triangles. This example addresses the problem of finding an equation that allows one to calculate the mass of such a fluorite specimen from a measurement of one of the edges.

## Experimental Data

Some data were obtained from direct measurement of five fluorite specimens:


Figure 2
Graph of mass vs edge length

Graphing this data in the ordinary manner is a good first step. The results suggest an equation such as $y=x^{2}$, or $y=x^{3}$. Of course, a constant multiplier would likely be present and we should use the variables for this problem, resulting in an equation such as $m=0.57 L^{2}$, or $m=3.9 L^{2}$.

Finally, if the exponent were a number such as $(3 / 2)$ or 2.716 , the graph would have the same basic shape. Quite often in physics, and particularly in simple situations such as this, the exponent will be either a small integer, or a ratio of two small integers.

## Data Analysis

All of the equations above are of the form $y=\mathrm{c} x^{\mathrm{k}}$, where c and k are two different constants. Many equations in physics, although certainly not all, are of this form.

If an initial graph or other factors make it reasonable to assume an equation of the form $y=\mathrm{c} x^{\mathrm{k}}$, the next task is to determine the values of c and k . Several methods exist for doing this. The first might be called "guess and test."

We might guess for the fluorite example that the exponent is 2 , so $m=c L^{2}$. This could be expressed in words as " $m$ is proportional to $L^{2}$." Making a new table and graph with $L^{2}$ results in the following:

Tip: A spreadsheet is an efficient way of creating such tables.

| $\mathbf{L}^{\mathbf{2}} \mathbf{( c m}^{\mathbf{2}} \mathbf{~}$ | $\mathbf{m}(\mathbf{g})$ |
| :---: | :---: |
| 0.6 | 0.8 |
| 1.7 | 3.3 |
| 4.0 | 12.0 |
| 7.3 | 29.5 |
| 13.7 | 75.9 |

Figure 3
Graph of mass vs edge length ${ }^{2}$


The graph of $m$ vs $L^{2}$ does not result in a straight-line (see Figure 3), as would have been the case if $m$ had been proportional to $L^{2}$. Instead, we can try an exponent of 3 . The corresponding graph from a table of $L^{3}$ and $m$ values is straight, and thus shows that $m$ is proportional to $L^{3}$ (see Figure 4).

Figure 4
Graph of mass vs edge length ${ }^{3}$

| $\mathbf{L}^{\mathbf{3}}\left(\mathbf{c m}^{\mathbf{3}}\right)$ | $\mathbf{m}(\mathbf{g})$ |
| :---: | :---: |
| 0.5 | 0.8 |
| 2.2 | 3.3 |
| 8.0 | 12.0 |
| 19.7 | 29.5 |
| 50.7 | 75.9 |



The analysis has established that the data is of the form $y=c x^{3}$. The constant of proportionality, c , is simply the slope of the graph. Picking two points on the line, $\left(x_{1}, y_{1}\right)$ and ( $x_{2}, y_{2}$ ), and using the formula for slope

$$
\begin{aligned}
& \left(y_{2}-y_{1}\right) /\left(x_{2}-x_{1}\right) \text { gives } \\
& c=(90 \mathrm{~g}-20 \mathrm{~g}) /\left(60.0 \mathrm{~cm}^{3}-13 \mathrm{~cm}^{3}\right)=1.49 \mathrm{~g} / \mathrm{cm}^{3}
\end{aligned}
$$

Notice that the value for c has dimensional units as part of its value. This will often be the case in science.

Substituting the value for c gives us an equation of

$$
y=\left(1.49 \mathrm{~g} / \mathrm{cm}^{3}\right) x^{3},
$$

and replacing the symbols $x$ and $y$ with our variables, $L$ and $m$, the final result is:

$$
m=\left(1.49 \mathrm{~g} / \mathrm{cm}^{3}\right) L^{3} .
$$

Although this is sometimes a tedious way to discover an equation representing data, a graph such as that in Figure 4 is a common and effective way to visually show the relationship. For this reason, it is valuable to understand the method, even when technology provides methods that are easier to use.

