General Physics Lab 2

Forces in Equilibrium

Objectives:

- To test the principle that the vector sum of forces acting on an object remaining at rest is zero (Newton's 1st Law of Motion)

Equipment:

- Table or Countertop
- Thread (2 lengths 50 cm each, 1 length 30 cm)
- Scotch Tape
- Duct Tape
- 4 Plastic Pens
- 5 Paper Clips
- 100 Pennies
- Spring Balance
- Printed 360^o Protractor
- Wood Block cut at 45^o angle
- Scissors

Physical Principles:

Vector Addition - Graphical Method

This method is also known as the polygon method. The vectors are repositioned so that the tail of each vector lies on the head (pointed end) of the previous vector (see Figure 1). The resultant, which represents the sum of the forces, is found by drawing a new vector from the tail of the first vector to the head of the last.

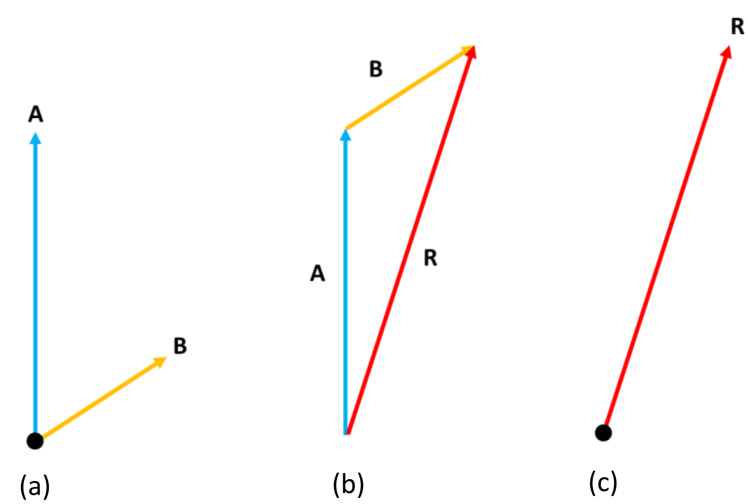

Fig. 1: Graphical Method for Adding Vectors

Vector Addition - Component Method

For this method, you resolve each vector into its components. Then the components are added to form the resultant components (see Fig. 2).

Resolve each vector into components.

$$A_x = A\cos\theta_A \quad , \quad B_x = B\cos\theta_B \qquad (1)$$

$$A_y = A\sin\theta_A \quad , \quad B_y = B\sin\theta_B \qquad (2)$$

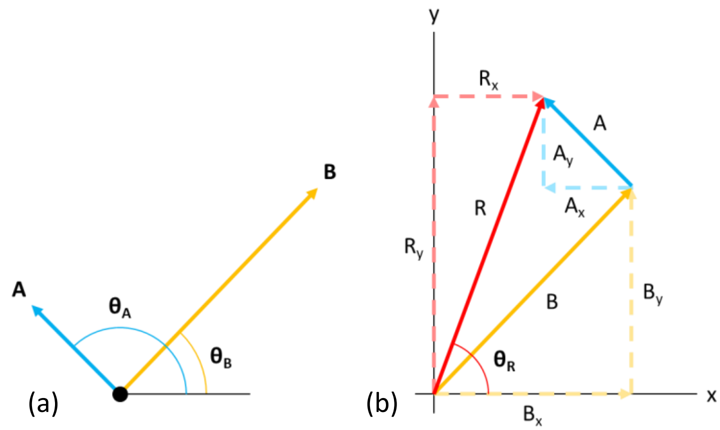

Fig. 2: Component Method for Adding Vectors

Add components to get resultant components.

$$R_x = A_x + B_x \qquad (3)$$

$$R_y = A_y + B_y \qquad (4)$$

Use Pythagorean Theorem to combine components of R.

$$R = \sqrt{R_x^2 + R_y^2} \qquad (5)$$

Use inverse tangent to determine the angle for R.

