General Physics Lab 11

Simple Harmonic Motion

Objectives:

- To observe acceleration as a function of time for a simple harmonic oscillator.
- To study the relationship between period and physical parameters for a simple harmonic oscillator.

Equipment:

Option 1 – Spring

- Smartphone with Phyphox App
- Duct Tape
- Spring
- Measuring Tape
- Spring Balance
- 1 Qt. Ziploc Bag (with twine) or lightweight shoe to hold phone, and weights
- Improvised Weights (each individually less than spring balance max)
    - 100 Pennies
    - Hot Wheels Cars
    - Steel Hex Nut
    - Wood Block
    - Measuring Tape
    - Etc.

Option 2 – Pendulum

- Smartphone with Phyphox App
- Duct Tape
- Twine
- 1 Qt. Ziploc Bag
- Measuring Tape
- Ruler

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

Physical Principles:

Spring

Hooke's Law states that the restoring force exerted by a spring when stretched a distance, x, from its equilibrium is linear with the distance, i.e.,

$$F = -kx \tag{1}$$

where k is the spring constant.

Such linear, Hooke's Law behavior characterizes all simple harmonic oscillators. The period of a mass, m, attached to the spring oscillates with a period T.

$$T = 2\pi\sqrt{\frac{m_{eff}}{k}} \tag{2}$$

The frequency is therefore given by,

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eff}}}\ . \tag{3}$$

Since the spring itself has mass with its own inertia, affecting the period, we have approximately included its effects with the following correction yielding the effective mass, $m_{eff.}$

$$m_{eff} = m + \frac{m_{spring}}{3} \tag{4}$$

The angular frequency, ω, is defined as,

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{(rad/s)}, \tag{5}$$

hence,

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(rad/s)}. \tag{6}$$

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

The displacement, x, of a harmonic oscillator is given by,

$$x(t) = A\sin(\omega t + \varphi_0) \ , \tag{7}$$

where A is the amplitude of the motion and φ_0 is the initial phase.

Using the previous equations, we can also show that the acceleration follows the same periodic pattern as the displacement.

$$a(t) = \frac{F(t)}{m} = -\frac{k}{m}x(t)$$

$$= -\omega^2 A \sin(\omega t + \varphi_0) \ . \tag{8}$$

This equation allows us to use an acceleration sensor (accelerometer) to measure the periodic motion of a spring and other simple harmonic oscillators, such as a pendulum.

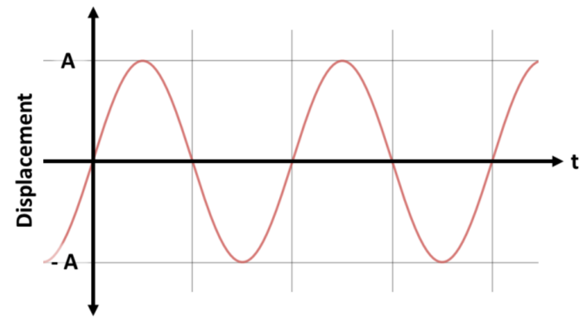

Fig. 1: Example graph of displacement vs. time

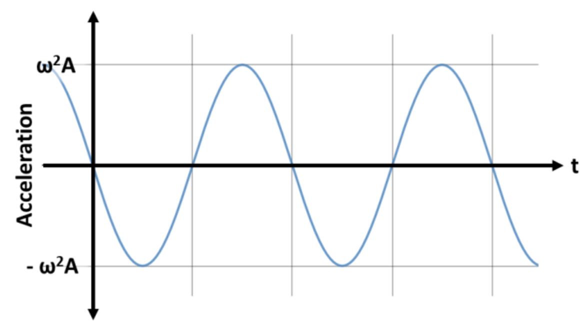

Fig. 2: Example graph of acceleration vs. time

Simple Pendulum

The simple pendulum is a point mass suspended from a long massless string. The restoring force when displaced to a small angle, θ, will be given by

$$F = -mg\sin(\theta) \approx -mg\theta \approx -mg\frac{x}{L} \ . \tag{9}$$

The form of Eq. (9) is similar to Hooke's Law for the spring. The period of the pendulum is thus similar to Eq. (2) but with k replaced by mg/L, that is,

$$T = 2\pi\sqrt{\frac{L}{g}} \ . \tag{10}$$

This expression holds true only for small amplitude oscillations.

Note: You may choose to study either the Spring (Option 1) or the Pendulum (Option 2). You are not required to do both experiments.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

Procedure (Option 1):
Spring

Setup

1. Find a “carrier”, such as a plastic bag or shoe, to hang from the spring and hold the various weights (smartphone, 100 pennies, Hot Wheels cars, wood block, etc.) (see Fig. 3). The mass of your carrier needs to be less than the spring balance maximum so that you can weigh it on the spring balance.

