General Physics Lab 6

Conservation of Momentum

Objectives:

- To test conservation of momentum in elastic and inelastic collisions

Equipment:

- Spring Balance
- Ruler
- 2 Racquetballs (option 1)
- Twine (option 1)
- 2 Hotwheels Cars (option 2)
- Hotwheels Track (option 2)
- 40 Pennies (option 2)
- Duct Tape and Scotch Tape
- Triangular Wood Block
- 2 Rubber Bands
- Smartphone (camera) or Webcam
- Tracker Video Analysis software
  available for free download at https://physlets.org/tracker/

Physical Principles:

Collisions between Two Objects

The Law of Conservation of Momentum states that in the absence of external forces, momentum is conserved. Thus, when two objects collide, the total momentum before the collision is equal to the total momentum after the collision. This holds for any type of collision – elastic or inelastic.

In one dimension one can write,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (1)$$

If initially, only object 1 is in motion and object 2 is stationary, this becomes,

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad (2)$$

Collisions may be divided into three different categories – all of which conserve momentum:

1. **Elastic** collisions conserve both momentum and kinetic energy such that,

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \quad (3)$$

2. **Inelastic** collisions conserve momentum but some kinetic energy is lost. A fraction of kinetic energy is converted into heat and/or deforming the objects.

3. **Totally Inelastic** collisions conserve momentum but lose the largest fraction of kinetic energy possible. In these collisions, the two colliding objects stick together and move off as a single unit with a common velocity, v_f. Kinetic energy is certainly **not** conserved in totally inelastic collisions. If initially, only object 1 is in motion and object 2 is stationary, conservation of momentum for the totally inelastic collision becomes,

$$m_1v_{1i} = (m_1 + m_2)v_f \quad (4)$$

Note: You may choose to study either Elastic Collisions (Option 1) or Inelastic Collisions (Option 2). You are not required to do both experiments.

Procedure (Option 1):

Elastic Collision between Two Rubber Balls

Setup

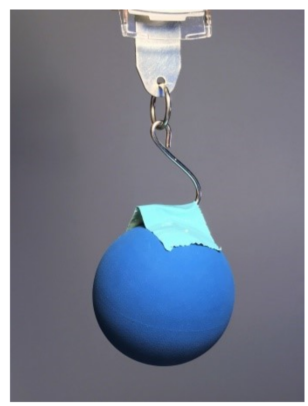

Fig. 1: Rubber ball with duct tape hanging from the spring balance hook.

1. Hang the spring balance by the top loop and zero it if necessary.
2. Use the spring balance to measure the mass of each racquetball in kg. This may be done by taping the ball to the spring balance hook with a small strip of duct tape (see Fig. 1). Keep the piece of duct tape small so that it doesn't contribute much extra mass to the reading. Record the masses in your eJournal and keep track which ball is which if they have different masses.
3. An elastic collision will be approximated with a simplified DIY version of the well-known executive toy called a Newton's Cradle. To construct this, you will need 4 items – twine, 2 racquetballs, and some duct tape.

Fig. 2: DIY Newton's Cradle Supplies (Twine, Racquetballs, and Duct Tape)

4. First, find a table with a straight edge and a light colored background underneath. You will want the background to have a clear, horizontal floor line and a clear vertical line from trim, doorjamb, etc. This is necessary for aligning the axes in Tracker.
5. For the first pendulum, begin by tearing off a half-width strip of duct tape about 5 cm long. Then tear it into 4 roughly equal sized pieces (see Fig. 3b). You will use 3 of these smaller strips in the following steps.
6. Tape one end of the twine to the top of the ball with about 2 cm extra sticking out past the tape (Fig. 3c).
7. Fold the extra twine back on top of the tape and stick another piece of tape over this. You should still have a short bit of twine sticking out past this tape (Fig. 3d).
8. Put one more piece of tape over this last short bit of twine (not the long twine) so that your ball and twine look like Fig. 3e.
9. Repeat these steps with the second ball, using the other end of the twine (no need to cut it). It would also be a good idea to label each ball as m1 and m2 (see Fig. 3f).

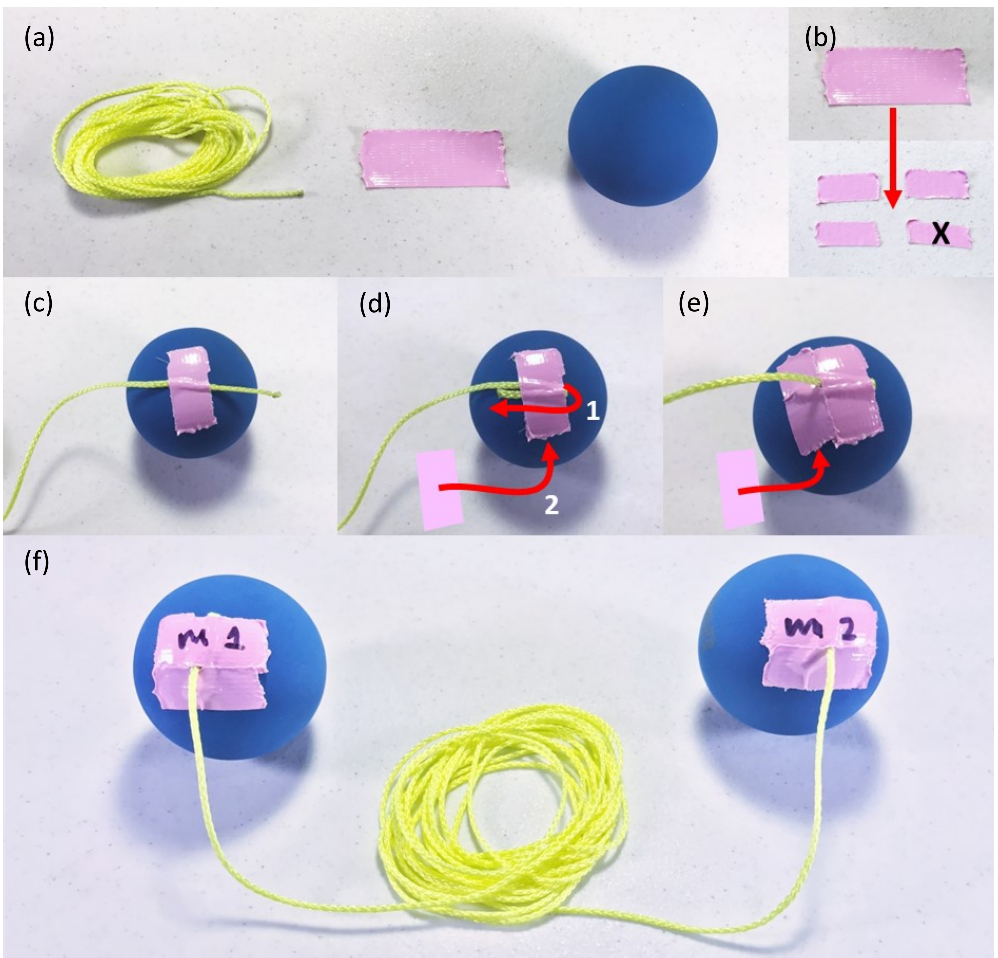

Fig. 3: Attach the twine to the racquetballs. (a) Gather supplies, (b) Tear the duct tape into smaller pieces, (c) Tape the twine to the top of the ball with some extra at the end, (d) Fold the twine back on itself and tape again, (e) Tape the extra tail of the twine, (f) Repeat with the other ball and label both balls on the tape.