## Another Method for Data Analysis

A second method is useful when a relationship such as $y=c x^{k}$ is suspected, but there is no clue what the exponent might be. This method is suggested by taking the logarithm of both sides of the equation:

$$
\begin{aligned}
& \log (y)=\log \left(c x^{k}\right), \text { and then simplifying, } \\
& \log (y)=\log (c)+k \log (x)
\end{aligned}
$$

If we regard $\log (y)$ and $\log (x)$ as two new variables, and if we understand that $\log (c)$ is just another constant, this new equation can be treated as a linear equation. In this case, k is the slope, and $\log (\mathrm{c})$ is the vertical intercept.

Creating a table with the logarithms of the original data (again, a spreadsheet would be helpful) and drawing another graph, we again see a linear relationship (see Figure 5).

| $\log (\mathrm{L})$ | $\log (\mathbf{m})$ |
| :---: | :---: |
| -0.1 | -0.1 |
| 0.11 | 0.52 |
| 0.30 | 1.08 |
| 0.43 | 1.47 |
| 0.57 | 1.88 |

## Figure 5

Graph of the log transformations of mass and length


The straight-line graph confirms that $y=c x^{k}$ was indeed the form of the equation (see Figure 5). The slope of this graph is approximately 3, showing that the exponent, $k$, is 3 .

The constant, c may be evaluated by various methods, but perhaps the best way is by solving the equation for c and substituting data from the original table.

Rearranging the equation, we have

$$
\begin{aligned}
& \mathrm{c}=y / x^{3}, \text { and substituting } \\
& x=2.7 \mathrm{~cm}, y=29.5 \mathrm{~g} \\
& \text { (from the next-to-last data pair; any point could have been used), }
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{c}=(29.5 \mathrm{~g}) /(2.7 \mathrm{~cm})^{3} \\
& \mathrm{c}=1.50 \mathrm{~g} / \mathrm{cm}^{3}
\end{aligned}
$$

Substituting this value and our variables, $L$ and $m$, the final equation is

$$
m=\left(1.50 \mathrm{~g} / \mathrm{cm}^{3}\right) L^{3} .
$$

As an alternative to calculating the logarithms, the data may be plotted on loglog graph paper, also called full logarithmic paper. The spacing between the lines on this paper is adjusted so that the appearance is the same as plotting the logarithms of the data on ordinary paper. The result is a straight line with a slope of 3 . Since the numbers on the paper are the same as the original data, calculating the slope requires first calculating the logarithms of the coordinates of two points.

Note: As remarked before, while computer programs such as DataStudio provide rapid methods of data analysis, logarithmic graphs such as that above are a common and effective way to visually show this type of relationship. For this reason, it is valuable to understand the method and gain familiarity with logarithmic graphs.

## Analysis Verification

Often, data analysis of this sort is done in the hope of confirming some hypothesis that has been proposed. In any case, some sort of check or comparison is in order. Frequently, this check first involves algebraic manipulation of either the equation developed or the hypothesized equation, to put them in the same terms.

In this example, we know that
mass = density * volume,
a math reference gives the volume for an octahedron,

$$
\text { volume }=(\sqrt{ } 2)(1 / 3) L^{3} \text {, where } L \text { is the length of an edge, }
$$

and another reference gives the density of fluorite,

$$
\text { density }=3.18 \mathrm{~g} / \mathrm{cm}^{3} .
$$

Combining these equations and simplifying gives the following.

$$
\begin{aligned}
& \text { mass }=\left(3.18 \mathrm{~g} / \mathrm{cm}^{3}\right)(\mathrm{V} 2)(1 / 3) L^{3} \\
& m=\left(1.499 \mathrm{~g} / \mathrm{cm}^{3}\right) L^{3}
\end{aligned}
$$

This result is in agreement with the results obtained by analysis of the experimental data.