   Suggested Carriers:
   a. Lightweight Shoe (hang by the laces)
   b. Ziploc Bag with Twine Handle (from Lab 10)
   c. Plastic Shopping Bag (suspend high enough to accommodate the size of the bag)

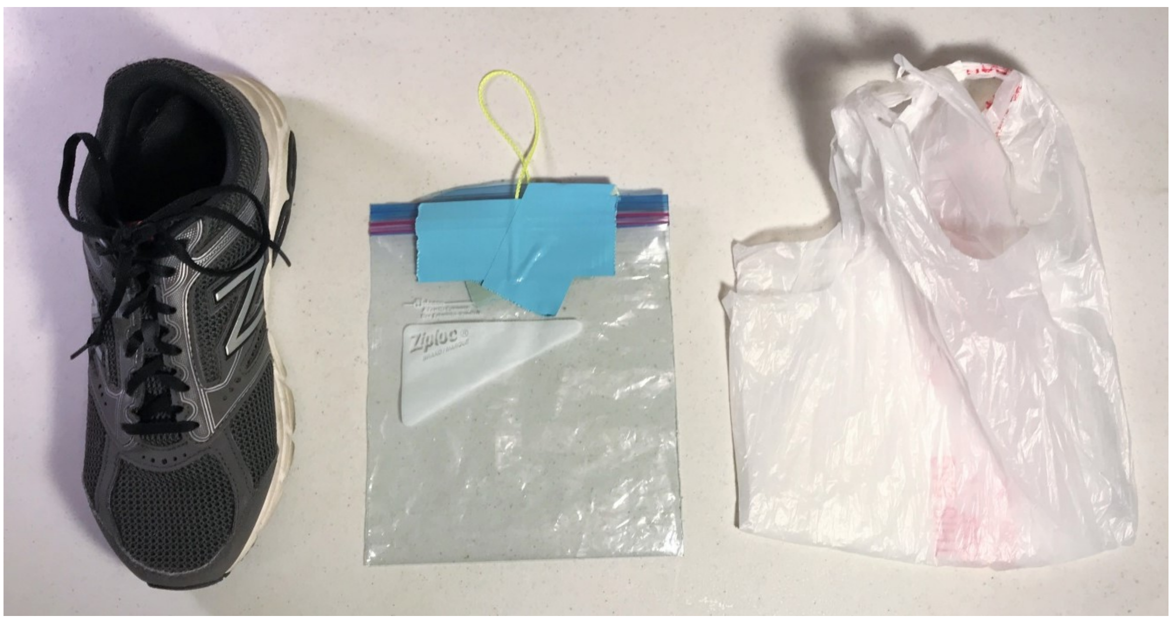

Fig. 3: Possible carriers to hang weights from the spring (shoe, bag from lab 10, shopping bag)

2. Next, find various weights (from the lab kit or otherwise) that will fit in the carrier, individually weigh less than the spring balance maximum, and together with the carrier and your smartphone, add up to around 0.8 – 1kg (see Fig. 4).

   Suggested Weights from your Lab Kit:
   a. 100 Pennies
   b. Hot Wheels Cars
   c. Steel Hex Nut
   d. Wood Block
   e. Measuring Tape

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

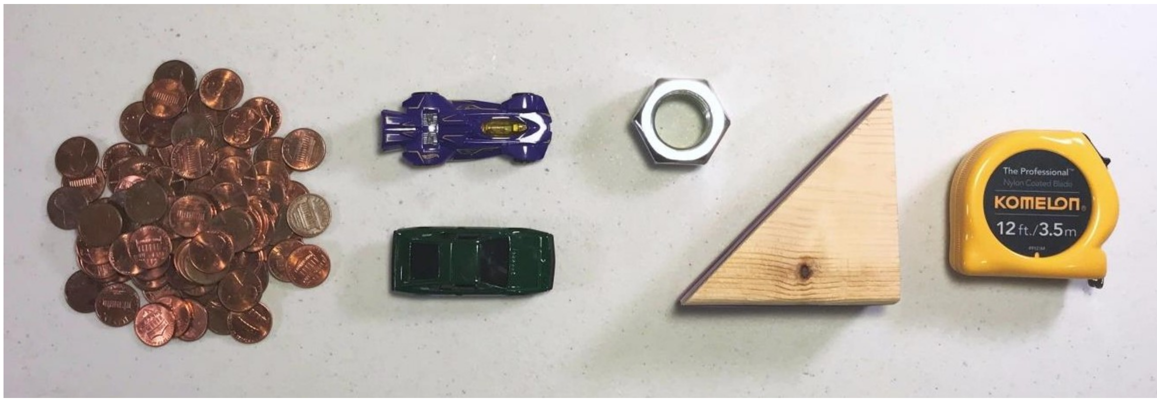

Fig. 4: Possible weights to hang from the spring (100 pennies, Hot Wheels cars, Hex Nut, Wood Block, Measuring Tape).