10. Hang the m1 ball by the twine and tape the twine to the edge of a table such that the ball hangs down close to the floor (see Fig. 4a). First test to make sure the tape will not damage the table's finish.
11. Hang the second ball next to the first so that the two strings are parallel and the balls just barely touch when at rest. Adjust the height of the second ball to match the first and tape the twine to the edge of the table (see Fig. 4b & d).
12. Add a strip of tape on either side of both strings and wrap the tape around under the edge of the table (see Fig. 4c). This will restrict the motion of the strings so they cannot slide behind the tape. Instead, each pendulum will swing from a single point.
13. Last, place a ruler below the two pendulums to use for distance calibration in Tracker.

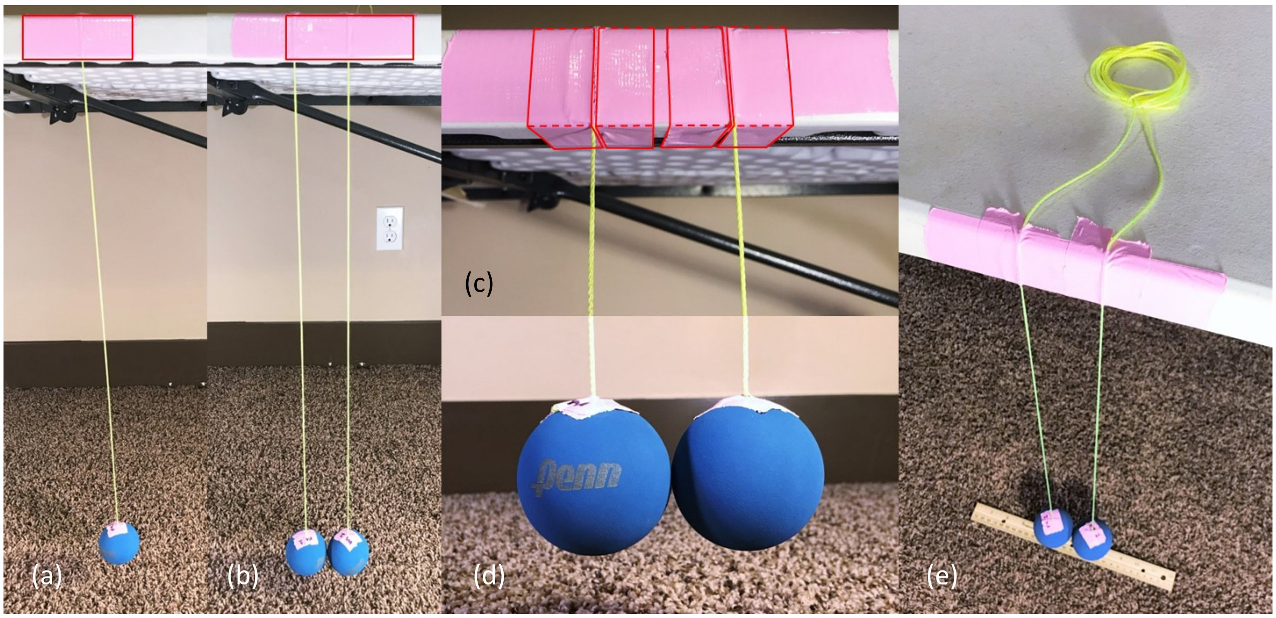

Fig. 4: Tape the racquetball pendulums to the table. (a) Hang the first pendulum from the table and secure with tape. (b) Hang the second pendulum, adjust the position to match the first, and secure with tape. (c) Add strips of tape on either side of the strings to keep them from sliding under the tape. (d) The two balls should just barely touch and they should hang at the same height. (e) Finished setup with two pendulums and a ruler below.

Video Capture

1. If you use a smartphone to film the pendulums, use the wood block and rubber bands to make a stand as shown (see Fig. 5). Horizontal orientation may be best for this setup. Refer to the Smartphone Stand Instructions document for more information.

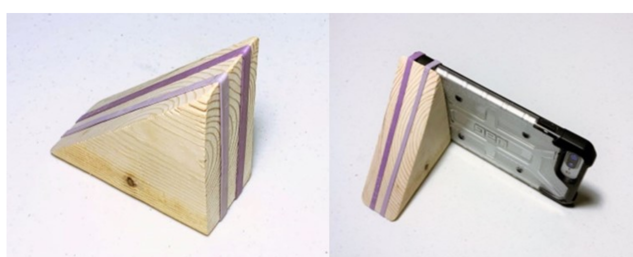

Fig. 5: Simple smartphone stand

2. Set the smartphone or webcam on a stable surface facing the pendulums and the table. The camera lens should be about the same height as the racquetballs (when they are hanging at rest), so set the height of the camera accordingly.
3. Move the camera out far enough to see a good portion of both pendulums' swings.
4. Make sure the camera is not angled relative to the pendulums and table or the tracking data will be inaccurate. The plane of the pendulums and the plane of the camera should be parallel to each other (see Fig. 6).
5. Also, make sure there is good lighting on the racquetballs so they are easily visible from the background when filmed through the camera. A lamp and a light colored backdrop would be ideal.
6. Hit record on the camera, pull back the ball on the left (mass 1) in a line along the edge of the table to about a 45° angle (less if your table is very tall). Release it so that it swings down and hits the other ball (mass 2).

Fig. 6: Smartphone with wood block stand facing the pendulums and framed to place the racquetballs in the center of the image and the ruler in view below.

7. The ideal collision is one that occurs entirely in 1 dimension, where the first ball swings down, hits the second ball and causes it to swing up, all moving within the same plane. Note that it is not easy to get this sort of ideal swing, but you can get close if you have patience and keep trying until you get a nice swing and collision.

