General Physics Lab 1

Uniform Motion: Terminal Velocity

Objectives:

- Film objects moving with constant velocity
- Learn to apply video analysis to motion
- Observe the distance-time relation for motion at constant velocity
- Show that for macroscopic objects moving through air at everyday speeds, the drag force is in opposition to the velocity and directly proportional to the square of the speed.

Equipment:

- 5 Coffee Filters
- Metric Measuring Tape or Meter Stick
- Triangular Wood Block
- 2 Rubber Bands
- Smartphone (camera) or Webcam
- Tracker Video Analysis software
  available for free download at https://physlets.org/tracker/
- Graphical Analysis software
  available for free download at https://www.vernier.com/graphical-analysis

Physical Principles:

Air Resistance and Terminal Velocity

In this experiment, you will be dropping coffee filters and analyzing their motion. When an object is in freefall with air resistance, there are two forces acting on it. Gravity pulls it downward and the air-resistance (drag force) opposes the motion increasing with the square of the speed of the object.

$F_{drag} = bv^2$ (direction of drag force vector opposite direction of velocity vector, $\vec{v}$) (1)

Fig. 1 shows a sequence of free-body diagrams of the object from the time it first begins to fall until it reaches terminal velocity.

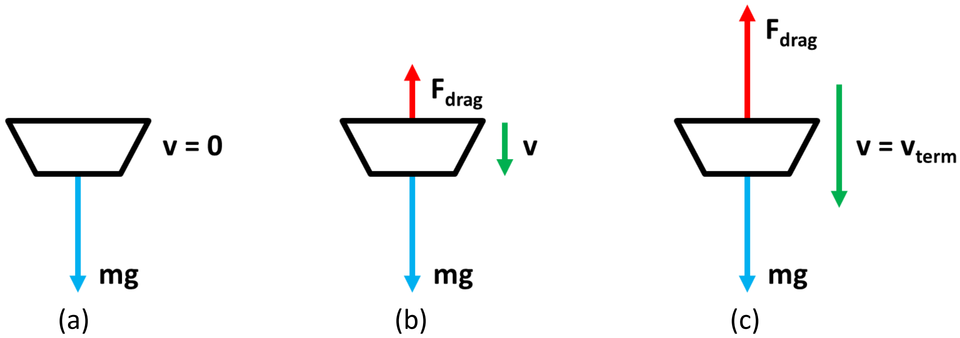


Fig. 1: Free-body diagrams of a coffee filter in free fall with air resistance as a drag force. The drag force increases with the increasing velocity, but in opposition to the motion.
(a) The coffee filter has just been released, but has no velocity yet – no air resistance acts.
(b) The coffee filter has picked up speed due to the pull of gravity and the drag force is beginning to oppose the velocity.
(c) The drag force is equal but opposite to gravity and the coffee filter falls with the constant terminal velocity.

The falling object initially accelerates downward ($a_y = -g$), but that acceleration rate is constantly reduced as air resistance grows until ultimately air resistance cancels the weight of the object and the object falls at a uniform speed equal to the terminal velocity.

A plot of displacement vs. time for an object falling from rest under the influence of both gravity and air resistance appears approximately as shown in Fig. 2a. A plot of velocity vs. time is shown in Fig. 2b.

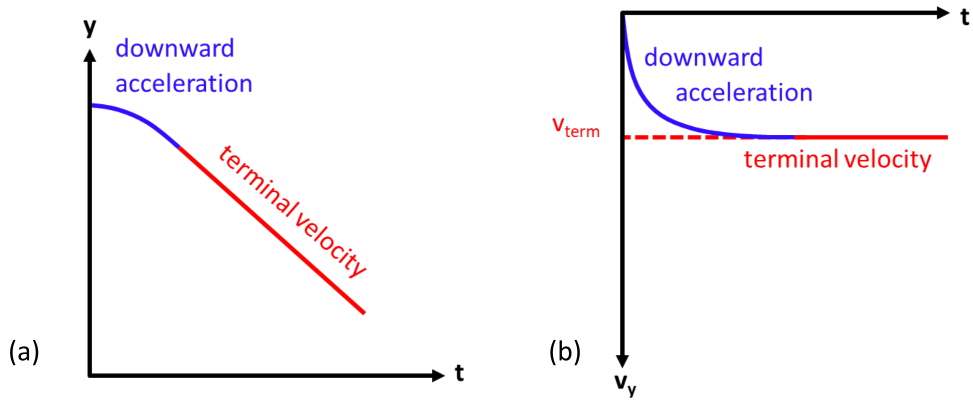


Fig. 2: (a) Displacement vs. time for an object falling with air resistance approaches a constant negative slope equal to the terminal velocity. (b) Velocity vs. time approaches constant terminal velocity.