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \qquad (6)$$

Equilibrium Conditions

Newton's first law asserts that a body will not accelerate when the net force acting on it is zero. So, for an object to be at rest (or in uniform motion), the resultant of the forces acting on it must be zero. That is, the sum of all forces acting on the body must be zero. In equation form, the above statement can be written as follows.

$$\sum \vec{F} = 0 \qquad (7)$$

If three forces act on an object at rest, Eq. (7) becomes,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0\,, \qquad (8)$$

or equivalently,

$$\vec{F}_3 = -\left(\vec{F}_1 + \vec{F}_2\right). \qquad (9)$$

In other words, the third force (equilibrant) must cancel the resultant of the other two.

Predictions: (Hypotheses)

Suppose you have

a) Two (non-equal) forces pulling at 90° to each other (Fig. 3a) with one directed along the positive x-axis ($\theta_1 = 0°$), and the other directed along the positive y-axis ($\theta_2 = 90°$).
b) Two (equal) forces pulling at 135° to each other (Fig. 3b) with one directed along the x-axis ($\theta_1 = 0°$), and the other directed 135° counter-clockwise from the x-axis ($\theta_2 = 135°$).

For each case, sketch and add the two vectors using the **graphical method**. Then find and sketch the equilibrant vector that would cancel out the two original vectors. Paste pictures of your prediction sketches in your eJournal.

Procedure:

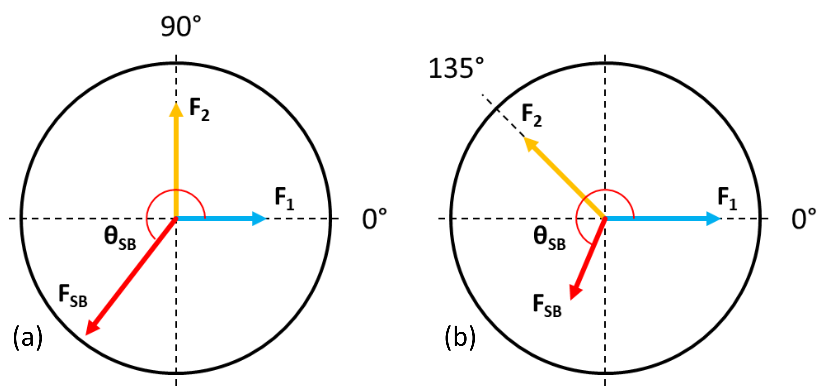

Fig. 3: Red line illustrates magnitude and direction of the Spring Balance equilibrant force.

Fig. 3 shows two forces, F_1 and F_2, acting in different directions. Fig. 3a shows F_1 and F_2 at right angles to one another and Fig. 3b shows F_1 and F_2 separated by 135°. In addition to these two forces, there is third force acting as the equilibrant, cancelling the vector sum of the first two forces.

For your experiment, F_1 and F_2 will be hanging weights comprised of pennies. These weights will pull in the different directions as shown in Fig. 3 and you will measure the equilibrant force with your spring balance.

The high-tech version of this lab uses a specialized force table, force sensors, and pulleys to accomplish these measurements (see Fig. 4). Without access to that equipment, you will be using a spring balance and everyday items to make your own force table. The setup will be a little more complicated, but the experiment results should be very similar to what you would get with the high-tech equipment.

Fig. 4: Specialized Force Table Setup

Weights and Strings

If you have not yet completed the Penny Weight-Set Instructions (PDF document provided by your instructor or on your class website) to bundle pennies into modular weights, please complete that before continuing.

After following the weight-set instructions, you should have 2 weights (50 pennies and 30 pennies) as well as a bundle of 20 pennies remaining.