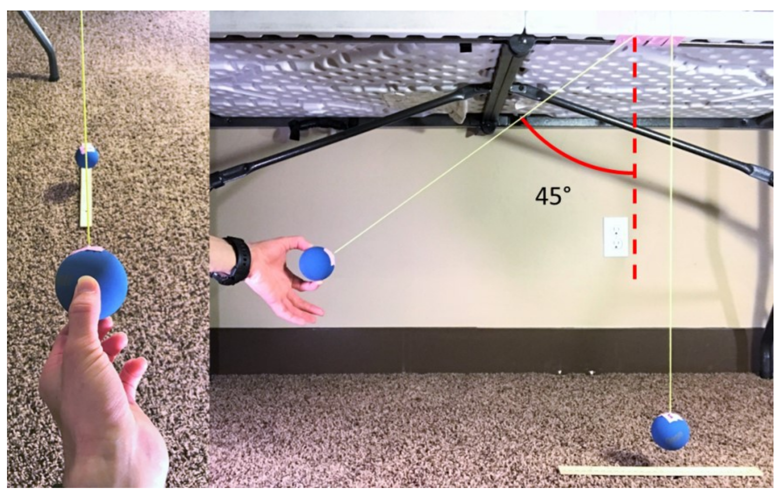

Fig. 7: Pull back the pendulum to a 45° angle and line it up with the stationary ball. Then, let go and allow it to swing down and hit the stationary ball in a 1-dimensional collision.

8. Review the video to make sure you got a good collision. Then if the video is long (more than 30 seconds), you may want to trim it shorter to include only the best collision.
9. Transfer the video to your computer via some means (email, Google Drive, iCloud, Dropbox, etc.) for analysis using Tracker Video Analysis software.

Tracking the Motion

Complete the following steps in Tracker.

1. Open the video file in Tracker.
2. Save the Tracker file.
3. If your video was filmed vertically, click Video > Filters > New > Rotate to make the video upright.
4. Using the Calibration Tools, add a new Calibration Stick, align the calibration stick on the ends of the ruler (Shift+Click on the end points), and set the length to the length of the ruler.
5. Click on the Coordinate Axes and place them on the video. Line up the origin with the intersection of two perpendicular lines in your image (ex. corner of a door) and rotate the axes so that the x and y axes match the angle of the floor and doorframe respectively (see Fig. 8). Then (without rotating the axes again) move the origin down into the bottom left corner.

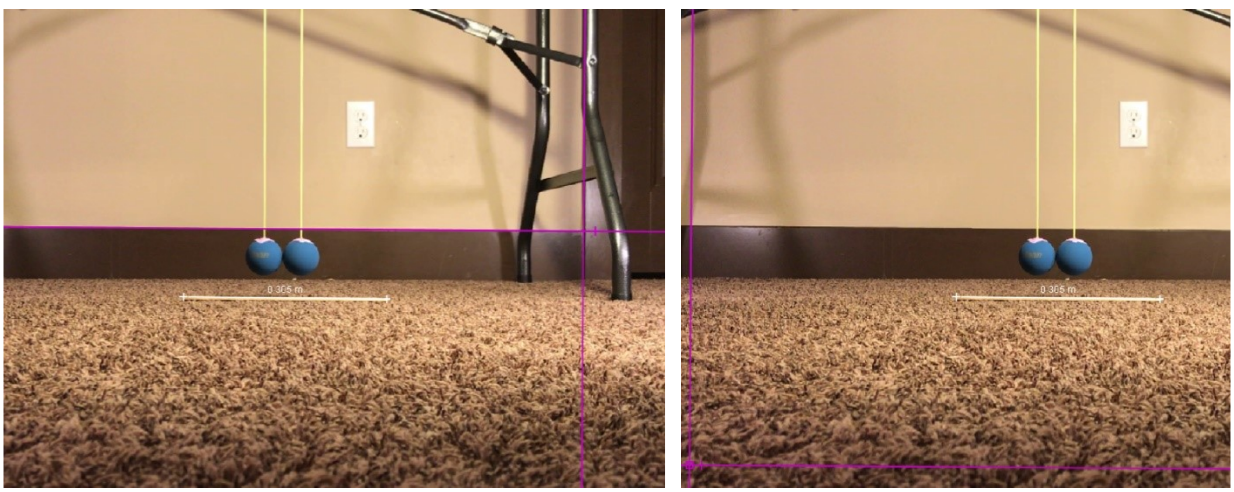

Fig. 8: Align the Calibration Stick to the ends of the ruler and set the length to the length of the ruler. Align the coordinate axes to some perpendicular lines in your image. Then move them to the bottom left corner.

6. Click Create > Point Mass and name the point mass “Mass 1” (right click on the original mass name to bring up an option menu where you can rename it).
7. Slide through the video to the point of collision. Then use the step buttons to step back at least 5 frames.
8. Add points to the center of left ball (mass 1) from this frame through at least 5 frames after the collision.
9. Create another point mass and name this one “Mass 2”.
10. Go back to the first frame where you added points for Mass 1 and add points to the center of the right ball for these same frames. Make sure you track the same frames for each ball (even when they are stationary) or they won’t have data for the same times.