## Experiment 2 Relating Mass and Diameter of Plastic Spheres

## Equipment

- Spheres S-V
- Ruler or Caliper
- Balance or Scale


## Procedure

1. Create a table to record the diameters and masses of the four spheres. Record the diameter in centimeters. If you are using a metric ruler, estimate to the nearest 0.01 cm when finding these dimensions. You should use two rectangular objects with the ruler to increase your accuracy (see Figure 6). Use the rules regarding significant figures or another appropriate expression of uncertainty.


Top View
2. Consider the diameter to be the independent variable (x-axis) and the mass to be the dependent variable ( $y$-axis) when graphing the data. Prepare a graph that shows the data for all four spheres.
3. Draw a best-fit line for the data, which may be a smooth curve. If it is not possible to represent the data well with a smooth curve, recheck your measurements for any data points that do not fit the pattern.
4. State a hypothesis regarding the form of equation that is likely to best describe the data.
5. Determine an equation that represents the data. Use one or more of the methods outlined in Prelab 2.
6. Your instructor may tell you which method(s) to use. Check the accuracy of the equation by using the following data from published sources:

```
volume of a sphere = (4/3)\pir r
radius = diameter / 2
density of sphere material = 1.18 g/cm}\mp@subsup{}{}{3
density = mass / volume
```

7. Algebraically combine this information to produce an equation giving the mass of the spheres in terms of their diameter.
8. Compare this result with the equation you determined experimentally. Are they in agreement, taking into account uncertainty?

## Prelab 3

Relating Mass and Diameter of Metal Cones

## Introduction

Mathematical equations of several variables are common in physics. Some examples are

$$
\begin{aligned}
& F=m a, \text { Newton's Second Law of Motion, } \\
& F=G\left(m_{1} m_{2}\right) / d^{2} \text {, the gravitational force between two objects, } \\
& a=v^{2} / r \text {, a formula for centripetal acceleration, and } \\
& T^{2}=\left(4 \pi r^{3}\right) /(G M) \text {, an equation relating orbital time of a satellite to the } \\
& \text { radius of its path and the mass of the body it orbits. }
\end{aligned}
$$

These equations and others may be discovered by organizing and analyzing experimental data. The following example leads you through the process of discovering a mathematical equation that describes experimental data. You will follow the same process in Experiment 3.

Suppose you are given a variety of solid cones made from a certain type of metal. You are asked to discover a formula that will let you calculate the mass of any cone of this metal if you know the diameter of the base. You are free to measure the mass and height and other dimensions of the cones you have been given. You should assume that you do not know any special mathematical formulas regarding cones.

## Experimental Data

First, you recognize that there are three variables involved: mass, diameter, and height. Since it is difficult to analyze data from experiments with more than two variables, you group the cones into two groups. In one group, all the cones have the same diameter. In the other group, they all have the same height. Two cones did not fit in either group. Measuring the cones gives the following results:

Group 1 - 2.0 cm diameter

| Height, h (cm) | Mass, m (g) |
| :---: | :---: |
| 3.0 | 85.6 |
| 4.0 | 114.1 |
| 5.0 | 142.7 |
| 6.0 | 171.2 |

Group $2 \mathbf{- 2 . 0} \mathbf{c m}$ height

| Diameter, d (cm) | Mass, m (g) |
| :---: | :---: |
| 2.0 | 57.1 |
| 3.0 | 128.4 |
| 4.0 | 228.3 |
| 5.0 | 356.7 |

## Cones not in either group

| Diameter, d (cm) | Height, h (cm) | Mass, m (g) |
| :---: | :---: | :---: |
| 1.0 | 4.0 | 28.5 |
| 1.0 | 6.0 | 42.8 |

## Data Analysis

Groups 1 and 2 each relate mass, which may be thought of as the dependent variable, to another variable that influences the mass.

Graphing the data from group 1, with mass on the vertical axis and height on the horizontal axis, we obtain a straight line that passes through the origin. This shows that the data is of the form $y=m x$, where $y$ is the variable on the vertical axis and $x$ is the variable on the horizontal axis.

Figure 7
Graph of mass vs height for group 1


The slope, m , is constant and could be determined to complete the equation. In this case, we do not need the slope. It is enough for us to see that $y$ is proportional to $x$, or, in this case, that

> mass is proportional to height.