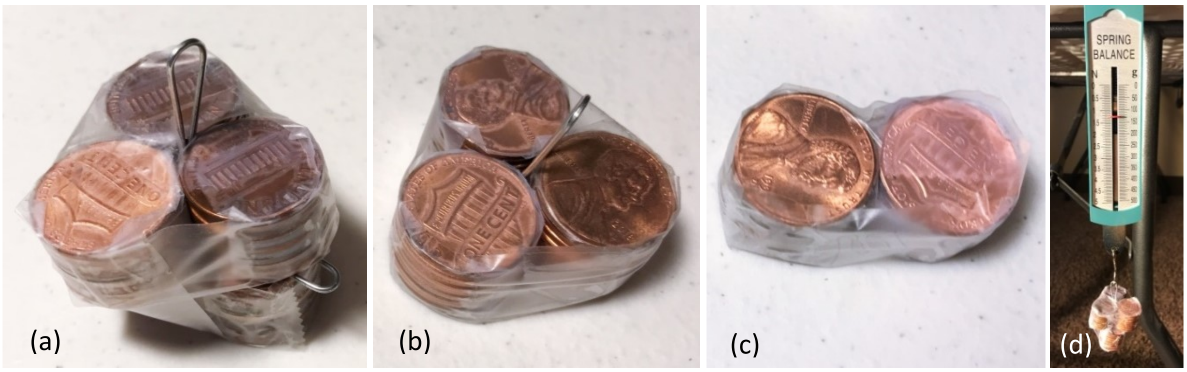

Fig. 5: Hanging weights made out of pennies. 50 pennies and 30 pennies will be for Part 1. For Part 2, you will combine the 30 and 20 pennies to make a second 50 penny weight. Weigh them on the spring balance.

1. Hang the spring balance by the top loop and zero it with nothing hanging from it (see Fig. 5d). Then use it to find the mass in kg of the 50 pennies and the 30 pennies. Calculate the weight (force) in N and record the mases and weights in your eJournal. It is your choice which one is F_1 and which is F_2.
2. Cut 3 lengths of thread. One should be about 30 cm long and two should be about 50 cm long. If it later turns out they are too long, you can shorten them.
3. Tie loops at the ends of each thread. Do not use slipknots because you may want to hook the loops onto things without the knot tightening. Instead, use a simple knot like the overhand loop knot (see Fig. 7a).
4. Once you have knots on each thread, hook a paper clip to one of the loops on each thread (see Fig. 7c).

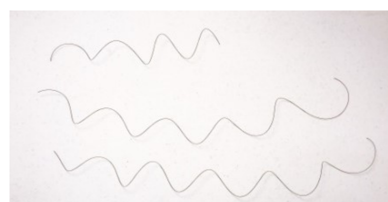

Fig. 6: Thread cut to lengths of 30 cm, 50 cm, and 50 cm.

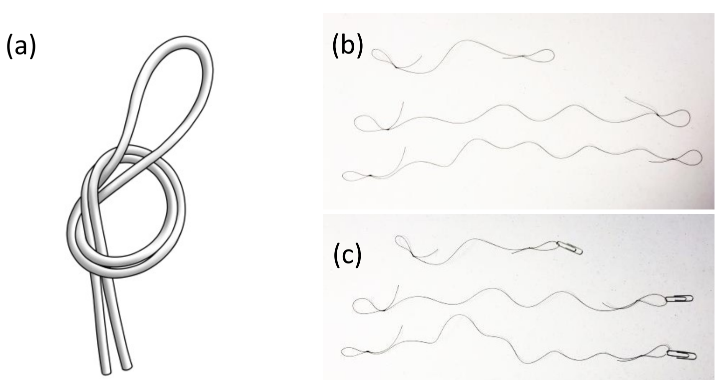

Fig. 7: (a) Overhand Loop Knot (b) Threads with knots tied on the ends and (c) paper clips added to the loops

5. Connect the loop on the shorter thread (30 cm) to the spring balance hook and connect the paper clip hooks on the longer threads (50 cm) to the loops on the weights as shown in Fig. 8.