11. Insert an image of your collision and tracking marks (such as Fig. 9) in your eJournal.

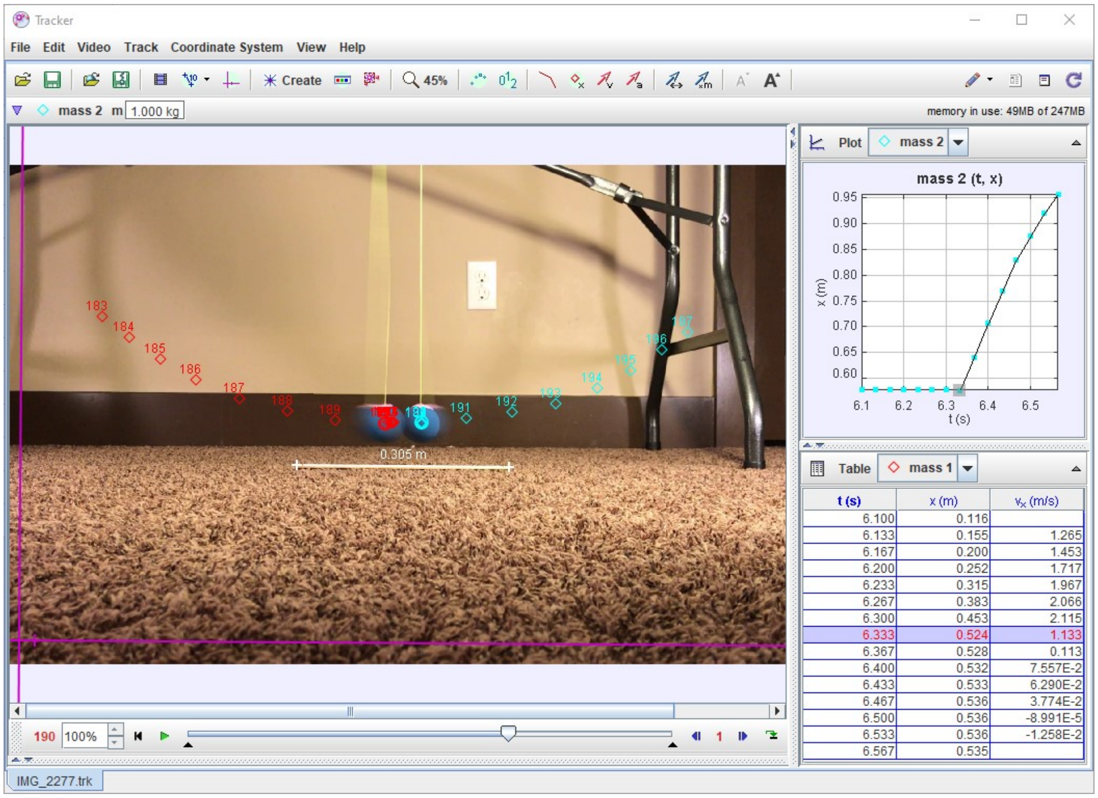

Fig. 9: Data tracked for each ball from 7 frames before the collision until 7 frames after the collision. Track the position of each ball for all of these frames, even when the ball is stationary.

Export the Data

After you have finished tracking the path of each ball, complete the following steps to export the data. You will need to do this for both Mass 1 and Mass 2. You can select which point mass is shown in the table using the dropdown next to the Table button.

1. Click Table and select only x and vx. You will only use vx for the calculations but x is useful to determine when the collision occurred.
2. Option 1 – Copy and paste the data
    a. Select all the data in the table
    b. Right Click > Copy Selected Cells > Full Precision
    c. Paste the data into Google Sheets or Excel

   Option 2 – Export a CSV file with the position and velocity data
    a. Click File > Export > Data File and set the following parameters:
       Cells = All Cells, Number Format = Full Precision, Delimiter = Comma
    b. Click Save As and name the file with the extension ".csv" (ex. "filename.csv").
    c. Using Google Sheets or Excel, import each CSV file.

Analysis (Option 1):

1. Open a blank Google Sheets or Excel document and import the data from Tracker.
    a. If you copied the data directly from the table in Tracker, you can simply paste it into the blank spreadsheet.
    b. If you exported a CSV file, you will need to import the data from that file into your spreadsheet.
2. Combine the two data sets (Mass 1 and Mass 2) by matching up the times, such that the position and velocity points for each mass at a given time are in the same row.
3. Reformat the cells into Number format and increase the decimal precision to include the full precision you copied from Tracker.
4. Using Google Sheets, Excel, or Graphical Analysis, generate plots of position vs time and velocity vs time with both positions (Mass 1 and Mass 2) in one graph and both velocities (Mass 1 and Mass 2) in another graph (see Fig. 10).

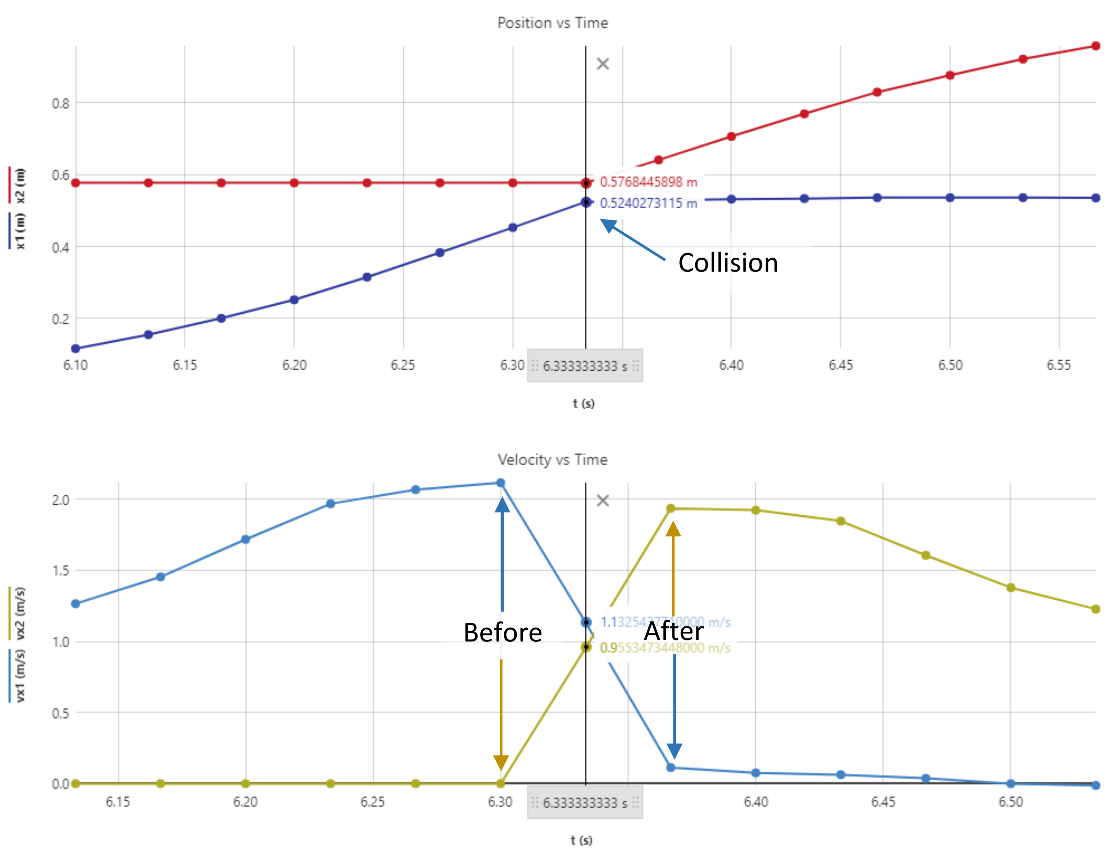

Fig 10: Plots of position vs time and velocity vs. time for two balls involved in an approximately elastic collision. On the position graph, the blue line is the incoming ball and the red line is the stationary ball. Notice that after the collision, the blue line goes constant and the red line continues the trajectory of the blue line. On the velocity graph, the blue line is the incoming ball and the yellow line is the stationary ball. Notice that the yellow line starts at zero (stationary). Then at the collision, the velocities switch and the blue line goes to zero and the yellow line continues the path of the blue.