A macroscopic object (like a coffee filter) traveling quickly (m/s) through a relatively non-viscous fluid (air) experiences a turbulent drag force proportional to the square of the speed as in Eq. (1). A plot of F vs. speed, v, will appear as in Fig. 3a. This relationship can be "linearized" (straight line) if one plots F vs. v^2 as in Fig. 3b.

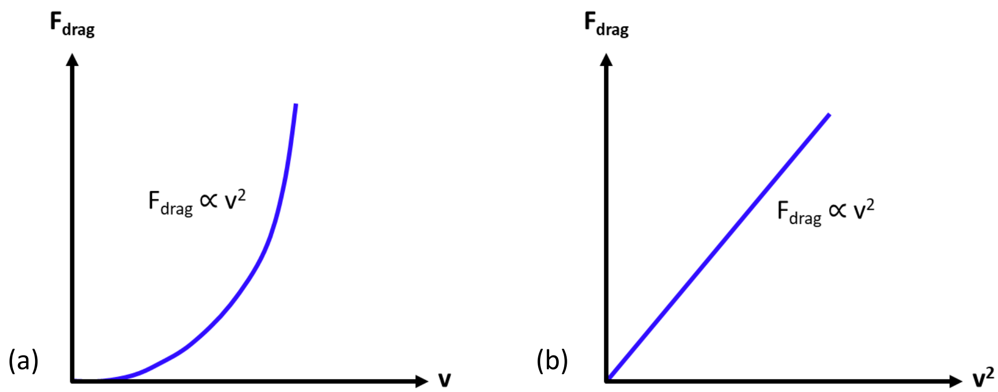


Fig. 3: (a) A plot of drag force vs. speed, v, is a second-order power law relation and curves upward. (b) A plot of drag force vs. v^2 is linear – straight line.

Life-science students should note that a microscopic object (like a paramecium/cell) traveling slowly (μm/s) in a viscous fluid (water/blood) experiences a smooth, laminar drag force directly proportional to just the speed.

$$F_{drag\ micro} = bv \qquad \text{(direction opposing motion)} \qquad (2)$$

Procedure:

Mini-Experiment/Demonstration – Surface Area and Terminal Velocity

Note: This is just a quick demonstration of the effect surface area has on terminal velocity. You will **not** be recording any data or writing about this. It is simply an interesting and easy way to see the effect that surface area has on a falling object. Do not spend long on this part as the main experiment will take much longer to complete.

Drag force (and therefore terminal velocity) is proportional to the surface area of a falling object. A larger surface area means more fluid particles can collide with the object. More collisions results in a larger drag force that will slow the object faster and yield a lower terminal velocity. For example, the terminal velocity of a skydiver without an open parachute is about 120 mph in the spread-eagle position and 180 mph in a tuck. A skydiver with an open parachute descends with a terminal velocity closer to 15 mph.

You can demonstrate this effect by comparing the fall time for 2 coffee filters.

1. Find one coffee filter that is very cylindrical. You could even squish it in to make it more cylindrical. Set this one aside.
2. Take a second filter and flatten it some so that its diameter is larger than the first one.
3. Hold both up above your head at the same height and drop them "nose first" at the same time.

You should see that the flattened coffee filter takes longer to hit the ground. The flattened filter falls slower because it has a larger surface area and a lower terminal velocity.

Setup

1. Find a dark (or at least not-too-bright) wall for a backdrop with sufficient lighting around to provide good contrast with the white coffee filters when filmed in motion. You will also want the background to have a clear, horizontal floor line and a clear vertical line from trim, doorjamb, etc.
2. Stand up your phone using the wood block and rubber bands as shown in Fig. 4 below. In this configuration, you can use either camera – front or back. Vertical orientation is best for this experiment. Refer to the Smartphone Stand Instructions document for more information.