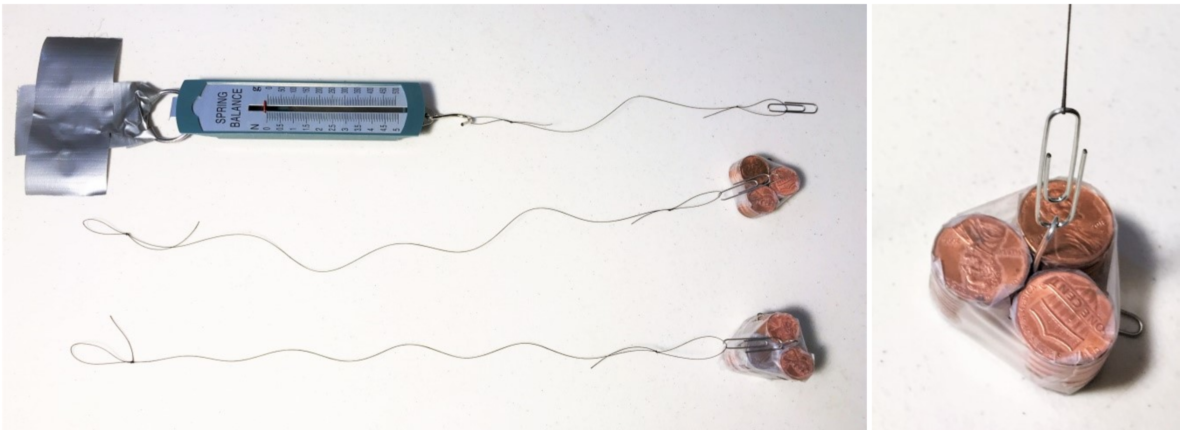

Fig. 8: Threads attached to spring balance and weights. Notice that the paper clip hook on the thread attaches to the paper clip loop on the pennies.

Table and "Pulleys"

1. Find a table, counter, etc. that has a 90° corner with smooth sides. Make sure you have enough space on top to set everything up and stretch the spring balance away from the corner. Ideally, you'll want a space of at least 50 cm x 50 cm although the exact size of your setup will vary so don't worry too much if your surface is small.

   Warning: Test the duct tape on a discrete area of your table to make sure the adhesive won't damage the table's finish. If it does, you may need to use a different kind of tape or find a different surface to work on.

2. Take one of the pens in your kit and remove the cap. Then duct tape it to the top edge of the table 10 cm away from the corner. Printed/textured words on the side of pen could add extra friction with the thread, so make sure to flip the pen over so the smooth side is up.

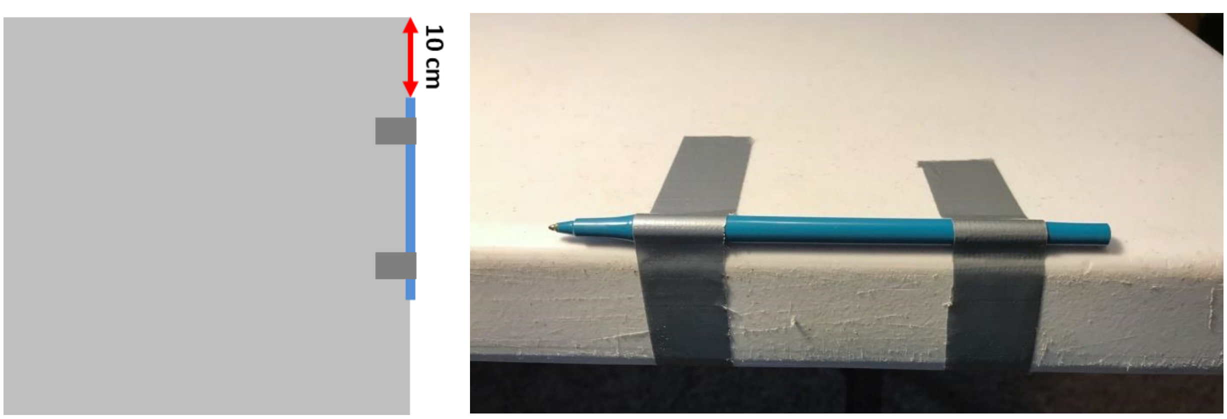

Fig. 9: Pen taped to the top edge of the table 10 cm away from the corner.

3. Take a second pen, remove the cap, and tape it just below/next to the first pen. It should nest into the gap between the tabletop and the first pen as shown in Fig. 10b.
4. Tape two more pens on the adjacent side of the table (10 cm from the corner). Once you finish this step, you should have two sets of pens taped to the edges near the corner as shown in Fig. 11. These pens will take the place of pulleys in your experiment. The top pens raise the thread above the table surface and the lower pens keep it out away from the edge of the table. The pens will add more friction to the system than pulleys would, but their friction is small compared to the weight of the pennies.

