General Physics Lab 7

Rotational Motion

Objectives:

- To probe the kinetic energy of a rotating object

Equipment:

- Hot Wheels Track (1 – 20” track segment)
- Stack of Books
- 1 Racquetball
- 1” Wood Dowel Segment
- 1” PVC Pipe Segment
- Duct Tape
- Metric Ruler
- Metric Measuring Tape
- Triangular Wood Block
- 2 Rubber Bands
- Smartphone (camera) or Webcam
- Tracker Video Analysis software
  available for free download at https://physlets.org/tracker/

Physical Principles:

Energy Considerations for an Object Rolling without Slipping

The kinetic energy of a solid, rotating object is the sum of kinetic energies of all the moving particles and can be written as follows

$$KE_{rot} = \frac{1}{2} I\omega^2 \qquad (1)$$

where I is the moment of inertia of the solid object about the axis of rotation,

$$I = \sum_i m_i r_i^2 \qquad (2)$$

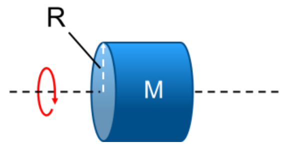
and

$\omega = \frac{\Delta\theta}{\Delta t}$ (rad/s) (3)

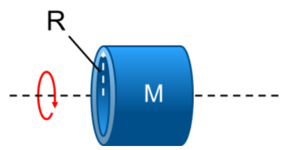
is the angular velocity of the rotation.

Moments of inertia of some commonly encountered objects of mass, M, and radius, R, are included here.

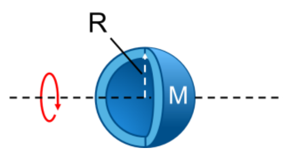
$I_{solid\ disk} = \frac{1}{2}MR^2$ (4)



$I_{hollow\ cylinder} = MR^2$ (5)



$I_{hollow\ sphere} = \frac{2}{3}MR^2$ (6)



For simplicity, we can define the fractional multiplier, *f*, multiplying MR^2, such that, in general,

$I_{object} = f_{object}\ MR^2$ (7)

where $f_{solid\ disk}$ = ½, $f_{hollow\ cylinder}$ = 1 and $f_{hollow\ sphere}$ = 2/3.

The radial symmetry of each object ensures that the mean location of mass will be at r = 0, i.e., directly on the axis of rotation. The product, fR^2, however, represents the variance of matter from the axis of rotation. Thus, an object with a larger fraction, f, has a greater fraction of its mass far from the axis of rotation and is thus harder to rotate.

When an object of radius, R, rolls without slipping with a velocity, *v*, the angular velocity and linear velocity are related as shown below.

$\omega = v/R$ (8)

The total kinetic energy of the rolling object is the sum of translational, KE_{trans}, and rotational, KE_{rot}.

$KE_{total} = KE_{trans} + KE_{rot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (9)

The speed that a solid object rolls down a hill depends on the moment of inertia, I_{object} .

Conservation of mechanical energy shows how this is so.

If the object is initially at rest and at height, h,

and ending at the bottom of the hill with speed, *v*,

then conservation of energy gives,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \ . \tag{10}$$

Rolling without slipping gives,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{2}fMR^2\left(\frac{v}{R}\right)^2 = \frac{1}{2}(1+f)Mv^2 \ . \tag{11}$$

Solving for v yields,

$$v = \sqrt{\frac{2gh}{(1+f)}} \ . \tag{12}$$

Thus, the larger the mulitplier, *f*, the larger the fraction of total energy goes into rotation – rather than translation - and the slower the speed of the rolling object.

Procedure:

Setup

1. This experiment begins much the same way as Lab 3 (Constant Acceleration) by building an inclined plane. Unlike Lab 3, you will only be using 1 Hot Wheels track segment for this setup. You are welcome to use two segments with extra support in the middle as you did in Lab 3, but using only one segment simplifies the setup, since it is stiff enough to support the weight of the rolling object without sagging significantly. However, if you are worried about sagging in the middle, you can still add additional support.
2. Start by finding a well-lit area with a flat level surface to work on. The surface must be level or the height measurement will be inaccurate.
3. Stack some books or other objects about 15 cm high on the surface to prop up one end of the track. You should prop up the connector end of the track so it won't be in the way at the bottom. Note that while you do not have to prop up the track exactly 15 cm, you shouldn't prop it up so high that the angle is too steep, causing the objects to slip instead of roll. The goal is for the objects to "roll without slipping".
4. Secure the top of the track with a little tape to keep it from sliding off (see Fig. 1).