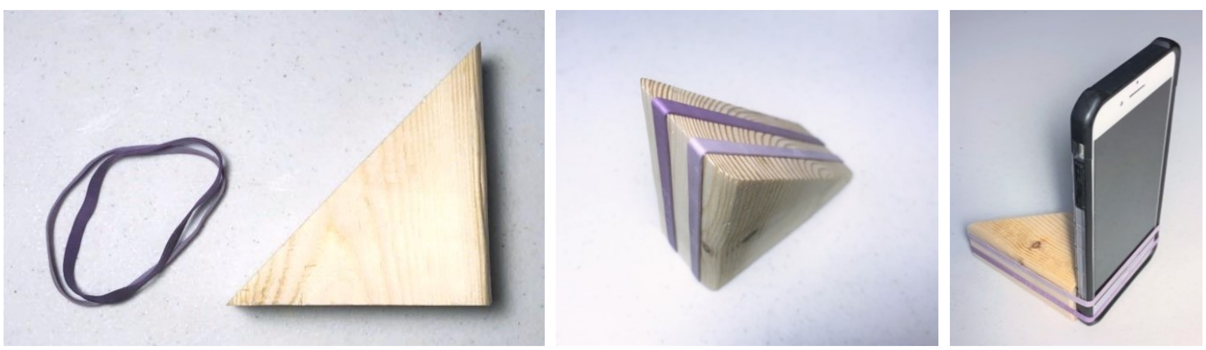
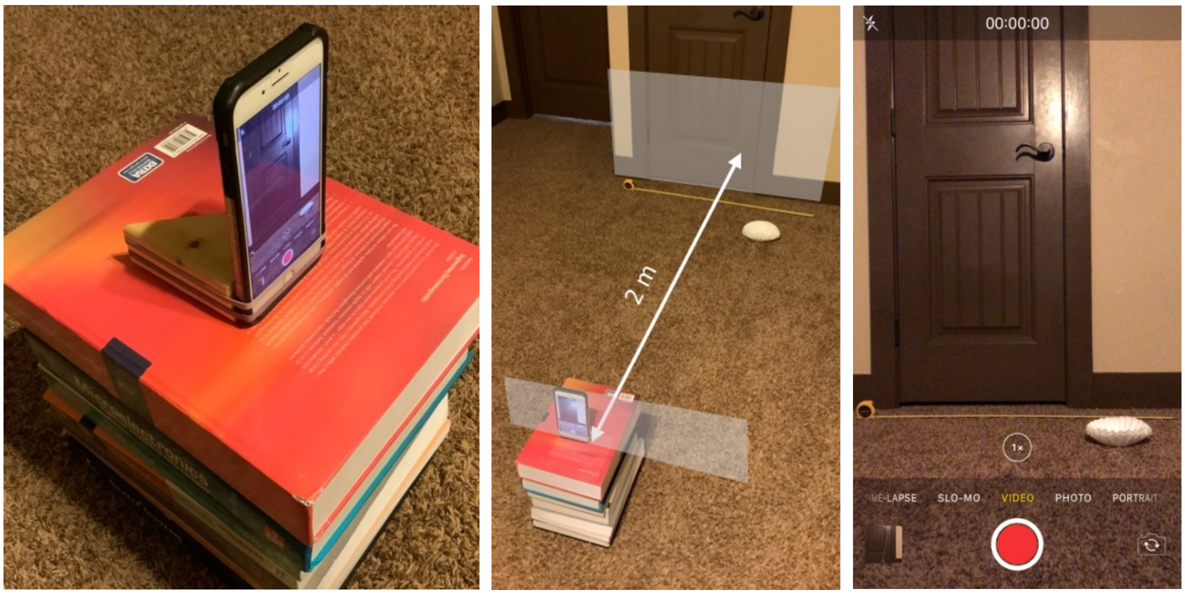


Fig. 4: Create a simple phone stand using the wood block and 2 rubber bands.
Orient your phone vertically as shown.

3. Set the phone (or webcam) on a stable raised surface (such as a stack of books) near the ground with the camera 2 meters from the wall and facing straight at the wall. Depending on the lens in your camera, you may need to place the camera closer or farther away to capture everything. In this position, the camera needs to have a clear view of the floor and the wall as shown in Fig. 5c. Make sure the camera is facing straight at the wall such that the plane of the phone is parallel to the plane of the wall (Fig. 5b). If it is angled relative to the wall, the measurements will not be as accurate.

   Alternatively, you can move the camera out even farther from the wall so that the entire path of the coffee filters (floor up to about 2 meters) is visible to the camera. This would allow you to track the entire path of each stack of coffee filters, but doing so takes more time than may be available during a lab period. Because of this, we recommend tracking only the last half-meter to a meter of the falling coffee filters.