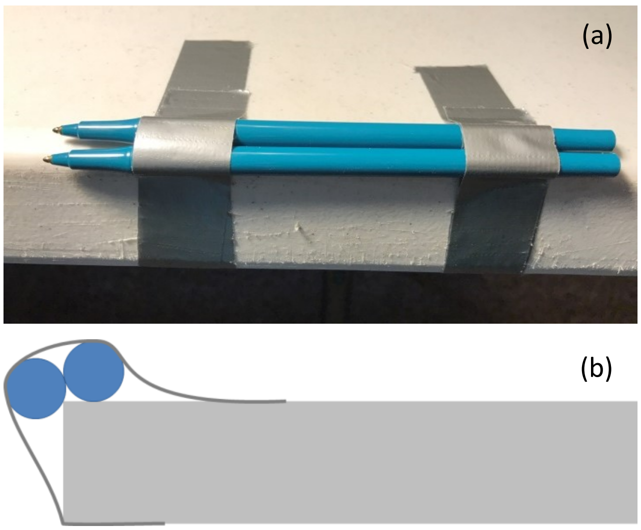

Fig. 10: Second pen taped below/next to the first pen. The second picture is a side-view diagram.

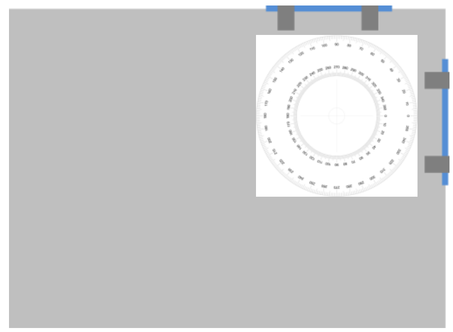

Fig. 11: Two sets of pens taped to the corners. These will take the place of pulleys in the experiment. The printed 360° protractor should be cut out and placed at the corner of the table between the pens.

5. Next, take the printed 360° protractor and cut off the extra margins. You don't have to cut out the circle if you don't want but at least cut it down to a square around the circle. This will make it easier to line up close to the edges of the table (see Fig. 11).
6. Attach some duct tape to the top ring of the spring balance. Don't stick it to the table yet but having the tape attached early will make the next steps easier.

Part 1 - Two Forces at Right Angles

1. Arrange the spring balance and weights as shown in Fig. 12. Note that the end of the paperclip from the spring balance will act as the intersection point of all three strings.
2. One weight will be F_1 and the other will be F_2. You can choose which is which.

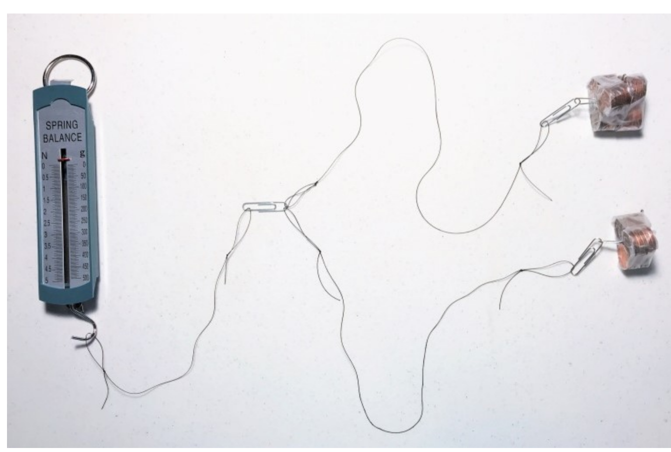

Fig. 12: Spring balance and weights connected by thread and paperclips.