3. Use the spring balance to measure the individual masses of the carrier, your smartphone, and each of the weights you chose (see suggested list above). Use as many weights as needed, such that the total mass (phone, carrier, & weights) adds up to the recommended 0.8 – 1kg. Remember that each individual mass must be less than the spring balance maximum and the sum of all the masses must add up to around 0.8 – 1kg. Record the individual masses and the total mass in kg.

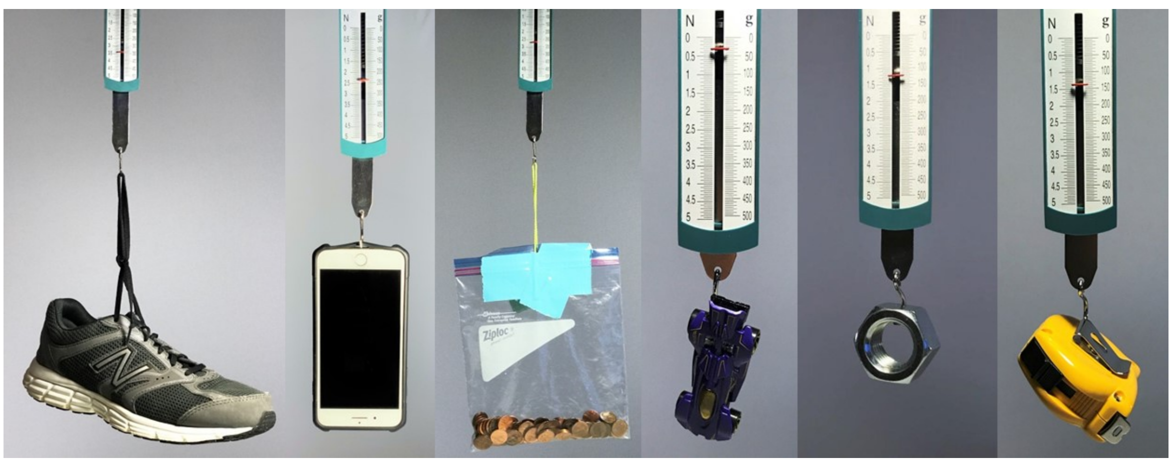

Fig. 5: Weigh the carrier (bag or shoe), your smartphone, and each of the weights you selected. The total mass of everything (carrier + phone + weights) should be around 0.8 – 1kg.

4. Use the spring balance to measure the mass of the spring. Record the mass in kg (see Fig. 6).
5. Suspend the spring vertically from a stable object. There are many ways to do this, but one option is to tape it to the edge of a table as shown in Fig. 7. Make sure that it will not slide down as you use it or your position measurements will be inaccurate.

Fig. 6: Weigh the spring on the spring balance.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

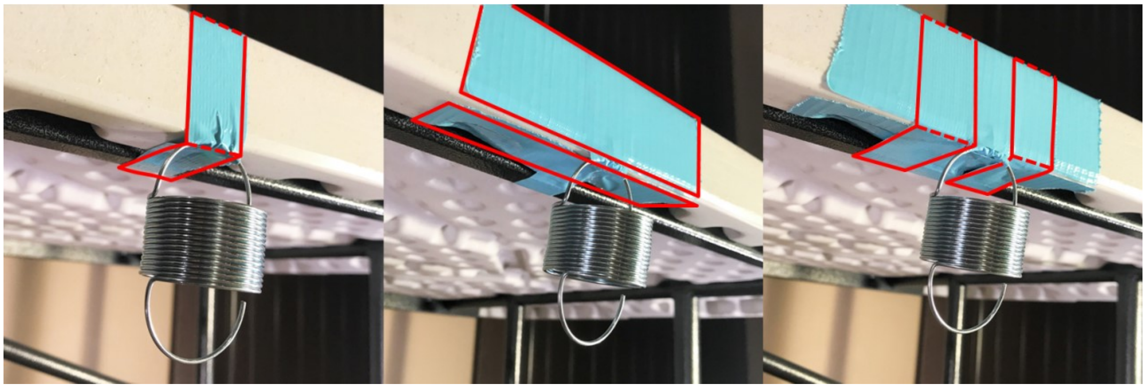

Fig. 7: Suggested setup to tape the spring to the edge of a table so that it won't slide down.