4. Extend the metric measuring tape to exactly 1 meter and lock it in place. Then lean it up against the wall or lay it on the floor below the path of the falling coffee filters so that you can see both ends of the 1 meter length (this is important for properly scaling distances in Tracker). See Fig. 5 for this setup. Alternatively, you could use a meter stick in place of the measuring tape.



(a) (b) (c)

Fig. 5: Smartphone with wood block stand on a stack of books facing the wall. The camera is framed so that both ends of the measuring tape are visible and the last meter of the coffee filters' fall (from the floor to 1m up the wall) is in the center of the image. This dark colored door works well because the white coffee filters will be clearly visible in front of it. The right angles of the door and trim make it easy to align the coordinate axes in Tracker.

5. Semi-flatten all the coffee filters together as you did to the one filter in the mini-experiment. This will increase the surface area and the drag force allowing the coffee filters to reach terminal velocity sooner. In order for the experiment to work, the coffee filters must have time to reach terminal velocity before they hit the floor. Increasing their surface area and dropping from high up will make this possible.
6. **Turn off any air conditioning, central heating, or fans and close windows that may cause disruptive air currents. These can negatively affect the experiment accuracy.**

Important Note to iPhone/iPad Users

Check your settings to make sure the camera is recording in "Most Compatible" format instead of "High Efficiency". Tracker does not accept the high efficiency format so you will need to switch formats before recording (easy) or convert the video later (not easy). You can change the format by going to the Settings App > Camera > Formats > Most Compatible.

Video Capture

For this next part, we recommend filming each set of coffee filters in separate clips. Tracker does not work well with long videos so it is best to record a separate video for each stack of filters. Also, try to keep the videos short. If they get too long, you may want to trim them later.

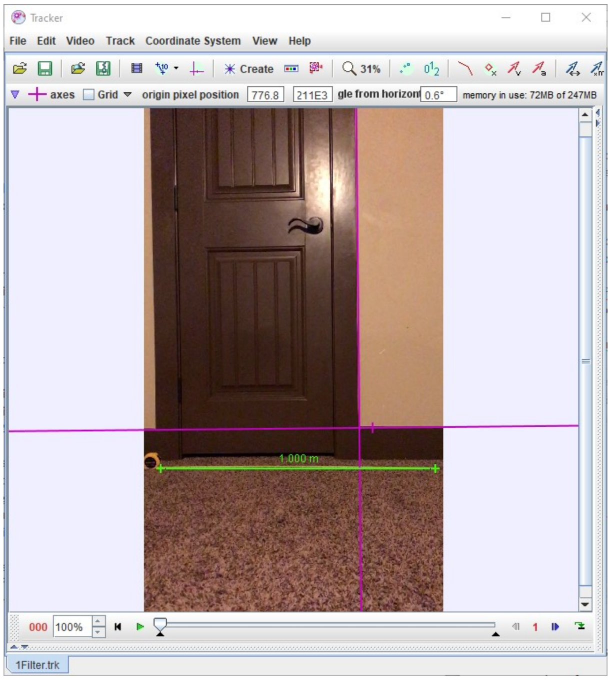
1. Start a video recording (ideally 30 fps frame rate) and hold up 1 finger in front of the camera to show you are dropping 1 filter. When you drop 2 filters, hold up 2 fingers, and so on. This is very useful later if the video clips get mixed up.
2. Raise a single filter as high up as you can reach and drop it “nose first” to the ground. It should fall reasonably straight down and land close to the wall and measuring tape. Try to drop it from about 2 meters up if you can, or stand on a chair if you can’t reach high enough. This is VERY IMPORTANT. If you do not drop it from high enough, it will not have time to reach terminal velocity and the experiment will not work.
3. Once it hits the ground, stop the video. Double check that you can see both ends of the measuring tape and where the coffee filter(s) land.
4. Stack a second coffee filter on top of the first and drop the stack as before. Continue dropping stacks of 3, 4, and 5 coffee filters as described in steps 1-3. Each time, make sure to drop the stack from 2 meters up and make sure it lands close to the measuring tape. As you increase the number of filters in the stack, you will notice that it falls faster each time. This is because as you add more filters, the weight (force due to gravity) increases.
5. Transfer the video clips to your computer via some means (email, Google Drive, iCloud, Dropbox, etc.) for analysis using Tracker Video Analysis software.