Figure 8
Graph of mass vs diameter for group 2

Figure 9
Graph of mass vs diameter ${ }^{2}$ for group 2

When we graph the data from group 2, we do not get a straight line. Instead, the graph suggests an equation of the form $y=c x^{k}$, where $c$ and $k$ are constants. This hypothesis may be tested and the constants evaluated using any of the three methods described in Prelab 2.


The result of such an analysis reveals that $y$ is proportional to $x^{2}$, or in this case, that
mass is proportional to diameter squared.
Evaluating the constant, c , is not needed.


## An Important Theorem:

If variable $\mathbf{z}$ is proportional to variable $\mathbf{x}$, and variable $\mathbf{z}$ is also proportional to variable $\mathbf{y}$, then variable $\mathbf{z}$ is proportional to the product of the variables $\mathbf{x}$ and $\mathbf{y}$.

In symbols:

$$
z=c x y
$$

Applying this theorem to the cone example makes the concept clearer.

Mass is proportional to height, and mass is proportional to diameter squared, so mass is proportional to height times diameter squared.

In symbols:

$$
m=c h d^{2}
$$

where c is a constant of proportionality to be determined.

Solving for c gives

$$
c=m /\left(h d^{2}\right) .
$$

Substituting any correlated set of data from the original data set, such as $d=2.0$ $\mathrm{cm}, h=6.0 \mathrm{~cm}, m=171.2 \mathrm{~g}$ (corresponding to the last cone in group 1) gives

$$
\begin{aligned}
& \mathrm{c}=171.2 \mathrm{~g} /\left[(6.0 \mathrm{~cm})(2.0 \mathrm{~cm})^{2}\right] \\
& \mathrm{c}=7.13 \mathrm{~g} / \mathrm{cm}^{3}
\end{aligned}
$$

The final equation, relating the mass, diameter, and height of all cones made of this particular metal, is

$$
m=\left(7.13 \mathrm{~g} / \mathrm{cm}^{3}\right) h d^{2}
$$

## Experiment 3

 Relating Mass to Length and Diameter of Plastic CylindersFigure 10
Using the rectangular solids to increase accuracy when measuring the diameter of a cylinder

## Introduction

In this activity you are given eight black plastic cylinders of different diameters. You are asked to group the cylinders, take measurements, organize and graph the data, and create an equation relating the mass to the length and diameter of the cylinders. A minimum of instructions are given. You should study and follow the examples in the previous prelabs, titled "The Speed of Sound," "The Mass of Fluorite Octahedra," and "Relating Mass and Diameter of Metal Cones."

## Equipment

- Cylinders K-R
- Ruler or Caliper
- Balance or Scale


## Procedure (Group 1)

Place the cylinders in two groups. In each group, mass and only one other quantity should vary.

1. For one group of cylinders, create a table to record the diameter and mass of each. The length of each cylinder in this group should be the same. Record the diameter in centimeters. If you are using a metric ruler, estimate to the nearest 0.01 cm when finding these dimensions. You should use two rectangular objects with the ruler to increase your accuracy (see Figure 10). Use the rules regarding significant figures or another appropriate expression of uncertainty.


Top View
2. Consider the diameter to be the independent variable and the mass to be the dependent variable when graphing the data. Prepare a graph that shows the data for all four cylinders.
3. Draw a best-fit line for the data, which may be a smooth curve. If it is not possible to represent the data well with a smooth curve, recheck your measurements for any data points that do not fit the pattern.
4. State a hypothesis regarding the form of equation that is likely to best describe the data.
5. Determine an equation that represents the data. Use one or more of the methods outlined in the previous examples. Your instructor may tell you which method(s) to use. It is not necessary to evaluate the constant in the equation at this time.