Measure the Spring Constant

1. With no weight hanging from the spring (m_0 = 0kg), measure the height (in meters) up from the floor to the bottom of the spring, x_0 (see Fig. 8).
    a. If you have a hard floor below the spring (hard wood, tile, etc.) you can simply measure from the floor up to the bottom of the spring.
    b. If the floor is soft (i.e. carpet), place a hard book or other hard flat object on the floor below the spring. Then measure up from this surface to the bottom of the spring.
2. Hang 5 different weights/combinations of weights (ex. pennies, shoe, shoe+cars, shoe+pennies, etc.) from the spring. For each weight/combo, measure and record the total hanging mass (m_1, m_2, m_3, …) and the distance (in meters) between the floor and the bottom of the spring (x_1, x_2, x_3, …) (see Fig. 8). Record the masses and heights in your eJournal.

    Try to evenly spread out the masses, starting from about 0.2kg and going up to at least 0.6kg. For example, don't choose 0.2kg, 0.23kg, 0.25 kg, 0.3kg, 0.33kg. Instead, you might do 0.22kg, 0.34kg, 0.42kg, 0.58kg, 0.71kg. Don't go lower than 0.2kg as masses close to zero will deviate from the linear trend of Eq. 1.

3. Later in the analysis section, you will use this data to find the spring constant.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

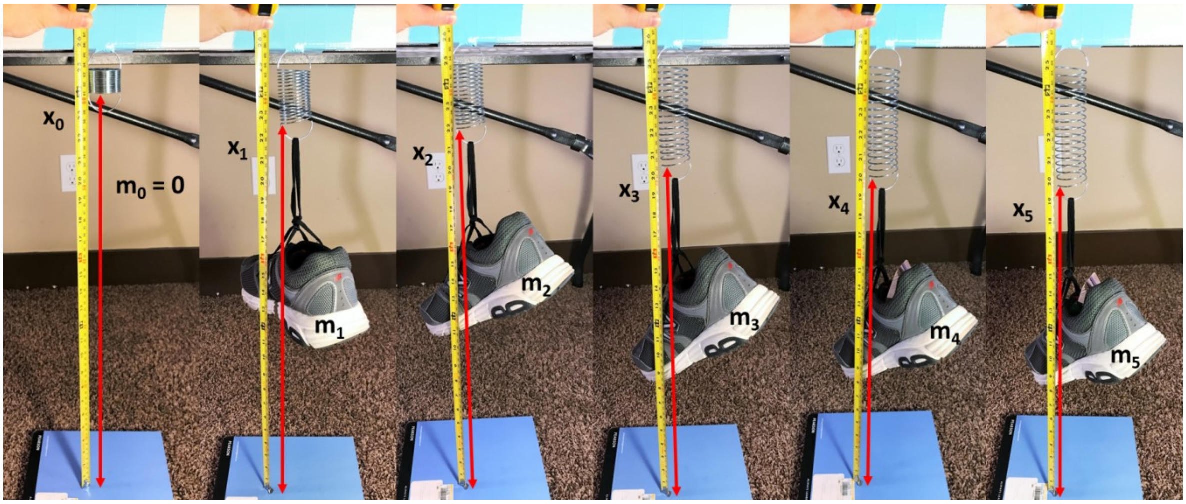

Fig. 8: Measure the distance from the floor up to the bottom of the spring with no mass hanging on it. Then measure the distance for 5 different masses ranging from about 0.2kg up to at least 0.6kg. Record the distances, x_i, and masses, m_i.

Measure the Period of Oscillation

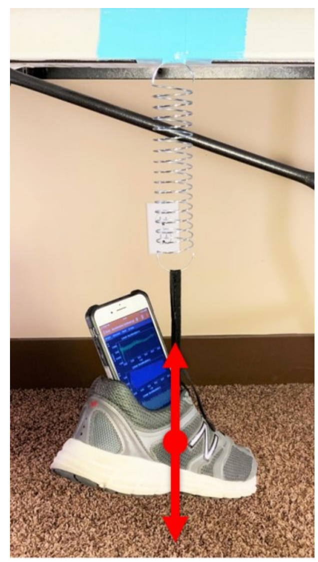

Fig. 9: Displace the weight and allow the spring to oscillate.