5. Look at your position graph to see at what time the collision occurred. Then look at the velocity graph to determine the initial velocities (just before the collision) and the final velocities (just after the collision). See Fig. 10 for an example of initial and final velocities.
6. From the masses and velocities of each ball, calculate the initial and final momenta (p = mv) and kinetic energies ($KE = ½mv^2$) for each ball.
7. Calculate the total initial and total final momentum and kinetic energy (ex. $KE_{i\ Total} = KE_{i\ 1} + KE_{i\ 2}$).
8. Calculate the percent loss of momentum and kinetic energy to determine how well these quantities were conserved during the collision. If you calculate these in a spreadsheet, you can skip the x100% and just format the cell as a percent.

$$\%Loss = \frac{|total\ initial - total\ final|}{total\ initial} \times 100\%$$

Don't be too disappointed if KE is not found to be conserved for this collision. Notice that if you drop one of the balls from a height, h, onto a hard surface, it does not return to the same height. That is because some of the ball's kinetic energy just before hitting the hard surface was lost to heat. Therefore, the collision was not totally elastic.

Procedure (Option 2):

Totally Inelastic Collision between Two Hot Wheels Cars

Setup

1. Start by taping stacks of pennies to the Hot Wheels cars. One car will have 10 pennies and the other will have 30.
2. Make sure that you have 4 stacks of 10 pennies each (see Fig. 11a). The construction of these was detailed in the Penny Weight-Set Instructions document.
3. For the car with 10 pennies, tape one stack to the middle by wrapping a long piece of Scotch tape around the car (see Fig. 11b), taking care to avoid the wheels (tape touching the wheels would add extra friction). Set this car aside for now.
4. For the car with 30 pennies, attach one stack to the middle, the same as you did for the previous car.
5. Use another strip of tape to attach two more penny stacks in front of and behind the first stack (see Fig. 11d). The tape should only go around the first stack, not under the car.
6. Next, tear off 2 short half-width strips of duct tape (see Fig. 11e).
7. Roll each piece of duct tape and stick them to the back of each car (see Fig. 11f). The idea here is to put the tape on the end of each car such that when the cars collide, the two rolls of tape are at the same height and will stick together.
8. Hang the spring balance by the top loop and zero it if necessary.
9. Weigh each car on the spring balance, and record the masses in kg. If you can, hang the cars on the spring balance hook by their axels (see Fig. 11g & h).

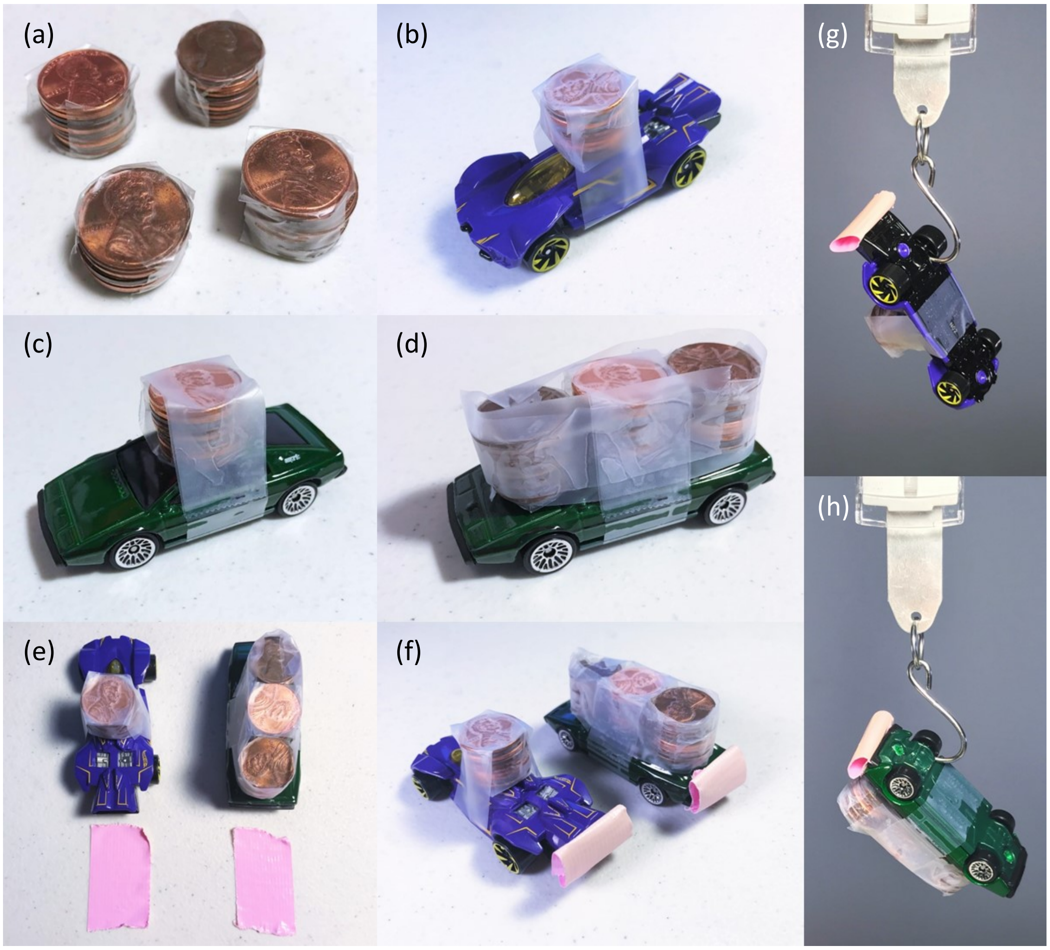

Fig. 11: Tape stacks of pennies to the two Hot Wheels cars, 10 pennies on one car and 30 pennies on the other. Add duct tape to the back of each car and then weigh them individually on the spring balance.

10. Connect 2 Hot Wheels track sections together to make one long track 40 inches in length.
11. Use 2 strips of Scotch tape to tape the sides of each track together (see Fig. 12). Fold each piece of tape over the edge of the track so it holds it together on the inside and outside edges. Smooth it down so no tape sticks out into the path of the car. The reason for the tape is to keep the edges together so the car can roll past without bumping into them. This is the same thing you did in Lab 3.
12. Lay the long track on a flat level surface, such as a table or counter.

