General Physics Lab 12

Resonance in a Tube

Objectives:

- To determine the relationship between harmonic number and frequency in an open tube.
- To measure the speed of sound in air.

Equipment:

- Sound Tube
- Measuring Tape
- Ruler
- Smartphone with Phyphox App

Physical Principles:

Resonant Frequencies of Vibrating Air Columns

The general relationship between wave speed, v, wavelength, λ, and frequency, f, is

$$v = \lambda f. \qquad (1)$$

Open Tube: Some musical instruments can be modeled as a tube of air, open at both ends. The flute, trumpet and trombone are examples of open tube resonators. Standing-wave resonances occur in an open tube when a half-integer number of sound wavelengths matches the length of the tube, L, as shown in Fig. 1.

For example, the first tube in Fig. 1 has ½ a wavelength across the length of the tube. The second has 1 wavelength across the tube. The third has 1.5 wavelengths across the tube. The fourth has 2 wavelengths and so on.

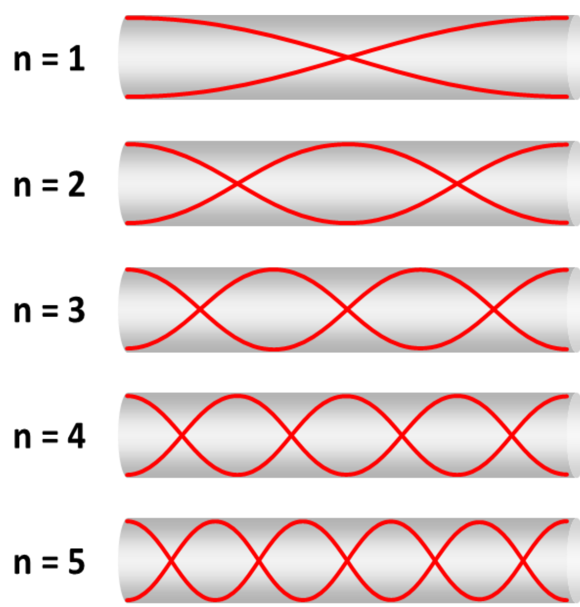

Fig. 1: Resonance conditions in an open tube. Antinodes (maximum air movement) form at each end of the tube.

The half-integer wavelength condition may be written as,

$$L = \frac{n}{2}\lambda_n \ , \ (n = 1, 2, 3, \ldots). \quad (2)$$

Combining Eqs. (1) & (2) gives the resonant frequencies for an open tube as

$$f_n = nf_1 = n\frac{v_s}{2L} \ , \text{(open tube)} \quad (3)$$

where f_1 is the fundamental frequency and v_s is the speed of sound in air. The frequency of a sound wave determines its pitch with high frequencies (short wavelengths) corresponding to high pitches.

End Correction

In practice, the actual frequencies will be slightly lower than the predictions of Eq. (3). It will appear as if the tube is longer than it really is. This happens because the antinodes form slightly past the end, almost as if the sound wave were bulging out the ends of the tube.

In order to correct for this, we must add an additional length, ΔL, to the tube. For an open tube, ΔL has been found to be 0.6 times the diameter, D, of the tube. The new corrected length then becomes

$$L_{cor} = L + \Delta L = L + 0.6D \ . \quad (4)$$

This corrected length is the tube length you will use in Eq. (3), which can be rewriten as

$$f_n = nf_1 = n\frac{v_s}{2L_{cor}} \ . \text{(open tube)} \quad (5)$$

This is the relationship we will explore in this experiment. By varying the harmonic number, n, and measuring the frequency, f_n, you will be able to determine the speed of sound in air.

Procedure:

Measure Length

1. Measure the length of your sound tube (Fig. 2a) and record it in your eJournal.
2. Measure the inner diameter of the tube opening (Fig. 2b) and record it in your eJournal.
3. Calculate the corrected length, L_{cor}, using Eq. (4) and record it in your eJournal.