1. Hang the carrier from the spring and load all of your selected weights into the carrier (don't put your phone in yet). The total mass of your phone, carrier, and weights (measured earlier) should be around 0.8 – 1kg.
2. On your phone, open the Phyphox app and use the "Acceleration with g" or "Acceleration without g" tool.
3. Start recording data in the app and place your phone into the carrier upright as shown in Fig. 9. If you do not orient your phone upright (y-axis vertical), then you may need to use a different acceleration graph when determining the period. It would be nice if you can position the phone in the carrier such that you can watch the graph while it collects data, but if that's not possible, don't worry about it.
4. Stretch the spring down a small amount (2-3 cm) and release it, allowing the spring to oscillate up and down (see Fig. 9). If it is also swaying side to side very much, stop and try again.
5. Once you have **at least** 4 periods of good oscillatory data, stop recording.
6. Determine the period from the acceleration data.
    a. Select the Acceleration Y graph to bring up the graph tools (see Fig. 10a). If you oriented your phone differently, you may need to use a different graph (x or z).
    b. Rotate your phone into landscape orientation, and use the Pan and Zoom tool to zoom in on a section of nice regular data. Scale the graph vertically so that the wave peaks are tall and easy to see. Scale and pan the graph horizontally so that

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

you can see at least 4 full periods of data. If you can include more, that's even better (see Fig. 10b).

c. Use the Pick data tool to select from one wave peak to another, stretching across all of the selected periods (see Fig. 10c). This should make a line between the starting and ending points and provide you with the difference in time, difference in acceleration, and the slope. The time difference is what you will use to calculate the period.

d. Divide the time difference by the number of periods between the two points to find the average period. Record the period in your eJournal.

e. Take a screenshot to save the graph and time data, and insert it in your eJournal.

Fig. 10: (a) Tap on the Acceleration Y Graph. (b) Use the Pan and zoom tool to scale the graph so that you can see at least 4 full periods. In this example, 10 periods were selected. (c) Use the Pick data tool to select from one wave peak across to another, spanning all of the periods you selected. Divide the provided time difference in seconds by the number of periods included to find the average period.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

Analysis (Option 1):

Determine the Spring Constant

1. Hooke's Law (Eq. 1) shows that the spring force, F, is directly proportional to the elongation, x, with the spring constant, k, being the constant of proportionality. A plot of ΔF vs. Δx should be a straight line with a slope equal to k.

   $$k = \frac{\Delta F}{\Delta x} \tag{11}$$

2. Convert the various masses you suspended from the spring into forces by computing the weights, where m_i is the mass (m_0, m_1, m_2, m_3, …) of the weights/weight combinations.

   $\Delta F_i = m_i g$ (Newtons) (12)

3. Compute the elongations of the spring, Δx_i,

   $$\Delta x_i = |x_i - x_0| \text{ (meters)} \tag{13}$$

   where x_i is the measured distance from the floor to the bottom of the spring and x_0 is the distance from the floor to the bottom of the spring with nothing hanging from it.

4. Plot ΔF (y-axis) vs. Δx (x-axis).
5. Fit a linear trendline to the data and record the slope as the spring constant, k (think about appropriate units). Any force very close to zero will likely deviate from the linear trend. For this reason, do not include the point (0,0) on the graph.
   Insert an image of the graph in your eJournal.

Analyze the Period of Oscillation

1. Use your measured spring constant, k, with Eq. (2) and Eq. (4) to compute a predicted, theoretical value of the period, T_{theory}. Note that the effective mass, m_{eff}, is the mass of the carrier + any objects included + 1/3 mass of spring.
2. Use a percent difference to compare the measured period to the theoretical period.

   $$\%Diff = \frac{|T_{theory} - T_{meas}|}{T_{theory}} \times 100\% \tag{14}$$

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

Procedure (Option 2):
Pendulum

Setup

1. Securely tape one end of the twine to the side of a Ziploc bag (see Fig. 11a-c). Don't cut the twine short; extra is okay.
2. Slide your smartphone into the bag with the phone lying on its side. Center the phone so that its center of mass is directly below the twine (see Fig. 11d).