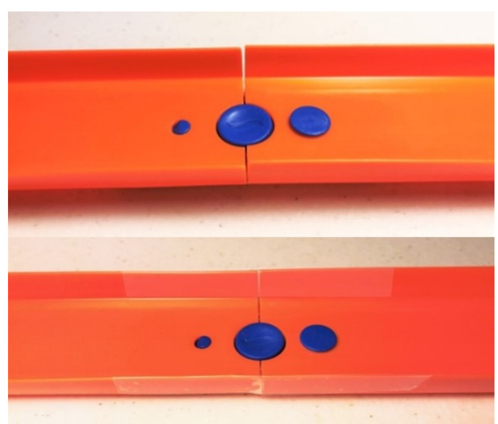

Fig. 12: Connect the two Hot Wheels track sections. Then tape the two edges on each side of the track.

13. Place the stationary car in the middle of the right track segment (see Fig. 13). It’s up to you which car is mass 1 and which is mass 2 but the experiment may work better if the heavier car is initially stationary.
14. Place the ruler next to the track with the center of the ruler lined up with the front of the stationary car (see Fig. 13). The ruler will be used for calibrating distances in Tracker.
15. The moving car (mass 1) will come from the left, pass over the junction of the two tracks, and collide with the stationary car (mass 2) at the middle of the right track.
16. Make sure the duct tape ends of the cars are facing each other for the collision.

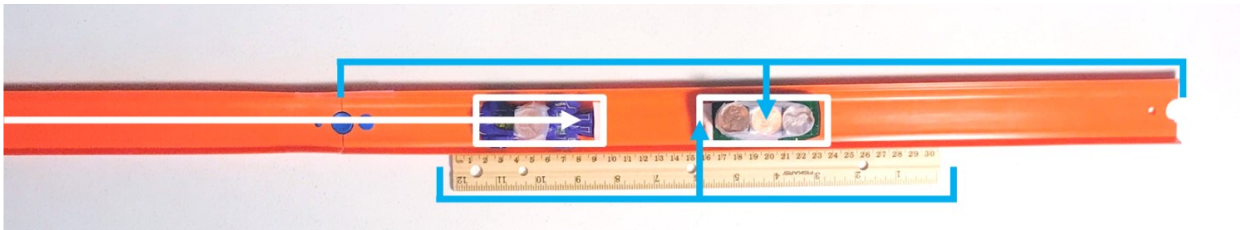

Fig. 13: Top-view setup diagram for the cars, track, and ruler. Place the stationary car (right) in the middle of the right track segment. Place the ruler next to the track and line up the center of the ruler with the end of the stationary car. Moving car will come from the left and collide with the stationary car. The duct tape ends of the cars should be facing each other for the collision.

Video Capture

1. If you use a smartphone to film the collision, use the wood block and rubber bands to make a stand as shown (see Fig. 14). Horizontal orientation will be best for this setup. Refer to the Smartphone Stand Instructions document for more information.
2. Set the smartphone or webcam on the table facing the track.
3. Move the camera out far enough that the camera can see the entire ruler and some track on either side.
4. Make sure the camera is not angled relative to the Hot Wheels track or the tracking data will be inaccurate. The phone, track, and ruler should all be parallel to each other (see Fig. 15).
5. Make sure to use a good light source (desk lamp, flashlight, open window, etc.) so the recorded video will have a clear contrast between the cars and the background.

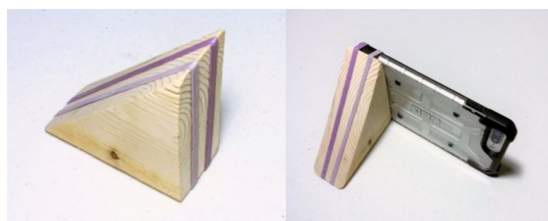

Fig. 14: Simple smartphone stand

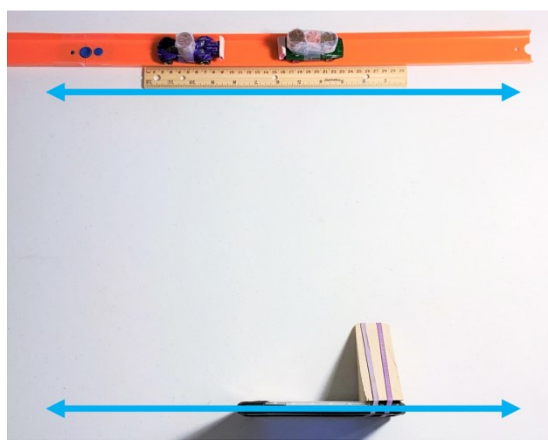

Fig. 15: Track, ruler, and camera should all be parallel and sit on the same level surface.

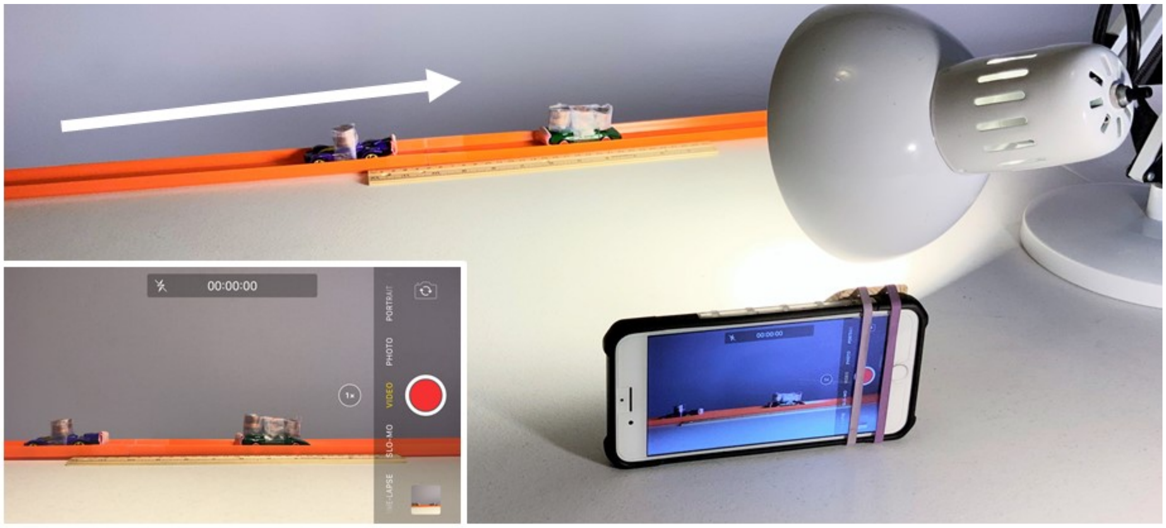

Fig. 16: Complete setup with track, cars, phone, and desk lamp (good light source).