3. Hold onto the spring balance and hang the two weights over the pen "pulleys". At this point, the strings should make a Y-shape with the spring balance and each weight pulling in different directions (see Fig. 13).
4. Adjust the spring balance in/out and side-to-side until the forces are reasonably balanced (see Fig. 13). Then tape the top ring of the spring balance down to the table.
5. Next, lift up the weights a small amount and gently release them. This will encourage the angles of all three forces to shift to their most balanced positions. You may have to do this a couple times if it doesn't work at first. When this is finished, the strings for each weight should cross the pens perpendicularly.
6. Place the printed protractor on the table under the threads and line it up so that the center of the circle is at the intersection of the three threads (at the end of the center paper clip.
7. Rotate the protractor until F_1 lines up at 0° and F_2 lines up at 90° (see Fig. 14). If the thread is slightly off from the desired angle, slide it over so that it crosses exactly at 0° or 90°.
8. Record the force in Newtons from the spring balance, and record the angles for F_1, F_2, and F_{SB} from the protractor.

   Note: Once the spring balance is taped down to the table and all the forces are balanced, don't wait too long to make the measurements or take pictures of your setup. The hanging weights will eventually pull the taped spring balance loose and cause everything to fall off the table.

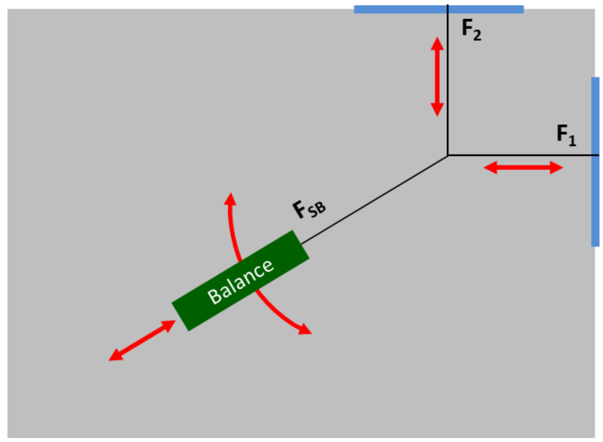

Fig. 13: Weights hang over the pens and the spring balance pulls in the opposite direction. Adjust the spring balance until it looks like all the forces are balanced.

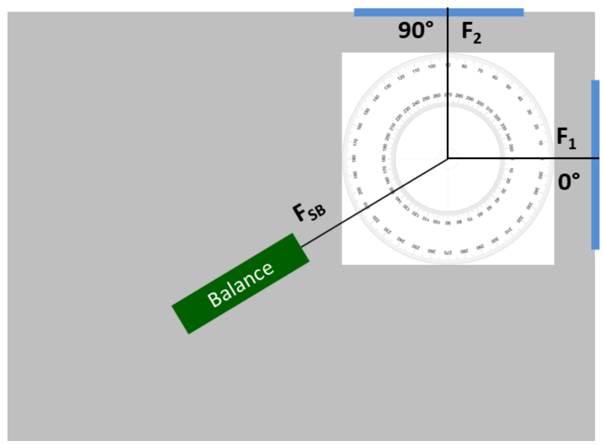

Fig. 14: Align the printed protractor under the threads so that the center is at the intersection of the threads, F_1 at exactly 0°, and F_2 at exactly 90°.

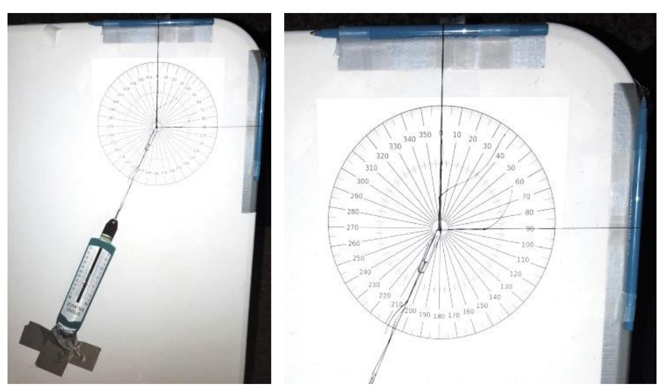

Fig. 15: Example setup with printed protractor aligned under the three threads.