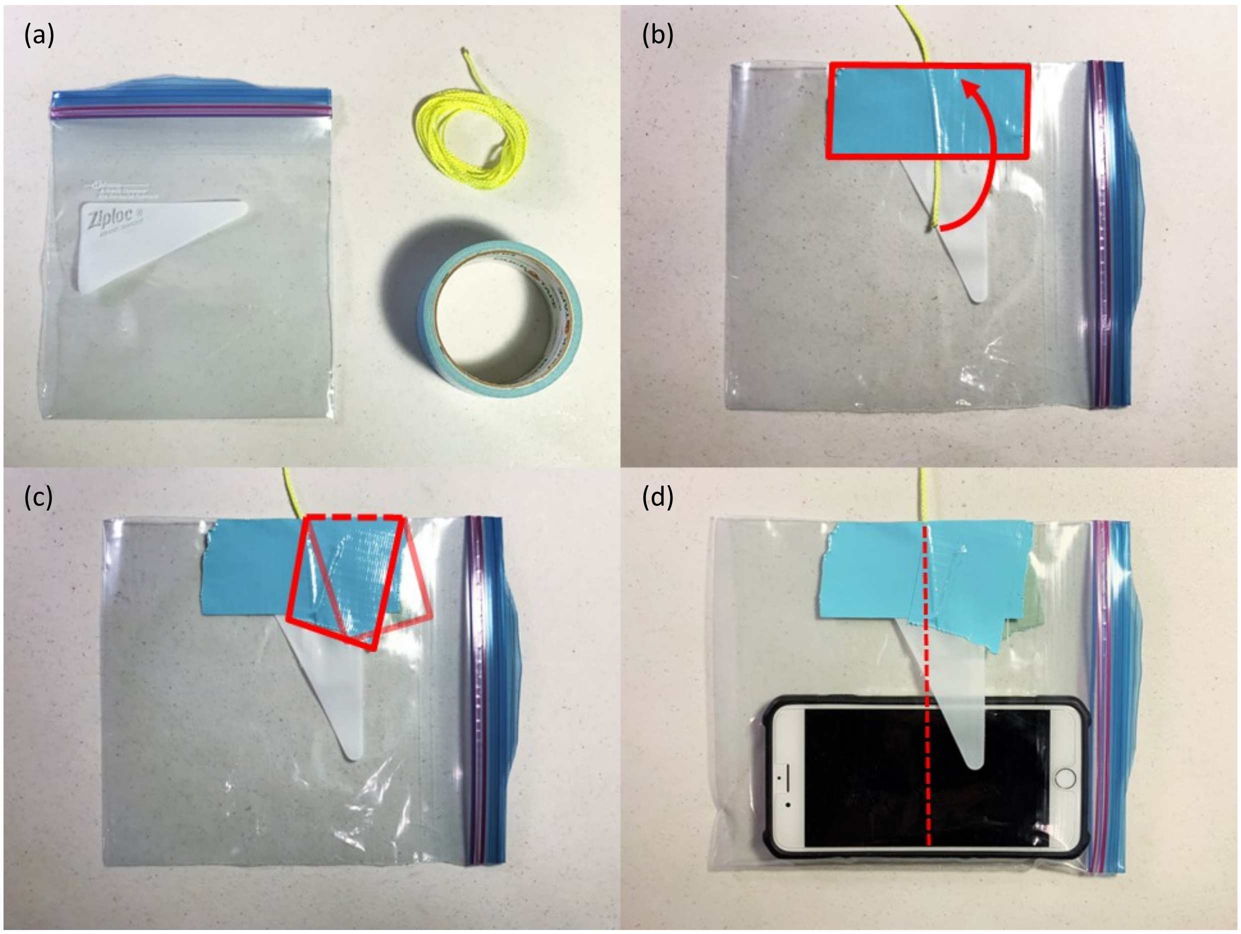

Fig. 11: (a) Use a zipper bag, twine, and duct tape to make a holder for your smartphone (pendulum weight). (b) Tape one end of the twine to the side of the bag, leaving a little bit extending past the tape. (c) Fold the twine end back over itself and tape it again to prevent it from pulling loose.
(d) Place your phone in the bag and line it up so that the phone is centered under the twine.

3. Hang the bag over the edge of a table such that the bag and phone almost touch the floor. Tape the twine to the edge and top surface of the table (see Fig. 12a-c).
4. Measure the length of the pendulum from the top of the twine (pivot point) down to the center of the phone (see Fig. 12d).