## Procedure (Group 2)

1. For the other group of cylinders, create a table to record the length and mass of each. The diameter of each cylinder in this group should be the same. Record the length in centimeters. If you are using a metric ruler, estimate to the nearest 0.01 cm when finding these dimensions. Use the rules regarding significant figures or another appropriate expression of uncertainty.
2. Consider the length to be the independent variable, and the mass to be the dependent variable when graphing the data. Prepare a graph that shows the data for all four cylinders.
3. Draw a best-fit line for the data. If it is not possible to represent the data well with a smooth line, recheck your measurements for any data points that do not fit the pattern.
4. State a hypothesis regarding the form of equation that is likely to best describe the data.
5. Determine an equation that represents the data. Use one or more of the methods outlined in the previous examples. Your instructor may tell you which method(s) to use. It is not necessary to evaluate the constant in the equation at this time.
6. Combine the equations that you have developed for the two groups of cylinders. You may follow the example in Prelab 3. At this time you should evaluate the constant in the equation, including dimensional units.

## Analysis Verification

Check the accuracy of the equation you have developed by using the following data from published sources:

```
volume of a cylinder = \pir r
radius = diameter/2
density of the cylinder material = 1.41-1.49 g/cm
density = mass/volume
```

Algebraically combine this information to produce an equation giving the mass of these cylinders in terms of their diameters.

Compare this result with the equation you determined experimentally. Are they in agreement, taking into account uncertainty?

Discover Density Set

## Teacher's Notes

## Finding the equation that describes the data

From the experimental data, the following relationship exists: mass is proportional to diameter cubed

Therefore:

$$
\text { mass }=\mathrm{c} * \text { diameter }^{3}, \quad \text { where } \mathrm{c} \text { is some constant }
$$

Solving for c :

$$
\mathrm{c}=\text { mass } / \text { diameter }^{3}
$$

Substituting values from sphere S:

$$
c=2.4 \mathrm{~g} /(1.58 \mathrm{~cm})^{3}=0.608 \mathrm{~g} / \mathrm{cm}^{3}
$$

Rechecking with values from sphere $U$ :

$$
\mathrm{c}=6.8 \mathrm{~g} /(2.22 \mathrm{~cm})^{3}=\mathbf{0 . 6 2 2} \mathrm{g} / \mathrm{cm}^{3}
$$

Final equation for the spheres:

$$
\mathrm{c}=\mathbf{0 . 6 1 5} \mathrm{g} / \mathbf{c m}^{3} * \text { diameter }^{3}
$$

## Calculating the theoretical value using known equations

The following formulas and values are known:

$$
\begin{aligned}
& \text { mass }=\text { density * volume } \\
& \text { volume }=4 / 3 \pi^{*} \text { radius }^{3} \\
& \text { radius }=\text { diameter } / 2 \\
& \text { density of acrylic }=1.18 \mathrm{~g} / \mathrm{cm}^{3}
\end{aligned}
$$

Then,

$$
\begin{aligned}
& \text { mass }=\text { density } * 4 / 3 * \pi *(\text { diameter } / 2)^{3} \\
& \text { mass }=1.18 \mathrm{~g} / \mathrm{cm}^{3} * 4 / 3 * \pi * \text { diameter }^{3} / 8 \\
& \text { mass }=\mathbf{0 . 6 1 8} \mathbf{g} / \mathbf{c m}^{\mathbf{3}}{ }^{*} \text { diameter }^{3}
\end{aligned}
$$

This result is in agreement with the experimentally determined equations, considering uncertainty due to measurement error.

## Teacher's Notes

## Finding the equation that describes the data

From the experimental data, the following relationships exist: mass is proportional to diameter squared mass is proportional to length

Therefore:
mass is proportional to (diameter ${ }^{2}$ * length) mass $=\mathrm{c}$ * diameter ${ }^{2}$ * length, where c is some constant

Solving for c :

$$
\left.\mathrm{c}=\text { mass } / \text { (diameter }{ }^{2} \text { * length }\right)
$$

Substituting values from cylinder L:

$$
c=11.6 \mathrm{~g} /\left[(1.91 \mathrm{~cm})^{2} * 2.86 \mathrm{~cm}\right]=1.11 \mathrm{~g} / \mathrm{cm}^{3}
$$