6. Hit record on the camera and push the car on the left (mass 1) toward the stationary car on the right (mass 2), making sure to release it well before the collision. The extra track on the left is for you to push the car along before it reaches the stationary car.
7. Push it at a reasonable speed (not too fast or too slow). The car should go fast enough that it is still moving at a reasonable speed just before the collision and still have enough kinetic energy left to carry both cars on for a short distance. You also don't want it to go so fast that you can't track the motion of the cars. If it doesn't work the first time, just try again (you may have to replace the duct tape if you repeat the collision).
8. At the collision, the tape should cause the cars to stick together and move off to the right as one unit. Stop the camera once the cars come to a halt.
9. Review the video to make sure you got a good collision and to see that both ends of the ruler are visible. Then if the video is long (more than 30 seconds), you may want to trim it shorter.
10. Transfer the video to your computer via some means (email, Google Drive, iCloud, Dropbox, etc.) for analysis using Tracker Video Analysis software.

Tracking the Motion

Complete the following steps in Tracker.

1. Open the video file in Tracker.
2. Save the Tracker file.
3. Using the Calibration Tools, add a new Calibration Stick, align the calibration stick on the ends of the ruler (Shift+Click on the end points), and set the length to the length of the ruler. Since the front edge of the ruler will appear larger than the back edge, place the end points of the calibration stick at the back corners of the ruler, closest to the track.

4. Click on the Coordinate Axes and place them on the video. Place the origin on the edge of the track and rotate the axes until the x-axis is aligned with the track edge (see Fig. 17).

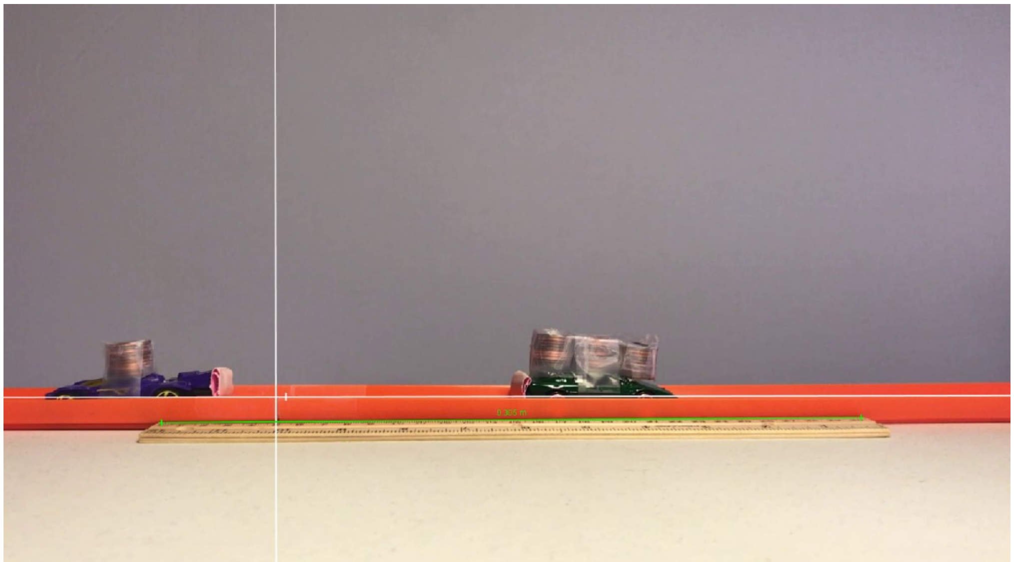

Fig. 17: Align the Calibration Stick to the back corners of the ruler (closest to the track) and set the length to the length of the ruler. Align the coordinate axes so that the x-axis is parallel to the track.

5. Click Create > Point Mass and name the point mass “Mass 1” (right click on the original mass name to bring up an option menu where you can rename it).
6. Slide through the video to the point of collision. Then use the step buttons to step back at least 5 frames.
7. Choose a point on the left car that will be easy to track (such as the corner of the pennies).
8. Add points (Shift+Click) to that spot on the left car (mass 1) for each frame from at least 5 frames before the collision until at least 5 frames after the collision.
9. Create another point mass and name this one “Mass 2”.
10. Choose a point on the right car that will be easy to track.
11. Go back to the first frame where you added points for Mass 1 and add points to the initially stationary car (mass 2) for these same frames. Make sure you track the same frames for each car (even when they are stationary) or they won’t have data for the same times.
12. Insert an image of your collision and tracking marks (such as Fig. 18) in your eJournal.

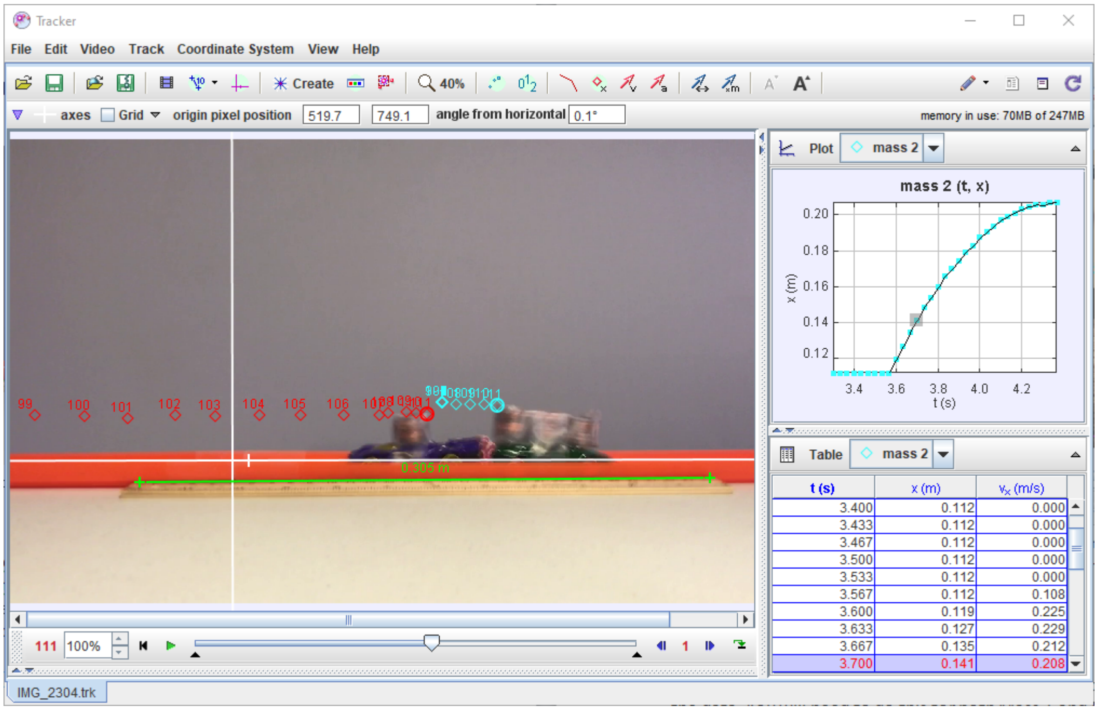

Fig. 18: Track the data for each car from at least 5 frames before the collision until at least 5 frames after the collision. Track the position of each car for all of these frames, even when they are stationary.