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

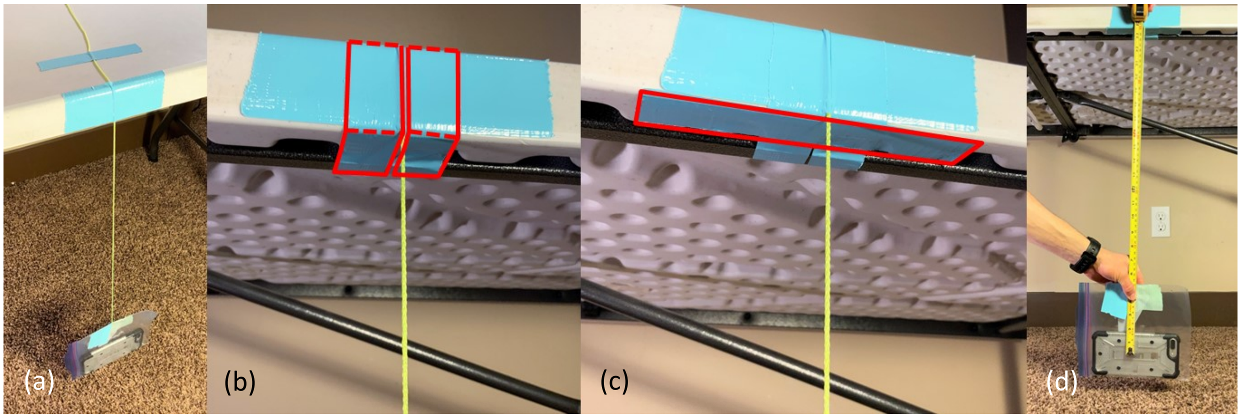

Fig. 12: (a) Tape the twine to the edge and top surface of the table. The piece on top will make it easier to adjust the pendulum length later. (b) Place tape on either side of the twine to keep it from sliding behind the tape. (c) Add another strip of tape behind the twine to secure the previous strips. (d) Measure the pendulum length from the pivot point (where the twine exits the tape) down to the center of the phone.

Measure the Period of Oscillation

1. Let the phone hang in the bag to see if it wants to spin (if the twine is twisted). Spinning could add unwanted vibrations to the acceleration data, so it is important to allow the twine to naturally untwist prior to recording data.
2. On your phone, open the Phyphox app and use the “Acceleration without g” tool.
3. Scroll down so you can watch the Acceleration z graph as the pendulum swings. Then start recording data.
4. Use a ruler to pull the pendulum weight (phone) back a small amount (about 5° off center). Then gently move the ruler away in the direction you want the phone to swing. You should release the phone so that the direction of motion goes through the phone screen (see Fig. 13).
5. As the phone swings back and forth, notice if it is moving in one direction or if it is moving in a circle/ellipse. Also notice if the phone is spinning or vibrating as it swings. Any of these unwanted motions can negatively affect the data and either decrease the accuracy or make it difficult to analyze. If any of these are happening, stop and try again so that you get smooth back and forth motion primarily in one direction.
6. Once the initial disturbance from releasing the pendulum has died down and you have **at least** 4 periods of good oscillatory data, stop recording.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

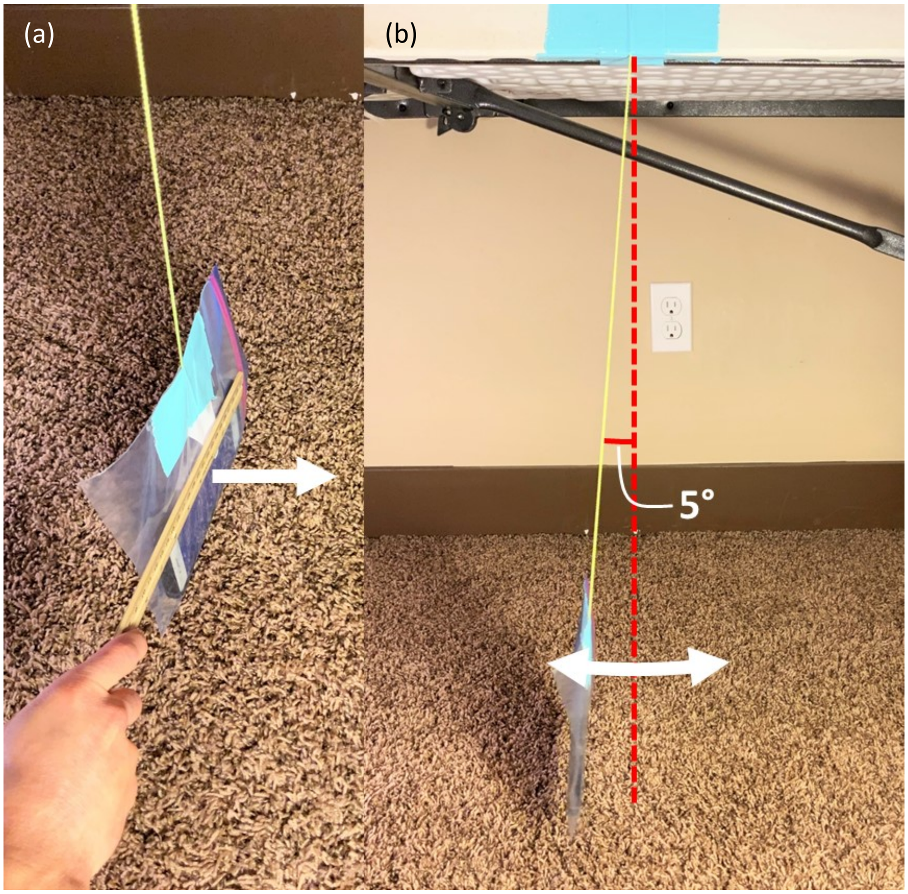

Fig. 13: (a) Use a ruler to pull back the phone pendulum about 5° off center. Then move the ruler gently to the right, encouraging the pendulum to swing in that direction. (b) The phone should swing back and forth in the direction of the screen (Phyphox z-axis). If the phone twists or spins very much, the motion won't stay in the z-axis direction and you won't be able to analyze the data very well.