Part 2 - Two Forces Separated by 135°

1. Remove the spring balance and weights and set them to the side.
2. Follow the weight-set instructions to combine the 30 penny weight and the 20 penny bundle into a second hanging weight of 50 pennies. These 2 groups of 50 pennies will be the weights for part 2.
3. Weigh the new 50 penny weight on the spring balance and record its mass in kg.
4. Remove the pens at 90° from the table edge. When you do this, the table should be back to how it was in Fig. 9.
5. On the same side of the table where you removed the pens, place the 45° wood block at least 22 cm from the corner (see 17a). Orient the block so that the long side of the triangle is against the edge of the table. Secure it to the table using a sufficient amount of duct tape (see Fig. 17b). Make sure that the top triangular surface of the block is roughly level with the table surface.
6. Tape one pen to the top edge of the block farthest from the table corner (see Fig. 17c).
7. Tape another pen below that one just like you did on the edge of the table (see Fig. 17d). In this case the edge of the wood block is acting as your new table edge. When you finish this step, the table should look like Fig. 17e.

Fig. 16: Two hanging weights with 50 pennies in each.

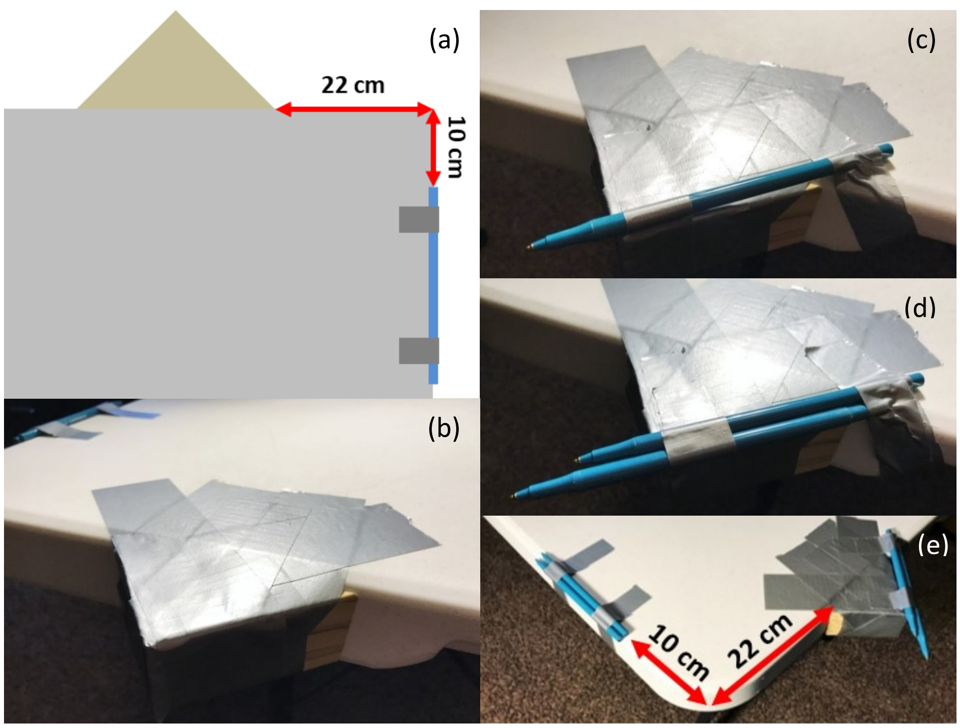

Fig. 17: Wood block taped to edge of the table and pens taped to the block. Note that 17a is a rotated top-down view of 17b-e.

8. Hold onto the spring balance and hang the two weights over the pens like you did in part 1. The threads should cross the pens perpendicularly.
9. Adjust the spring balance in/out and side-to-side until the forces are reasonably balanced (see Fig. 18). Then tape the top ring of the spring balance down to the table.
10. Lift up the weights a small amount and gently release them. This will encourage the angles of all three forces to shift to their most balanced positions. You may have to do this a couple times if it doesn't work at first. When this is finished, the strings for each weight should cross the pens perpendicularly.
11. Place the printed protractor on the table under the threads and line it up so that the center of the circle is at the intersection of the three threads.
12. Rotate the protractor until F_1 lines up at 0° and F_2 lines up at 135° (see Fig. 19). If the thread is slightly off from the desired angle, slide it over so that it crosses exactly at 0° or 135°.
13. Record the force in Newtons from the spring balance, and record the angles for F_1, F_2, and F_{SB} from the protractor.