Rechecking with values from cylinder R:

$$
c=14.1 \mathrm{~g} /\left[(2.22 \mathrm{~cm})^{2} * 2.54 \mathrm{~cm}\right]=1.13 \mathrm{~g} / \mathrm{cm}^{3}
$$

Final equation for the cylinders:

$$
\text { mass }=\mathbf{1 . 1 2 ~ g / c m}{ }^{\mathbf{3}} \text { * diameter }{ }^{2} \text { * length }
$$

## Calculating the theoretical value using known equations

The following formulas and values are known:

$$
\begin{aligned}
& \text { mass }=\text { density * volume } \\
& \text { volume }=\pi * \text { radius }{ }^{2} \text { * length } \\
& \text { radius }=\text { diameter } / 2 \\
& \text { density of plastic }=1.42 \mathrm{~g} / \mathrm{cm}^{3}
\end{aligned}
$$

Then,

$$
\begin{aligned}
& \text { mass }=\text { density } * \pi^{*}(\text { diameter } / 2)^{2} * \text { length } \\
& \text { mass }=1.42 \mathrm{~g} / \mathrm{cm}^{3} * \pi^{*} \text { diameter }{ }^{2} / 4 * \text { length } \\
& \text { mass }=1.12 \mathrm{~g} / \mathrm{cm}^{3}{ }^{*} \text { diameter }{ }^{2} \text { length }
\end{aligned}
$$

This result is in agreement with the experimentally determined equations, considering uncertainty due to measurement error.