Tracking the Motion

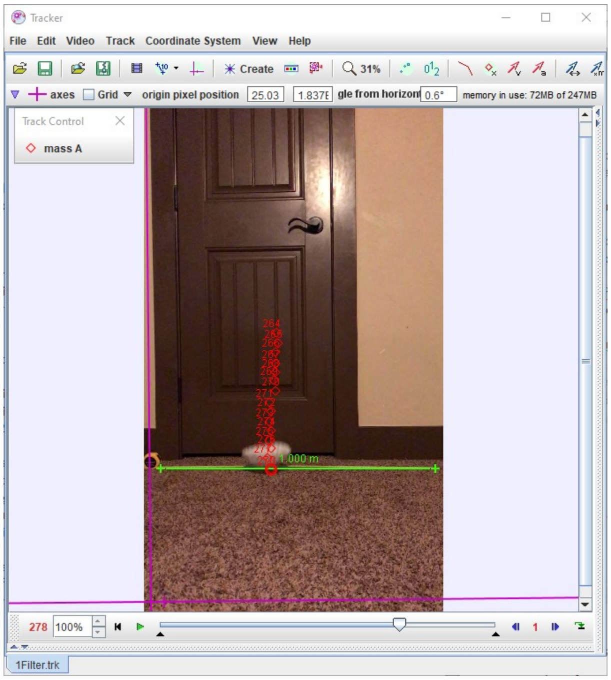
The following steps apply to each video of coffee filters falling. Repeat the steps for each video to collect the tracking data.

1. Open the first video file in Tracker
2. Save the file with an appropriate name to represent the number of filters (ex. “1Filter”, “2Filters”, etc.).
3. If your video was filmed vertically, click Video > Filters > New > Rotate to make the video upright.
4. Using the Calibration Tools, add a new Calibration Stick, align the calibration stick on the tape portion of the tape measure (Shift+Click on the end points), and set the length to 1 meter.
5. Click on the Coordinate Axes and place them on the video. Line up the origin with the intersection of two perpendicular lines in your image (ex. corner of a door) and rotate the axes so that the x and y axes match the angle of the floor and doorframe respectively (see Fig. 6a). You can then leave the axes there or move them off to the side (a good place might be the bottom left corner).
6. Click Create > Point Mass.

7. Slide through video to find the frame where the coffee filters touch down on the ground. Then use the step buttons to step back 15-20 frames.
8. Shift+Click on the bottom edge of the coffee filter stack to mark its position. This automatically advances the video to the next frame where you can Shift+Click again on the bottom edge of the stack. Continue doing this until the stack touches the ground (don't mark that frame). This will give you 15-20 points marked.



(a)



(b)

(c)

Fig. 6: Use Tracker to follow the motion of the falling coffee filters.
(a) Align the calibration stick to the tape portion of the measuring tape and align the coordinate axes to the vertical and horizontal lines in the image.
(b) Track the bottom edge of the falling coffee filters until it reaches the floor.
(c) Example tracking mark aligned to the bottom edge of a coffee filter.

Export the Data

After tracking the positions of each stack of coffee filters, complete the following steps for each video to export the data:

1. Click Table and select only y and vy. You will only be using vy for the analysis, but y is useful for troubleshooting, so include that just in case.
2. Option 1 – Copy and paste the data
    a. Select all the data in the table
    b. Right click > Copy Selected Cells > Full Precision
    c. Paste the data into Google Sheets or Excel

   Option 2 – Export a CSV file with the velocity data
    a. Click File > Export > Data File and set the following parameters:
       Cells = All Cells, Number Format = Full Precision, Delimiter = Comma
    b. Click Save As and name the file as “1Filter.csv”, “2Filters.csv”, etc.
    c. Using Google Sheets or Excel, import each CSV file.

Analysis:

Extracting Terminal Velocity from Data

If you were to track the entire path of the falling coffee filters, from the time they are released until they hit the ground, you would see graphs similar to Fig. 2. In order to simplify things, we only asked you to track the filters close to the ground where they should have already reached terminal velocity. At terminal velocity, the position will decrease linearly and the velocity should be roughly constant (see Fig. 7). By averaging these final velocity points, you will be able to find the terminal velocity.