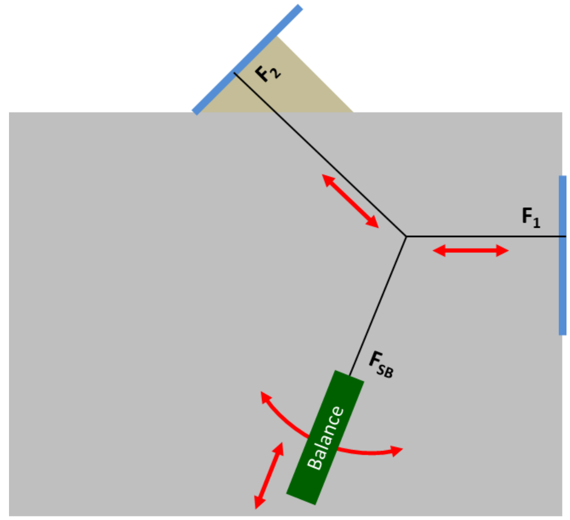

Fig. 18: Weights hang over the pens and the spring balance pulls in the opposite direction. Adjust the spring balance until it looks like all the forces are balanced.

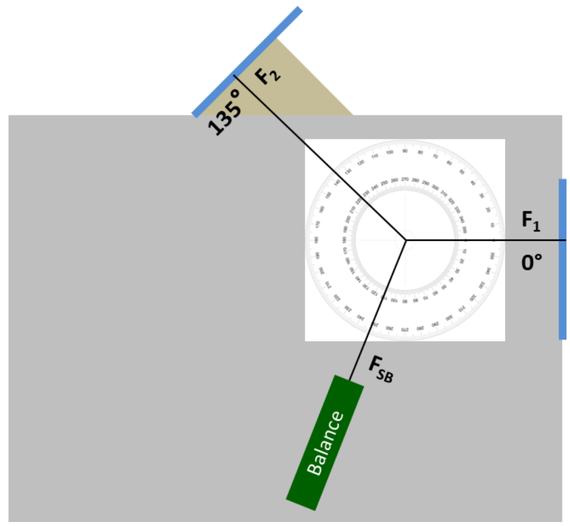

Fig. 19: Align the printed protractor under the threads so that the center is at the intersection of the threads, F_1 at exactly 0°, and F_2 at exactly 135°.

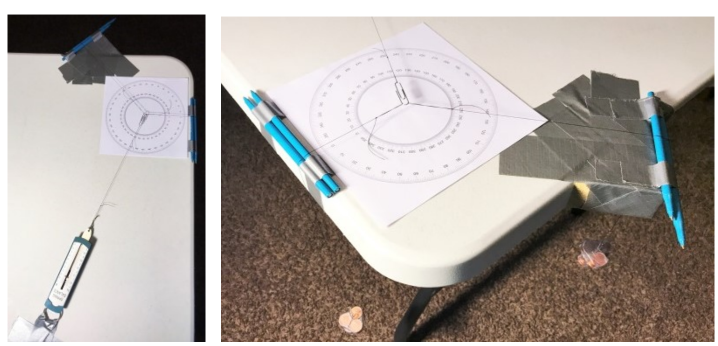

Fig. 20: Example setup with printed protractor aligned under the three threads.

Analysis:

Complete the following steps both for Part 1 and for Part 2:

1. Resolve F_1 and F_2 into their x and y components using Equations 1 and 2.
2. Calculate the resultant components R_x and R_y using Equations 3 and 4.
3. Calculate the predicted resultant magnitude (R) using Equation 5
   and the predicted angle (θ_R) using Equation 6.
4. Calculate the predicted equilibrant direction, $\theta_{eq} = \theta_R + 180^o$, since it must cancel the resultant.
5. Calculate % differences between predicted and measured magnitudes,

   $$\%\Delta_{mag} = \frac{|R - F_{SB}|}{R} \times 100\%, \qquad (10)$$

   and angles,

   $$\%\Delta_{\theta} = \frac{|\theta_{eq} - \theta_{SB}|}{\theta_{eq}} \times 100\%. \qquad (11)$$