7. Determine the period from the acceleration data.
    a. Tap on the Acceleration Z graph to bring up the graph tools (see Fig. 14a).
    b. Rotate your phone into landscape orientation, and use the Pan and Zoom tool to zoom in on a section of nice periodic data. Scale the graph vertically so that the wave peaks are tall and easy to see. Scale and pan the graph horizontally so that you can see **at least** 4 full periods (see Fig. 14b). More would be better.
    c. Use the Pick data tool to select from one wave peak to another, spanning across all of the selected periods (see Fig. 14c). This should make a line between the starting and ending points and provide you with the difference in time, difference in acceleration, and the slope. You will use the time difference to find the period.
    d. Divide the time difference by the number of periods between the two points to find the average period. Record the period in your eJournal.
    e. Take a screenshot to save the graph and time data.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

Fig. 14: (a) Tap on the Acceleration Z Graph. (b) Use the Pan and zoom tool to scale the graph so that you can see at least 4 full periods. In this example, 8 periods were selected. (c) Use the Pick data tool to select from one wave peak across to another, spanning the full periods you selected. Divide the provided time difference in seconds by the number of periods included to find the average period. The reason for the decrease in amplitude was a slight rotating of the phone as it was swinging.

8. Repeat the previous steps 4 more times, each time adjusting the pendulum to a new length (see Fig. 15). When you finish, you should have 5 total runs, with pendulum lengths ranging evenly between the maximum (run 1) down to about 30 cm at the minimum. For each run, record the period and the new pendulum length.

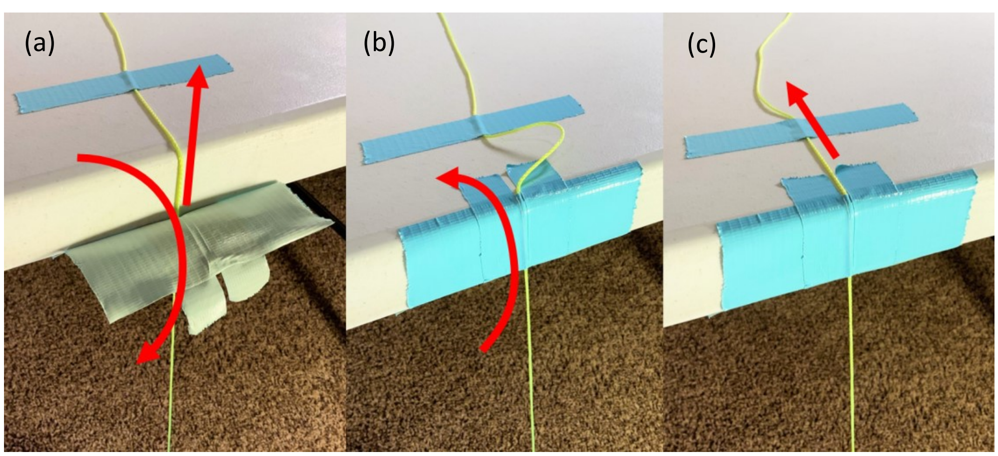

Fig. 15: Adjust the pendulum length using the following steps: (a) Partially remove the tape on the edge and pull the twine up a small amount. (b) Replace the tape on the edge. (c) Re-tape the twine on top to remove the slack.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.

Analysis (Option 2):

Period/Length Relationship for a Pendulum

1. Eq. (10) suggests that since $T \propto \sqrt{L}$, a plot of T vs. $\sqrt{L}$ should yield a straight line with a slope equal to $\frac{2\pi}{\sqrt{g}}$.
2. Use a spreadsheet or a calculator to calculate $\sqrt{L}$ for each of the pendulum lengths. Record these in your eJournal.
3. Plot T (y-axis) vs. $\sqrt{L}$ (x-axis).
4. Fit a linear trendline to the data and record the slope, m, and correlation coefficient, R. Insert an image of the graph in your eJournal.
5. Use the slope = $\frac{2\pi}{\sqrt{g_{meas}}}$ to determine g_{meas}.
6. Use a percent error to compare the measured acceleration of gravity to the generally accepted value of 9.80 m/s^2.

$$\%Error = \frac{|g_{theory} - g_{meas}|}{g_{theory}} \times 100\% \qquad (15)$$

7. Based on the correlation coefficient, how accurate were your experimental results? Remember that R = +1 is a perfect positive correlation, R = -1 is a perfect negative correlation, and R = 0 is no correlation.

Copyright © 2020 Physics Enterprises, Andrews University. All rights reserved.