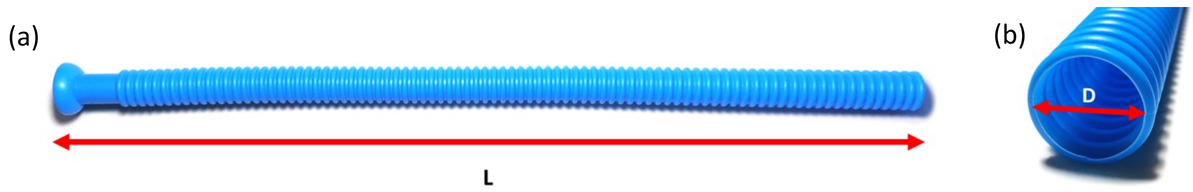

Fig. 2: (a) Measure the length of the tube including the handle end. (b) Measure the inner diameter of the tube opening. Don't worry about the handle end being different. Just measure the small end.

Sound Tube Explained

To produce sound with the tube, hold it by the handle end (see Fig. 3a), and swing the tube around in a circle over your head (Fig. 3b). This should produce a sound at one of the resonant frequencies. The faster you spin it, the higher the frequency or pitch of the sound.

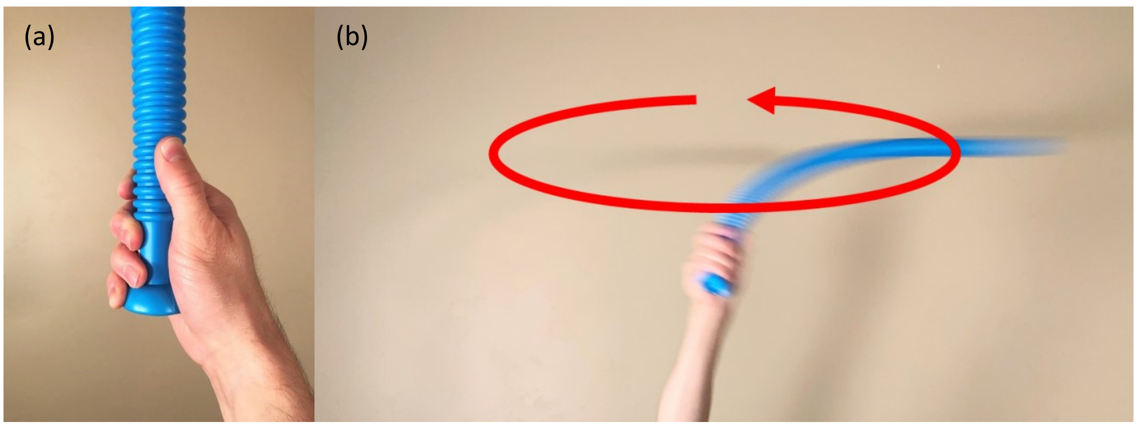

Fig. 3: (a) Hold the sound tube by the handle. (b) Swing the tube around in a circle. As you swing it faster/slower, the frequency of the sound will increase/decrease.

The sound is caused by the air moving through the tube as you swing it. According to the Bernoulli Principle, the moving end experiences a lower pressure relative to the handle end. This difference in pressure sucks the air in the handle, through the tube, and out the moving end (see Fig. 4). As you swing the tube faster, the pressure difference increases, causing the air to move faster through the tube.

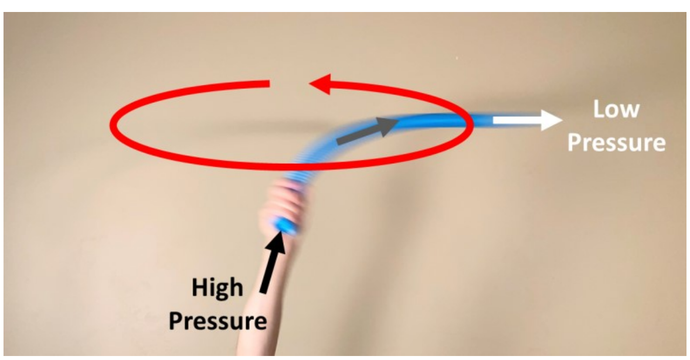

Fig. 4: According to the Bernoulli Principle, the moving end of the tube experiences a lower air pressure because of the tube moving past the air (same as if the air were moving past the tube). The pressure difference pulls air in the non-moving end, through the tube, and out the moving end.