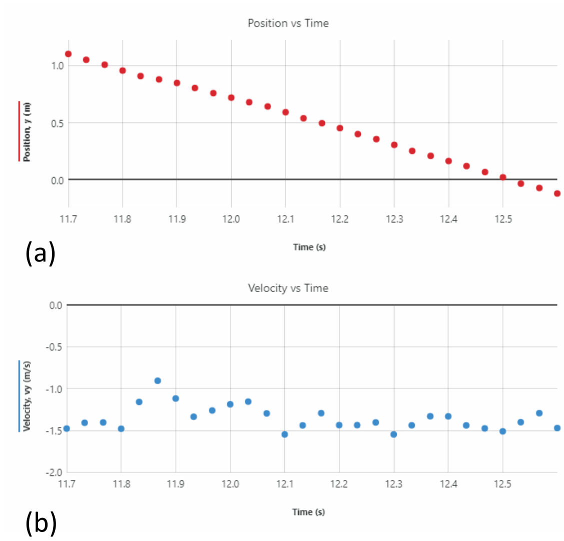


Fig. 7: (a) Position vs time and (b) velocity vs time graph for a coffee filter falling at terminal velocity. Position falls linearly and velocity becomes constant.

1. In Google Sheets or Excel, generate a plot of velocity vs time to see if the data is still trending downward (Fig. 2b) or if it has leveled out to its terminal velocity (Fig. 7b).
2. Calculate the average of the velocity points where the velocity seems roughly constant.
    a. If the data randomly fluctuates around a central value (Fig. 7b), then it is probably at terminal velocity and you can average all of those points.
    b. If some of the data looks messy (left half of Fig. 7b), you should exclude this from the velocity average.
    c. If the data trends downward and then levels out (Fig. 2b), only average the data that is flat (constant).
    d. If the data never flattens out, you may want to redo that video.

3. To calculate the average final velocity (terminal velocity), you can follow these steps:
    a. In a cell, type =AVERAGE(
    b. Then click and drag to select the cells you want to average
    c. Close the parentheses so that you have something like this =AVERAGE(C11:C20)

4. Repeat the last two steps for each set of data (1 filter, 2 filters, etc.)
5. Make a new tab in your spreadsheet document to record the combined results from each data set.
6. Make 2 columns for # of filters and terminal speed, $|v_{term}|$ (absolute value of terminal velocity). You can calculate absolute value with the formula =ABS( ).
7. Copy the values in for # of Filters and $|v_{term}|$. Note that the graph analysis will not work properly if you use v_{term} instead of $|v_{term}|$.
8. Make a new column for velocity squared. You can calculate this in your spreadsheet by typing for example =B2^2 where B2 is the cell you want to square.
9. Copy and paste the data for # of filters, $|v_{term}|$, and v_{term}^2 into the provided table in your eJournal.

Graphing Results

One of the stated objectives of this lab is to graphically verify the relationship of Eq. 1. Terminal speeds were determined as described above. It is difficult to measure the drag force directly, but we can use the fact that at terminal velocity, the drag force equals the weight of the filters.

$$F_{drag} = Weight_{filters} \quad (3)$$

where the weight of the filters is directly proportional to the number of filters in the stack.

Verify Eq. 1 by plotting:

a) # Filters vs. Terminal Speed (an expected power law graph – Fig. 3a)
b) # Filters vs. Terminal Speed Squared (an expected linear graph – Fig. 3b)

Generate these graphs by following these steps.

1. Use **Graphical Analysis** to plot # Filters (y-axis) vs. terminal speed, $|v_{term}|$ (x-axis). Include the point (0,0) on your graph. This represents the fact that if there is no weight, the object would not be pulled down and would have zero velocity.
2. Add a trendline with a power law fit ($y = ax^b$) and display the fit equation on your graph. Note that **Graphical Analysis** seems to handle power law fits better than Google Sheets or Excel so it is best if you use that for making this graph.
    a. Please note: a power fit is **<u>not</u>** the same as a quadratic or polynomial fit.

3. Compare the exponent with 2 (as expected for F_{drag} proportional to v^2) using a percent difference. If the exponent is very far off, try excluding the point for 5 filters from the power fit. Sometimes this point is less accurate than the others and must be left out.

$$\%Diff = \frac{|exponent - 2|}{2} \times 100\%$$

4. Plot # Filters vs. v^2. Include the point (0,0).
5. Add a linear fit trendline (y = mx + b) with the fit equation displayed on the graph.
6. Record the correlation coefficient, R, of your linear fit and discuss briefly how well your data correlates to a straight line.
   Note that R = +1 indicates a perfect positive correlation between the data and the fit line, R = -1 indicates a perfect negative correlation, and R = 0 indicates no correlation between the data and the fit line.
7. Insert images of both graphs into your eJournal.