Notes

## Specifications

| Material |  | Mass | Dimensions | Volume | Density |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Aluminum | A | $16.1 \pm 0.3 \mathrm{~g}$ | $1.26 \times 1.89 \times 2.51 \pm 0.04 \mathrm{~cm}$ | $5.99 \pm 0.10 \mathrm{~cm}^{3}$ | $2.69 \pm 0.01 \mathrm{~g} / \mathrm{cm}^{3}$ |
|  | B | $26.0 \pm 0.2 \mathrm{~g}$ | $1.26 \times 1.89 \times 4.06 \pm 0.03 \mathrm{~cm}$ | $9.68 \pm 0.06 \mathrm{~cm}^{3}$ |  |
|  | C | $38.2 \pm 0.1 \mathrm{~g}$ | $1.26 \times 1.89 \times 5.98 \pm 0.02 \mathrm{~cm}$ | $14.22 \pm 0.05 \mathrm{~cm}^{3}$ |  |
|  | D | $34.7 \pm 0.2 \mathrm{~g}$ | $1.91 \times 1.91 \times 3.56 \pm 0.02 \mathrm{~cm}$ | $12.92 \pm 0.06 \mathrm{~cm}^{3}$ |  |
| Gray Plastic | E | $8.8 \pm 0.2 \mathrm{~g}$ | $1.65 \times 1.65 \times 2.35 \pm 0.03 \mathrm{~cm}$ | $6.46 \pm 0.19 \mathrm{~cm}^{3}$ | $1.37 \pm 0.01 \mathrm{~g} / \mathrm{cm}^{3}$ |
|  | F | $16.3 \pm 0.6 \mathrm{~g}$ | $1.65 \times 1.65 \times 4.44 \pm 0.03 \mathrm{~cm}$ | $11.98 \pm 0.48 \mathrm{~cm}^{3}$ |  |
|  | G | $23.8 \pm 0.7 \mathrm{~g}$ | $1.65 \times 1.65 \times 6.38 \pm 0.03 \mathrm{~cm}$ | $17.45 \pm 0.51 \mathrm{~cm}^{3}$ |  |
|  | H | $29.0 \pm 0.3 \mathrm{~g}$ | $1.94 \times 1.94 \times 5.57 \pm 0.02 \mathrm{~cm}$ | $21.01 \pm 0.26 \mathrm{~cm}^{3}$ |  |
| Clear Plastic | I | $5.1 \pm 0.1 \mathrm{~g}$ | $1.22 \times 1.22 \times 2.86 \pm 0.02 \mathrm{~cm}$ | $4.26 \pm 0.05 \mathrm{~cm}^{3}$ | $1.20 \pm 0.01 \mathrm{~g} / \mathrm{cm}^{3}$ |
| Black Plastic | J | $11.3 \pm 0.2 \mathrm{~g}$ | $1.72 \times 1.95 \times 2.40 \pm 0.02 \mathrm{~cm}$ | $8.01 \pm 0.10 \mathrm{~cm}^{3}$ | $1.41 \pm 0.01 \mathrm{~g} / \mathrm{cm}^{3}$ |
| Black Plastic (cylinders) | K | $8.0 \pm 0.1 \mathrm{~g}$ | $\varnothing 1.59 \times 2.87 \pm 0.03 \mathrm{~cm}$ | $5.65 \pm 0.02 \mathrm{~cm}^{3}$ | $1.42 \pm 0.01 \mathrm{~g} / \mathrm{cm}^{3}$ |
|  | L | $11.6 \pm 0.1 \mathrm{~g}$ | $\varnothing 1.90 \times 2.87 \pm 0.03 \mathrm{~cm}$ | $8.19 \pm 0.03 \mathrm{~cm}^{3}$ |  |
|  | M | $16.0 \pm 0.2 \mathrm{~g}$ | $\varnothing 2.22 \times 2.87 \pm 0.03 \mathrm{~cm}$ | $11.27 \pm 0.10 \mathrm{~cm}^{3}$ |  |
|  | N | $20.2 \pm 0.2 \mathrm{~g}$ | $\varnothing 2.55 \times 2.87 \pm 0.03 \mathrm{~cm}$ | $14.43 \pm 0.15 \mathrm{~cm}^{3}$ |  |
|  | 0 | $5.6 \pm 0.1 \mathrm{~g}$ | $\varnothing 2.22 \times 1.02 \pm 0.01 \mathrm{~cm}$ | $3.95 \pm 0.06 \mathrm{~cm}^{3}$ |  |
|  | P | $8.3 \pm 0.1 \mathrm{~g}$ | $\varnothing 2.22 \times 1.51 \pm 0.02 \mathrm{~cm}$ | $5.87 \pm 0.09 \mathrm{~cm}^{3}$ |  |
|  | Q | $11.2 \pm 0.2 \mathrm{~g}$ | $\varnothing 2.22 \times 2.04 \pm 0.03 \mathrm{~cm}$ | $7.92 \pm 0.14 \mathrm{~cm}^{3}$ |  |
|  | R | $14.1 \pm 0.2 \mathrm{~g}$ | $\varnothing 2.22 \times 2.54 \pm 0.03 \mathrm{~cm}$ | $9.85 \pm 0.11 \mathrm{~cm}^{3}$ |  |
| Clear Plastic (spheres) | S | $2.4 \pm 0.1 \mathrm{~g}$ | $\varnothing 1.58 \pm 0.01 \mathrm{~cm}$ | $2.08 \pm 0.02 \mathrm{~cm}^{3}$ | $1.18 \pm 0.01 \mathrm{~g} / \mathrm{cm}^{3}$ |
|  | T | $4.3 \pm 0.1 \mathrm{~g}$ | $\varnothing 1.90 \pm 0.01 \mathrm{~cm}$ | $3.61 \pm 0.03 \mathrm{~cm}^{3}$ |  |
|  | U | $6.8 \pm 0.1 \mathrm{~g}$ | $\varnothing 2.22 \pm 0.01 \mathrm{~cm}$ | $5.74 \pm 0.02 \mathrm{~cm}^{3}$ |  |
|  | V | $10.1 \pm 0.1 \mathrm{~g}$ | $\varnothing 2.53 \pm 0.01 \mathrm{~cm}$ | $8.51 \pm 0.07 \mathrm{~cm}^{3}$ |  |

Specifications obtained by averaging measurements from 10 samples of each part. Uncertainties reflect one standard deviation of sampled values. Some uncertainties rounded up to match place value of other uncertainties.


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