As the air moves through the tube, it resonates at one of the frequencies described by Eq. (5). When you swing the tube faster, the pitch/frequency increases. The air can only resonate at certain frequencies (Eq. 5) so you will notice it suddenly jump from one frequency to another as you swing the tube faster or slower. As the air moves through the tube, the air particles hit the ridges, and allow you to hear the sound of the resonating air.

Measure Fundamental Frequency

If you try to make the fundamental frequency by swinging the tube, the sound produced is very quiet and hard to sustain for very long. Instead, you will do this one differently.

1. Find a relatively quiet room with no music or talking.
2. Open the Phyphox app on your smartphone and go to the Audio Spectrum tool.
3. In the settings tab, change the number of samples to 8192 (see Fig. 5). A higher sample rate measures more accurately, but it also updates less often. 8192 is a good compromise between accuracy and speed.
4. Go to the History tab and tap on the Peak-Frequency vs Time graph. Then start recording.
5. Slap/bounce your palm repeatedly on and off the small end of the tube (see Fig. 6a) to cause short bursts of sound at the fundamental frequency or first harmonic (n = 1).
6. As you watch the graph, you should see a series of points start forming a line as you make the sound repeatedly. Once you have identified the points that represent the fundamental frequency, stop recording.
7. Use the Pick data tool to select the frequency (see Fig. 6b) and read the value. It should be between 200 – 300 Hz. You may need to scale the graph first if it is very zoomed out. Take a screenshot to save the value and record the frequency, f_n, and harmonic number (n = 1), in your eJournal.

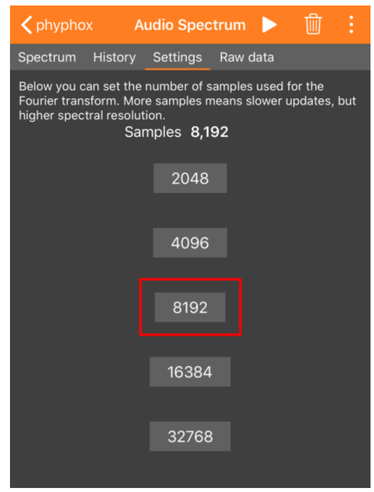

Fig. 5: Change the # of samples to 8192. If you close the tool, it will reset.

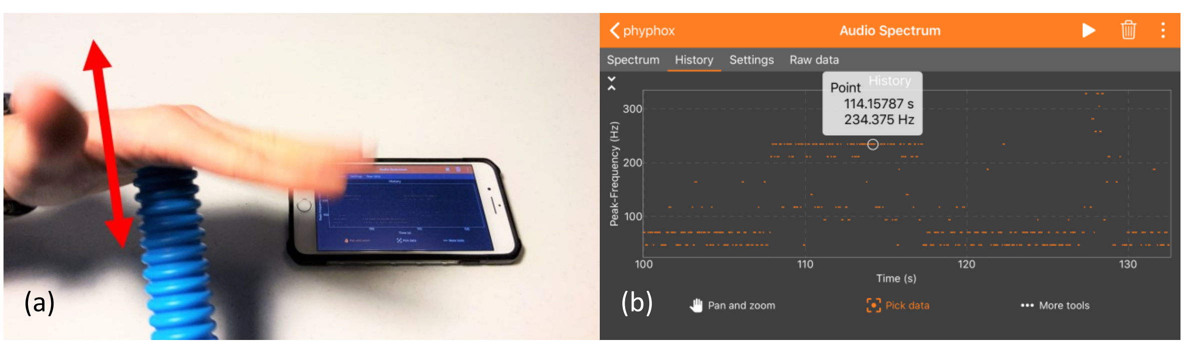

Fig. 6: (a) Slap/bounce your palm repeatedly on/off the small end of the tube to make a sound at the fundamental frequency. (b) Use the Pick data tool to select a point in the peak-frequency graph to determine the fundamental frequency. It should be between 200 – 300 Hz.