Export the Data

After you have finished tracking the path of each car, complete the following steps to export the data. You will need to do this for both Mass 1 and Mass 2. You can select which point mass is shown in the table using the dropdown next to the Table button.

1. Click Table and select only x and vx. You will only use vx for the calculations but x is useful to determine when the collision occurred.
2. Option 1 – Copy and paste the data
    a. Select all the data in the table
    b. Right Click > Copy Selected Cells > Full Precision
    c. Paste the data into Google Sheets or Excel

   Option 2 – Export a CSV file with the position and velocity data
    a. Click File > Export > Data File and set the following parameters:
       Cells = All Cells, Number Format = Full Precision, Delimiter = Comma
    b. Click Save As and name the file with the extension ".csv" (ex. "filename.csv").
    c. Using Google Sheets or Excel, import each CSV file.

Analysis (Option 2):

1. Open a blank Google Sheets or Excel document and import the data from Tracker.
    a. If you copied the data directly from the table in Tracker, you can simply paste it into the blank spreadsheet.
    b. If you exported a CSV file, you will need to import the data from that file into your spreadsheet.
2. Combine the two data sets (Mass 1 and Mass 2) by matching up the times, such that the position and velocity points for each mass at a given time are in the same row.
3. Reformat the cells into Number format and increase the decimal precision to include the full precision you copied from Tracker.
4. Using Google Sheets, Excel, or Graphical Analysis, generate plots of position vs time and velocity vs time with both positions (Mass 1 and Mass 2) in one graph and both velocities (Mass 1 and Mass 2) in another graph (see Fig. 19).

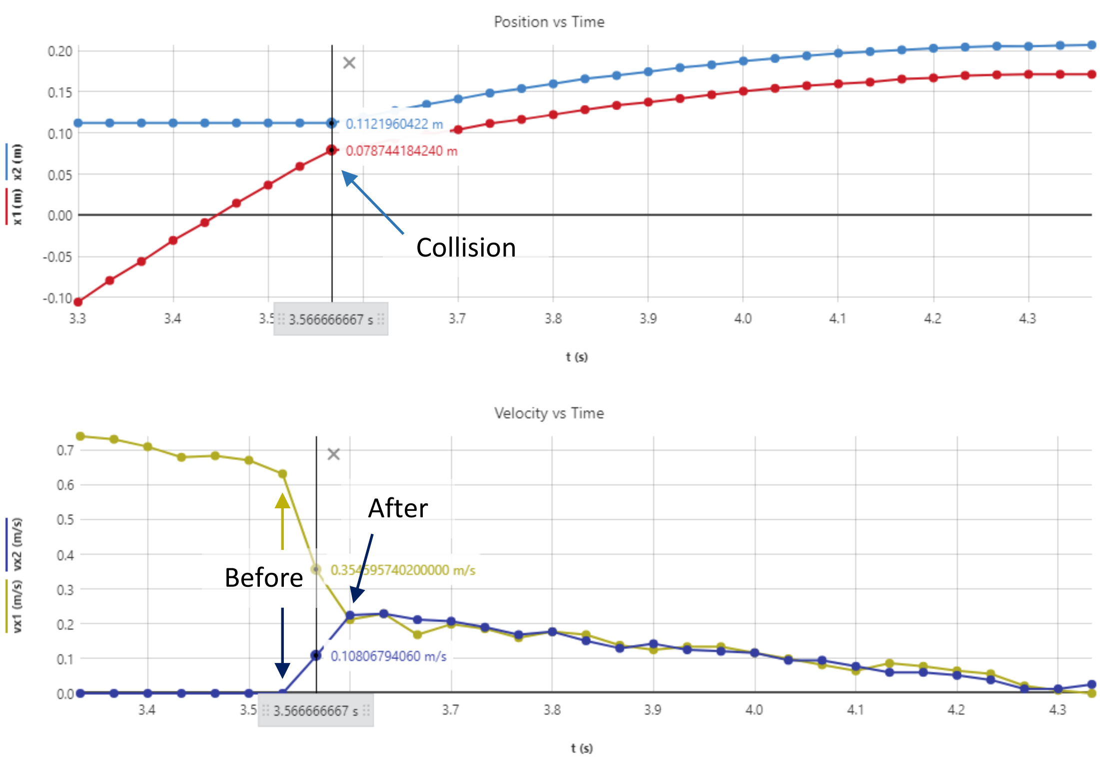

Fig 19: Plots of position vs time and velocity vs. time for two cars involved in an inelastic collision. On the position graph, the red line is the moving car and the blue line is the stationary car. Notice that after the collision, the two lines continue along together. On the velocity graph, the yellow line is the moving car and the blue line is the stationary car. Notice that the yellow line initially has some velocity and the blue line is at zero (stationary). Then at the collision, the two lines join and continue at the same velocity, eventually slowing down to a stop.

5. Look at your position graph to see at what time the collision occurred. Then look at the velocity graph to determine the initial velocities (just before the collision) and the final velocities (just after the collision). Note that since the cars stuck together during the collision, their final velocities should be the same. See Fig. 19 for an example of initial and final velocities in this experiment.
6. From the masses and velocities of each car, calculate the initial and final momenta (p = mv) and kinetic energies (KE = ½mv^2) for each car.
7. Calculate the total initial and total final momentum and kinetic energy (ex. $KE_{i\,Total} = KE_{i\,1} + KE_{i\,2}$).
8. Calculate the percent loss of momentum and kinetic energy to determine how well these quantities were conserved during the collision. If you calculate these in a spreadsheet, you can skip the x100% and just format the cell as a percent.

$$\%Loss = \frac{|total\ initial - total\ final|}{total\ initial} \times 100\%$$

Note that in an inelastic collision, KE will not be conserved, so you should expect a higher percent loss for KE. This loss in KE is due to heat and deformation that occur during the collision.