Measure Higher Harmonic Frequencies

1. Go to the Spectrum tab and you should see a graph of FFT Mag vs Frequency. This is essentially a graph of loudness vs frequency. You should also see the Peak-Frequency listed above the graph. This is the current peak (loudest) frequency. Unless there is other noise or music that the sensor can hear, this peak frequency will be the frequency produced by the sound tube.
2. Hit record on the app, swing the tube around over your head (Fig. 3) to produce the second harmonic (n = 2). When you see the peak-frequency spike on the graph (Fig. 7), hit pause to capture it, and then stop swinging the tube.
3. Take a screenshot of the graph and peak frequency to save it. Record the frequency, f_n, and harmonic number (n = 2), in your eJournal.

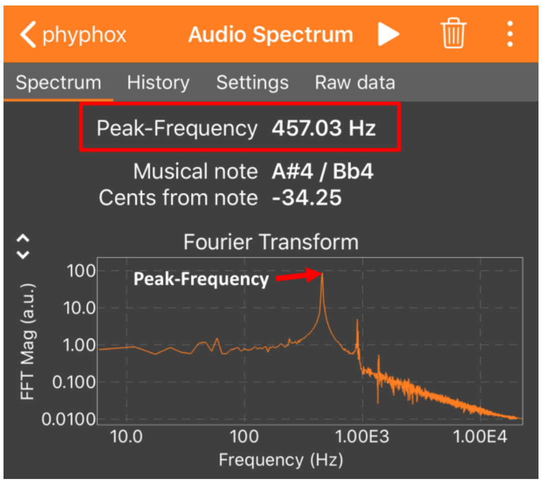

Fig. 7: Swing the tube around to produce sound at one of the harmonic frequencies. Pause the recording and then stop swinging the tube. Look at the Peak-Frequency for the frequency of the note you just produced. Repeat this process for each of the higher harmonics.

4. Repeat this process for the third harmonic (n = 3), fourth harmonic (n = 4), and if possible, the fifth harmonic (n = 5). The fifth harmonic can sometimes be difficult to make with these tubes, so do your best. If you cannot sustain the fifth harmonic long enough to capture it live on the FFT graph, use the Peak-Frequency vs Time graph in the History tab to identify it after you stop recording (same as you did for the fundamental).

Challenge: If you spin the tube fast enough, you may even be able to produce a sixth harmonic or higher. It is very difficult to swing it that fast, but some people can do it.

Analysis:

According to Eq. (5), a plot of f_n vs n should yield a linear graph with a slope of $v_s/2L_{cor}$. From this we can determine the measured speed of sound, $v_{s\ meas}$.

1. Generate a plot of f_n (y-axis) vs n (x-axis) and fit a linear trendline to the data.
2. Record the slope, m, and correlation coefficient, R.
3. By rearranging the slope from Eq. (5), you can determine the measured speed of sound, $v_{s\ meas}$. This is written below as Eq. (6).

$$v_{s\ meas} = slope \times 2L_{cor} \quad (6)$$

4. Use a percent difference to compare your measured speed of sound with the speed of sound at 20°C room temperature (v_s = 343 m/s). Or, if you want to be more accurate, you could measure your room temperature and use an online calculator to find the speed of sound at that temperature.

$$\%Diff = \frac{|v_s - v_{s\ meas}|}{v_s} \times 100\% \quad (7)$$

We can also compare the slope from our graph to the measured fundamental frequency. According to Eq. (5), the slope of an f_n vs n graph should also be equal to the fundamental frequency, f_1.

5. Use a percent difference to compare the slope with your measured fundamental frequency, f_1.

$$\%Diff = \frac{|f_1 - slope|}{f_1} \times 100\% \quad (8)$$

6. Based on the correlation coefficient, R, how linear are your results? What does this say about the accuracy of your experiment? Remember that R = +1 is a perfect positive correlation, R = -1 is a perfect negative correlation, and R = 0 is no correlation.