5. Use the measuring tape to measure the height, h, of the book stack relative to the table surface (see Fig. 1). This will be the starting height for each object. Note that even though the center of mass of the objects will be higher than this point, h is still the change in height each object will experience, as long as you start them over this point.
6. Place the ruler on the table right next to the end of the track. This will be used to calibrate the distances in Tracker.

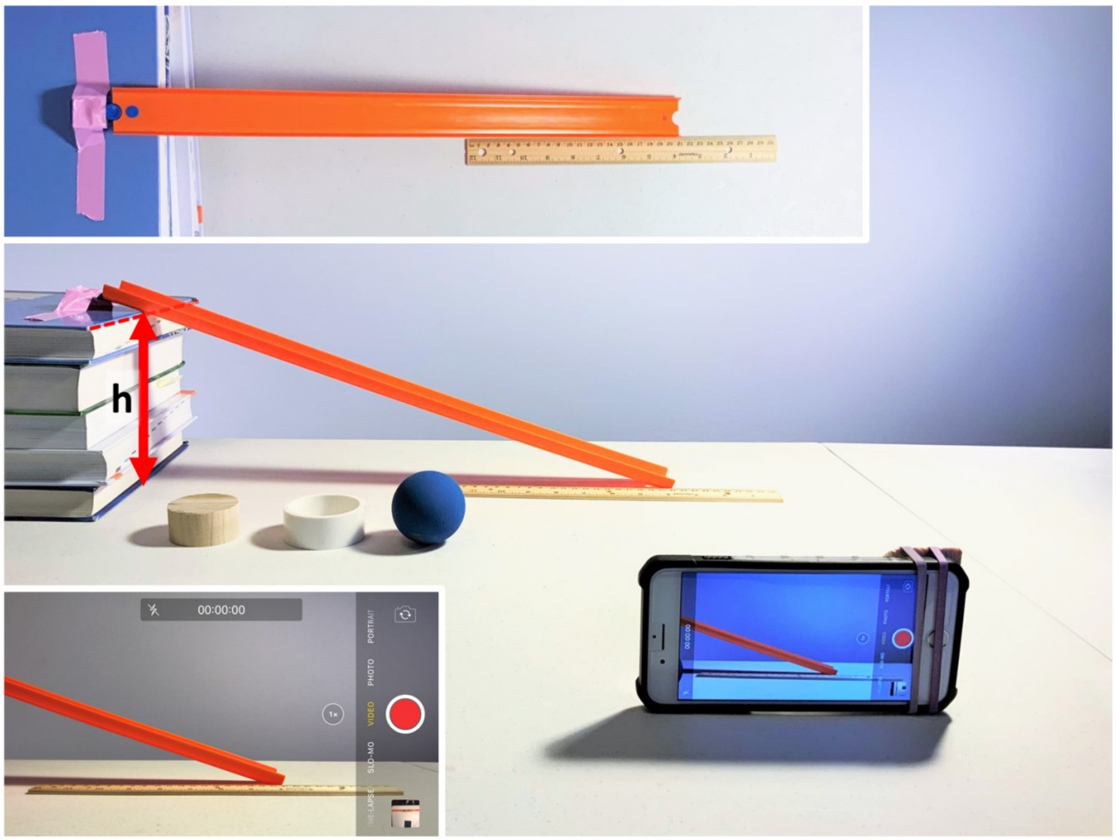


Fig. 1: Hot Wheels track setup
The connector end of the track is propped up on a 15 cm high stack of books and secured in place with duct tape. The ruler is placed on the table next to the end of the track and parallel to the track. The smartphone camera is placed on the table facing the end of the track and framed to include both ends of the ruler and a sufficient length of the track.

7. Select **any two** of the objects – solid disk (wood dowel segment), hollow cylinder (PVC pipe), and/or hollow sphere (racquetball) – to roll down the track.
8. Zero your spring balance. Then measure and record the mass, M, of each object in kg.

9. If you use a smartphone to film the rolling objects, use the wood block and rubber bands to make a stand as shown (see Fig. 2). Horizontal orientation is best for this experiment. Refer to the Smartphone Stand Instructions document for more information.

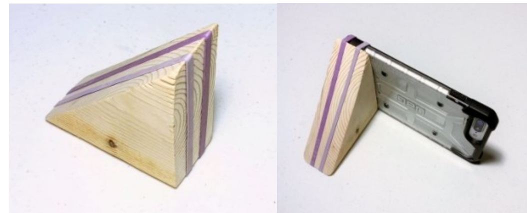


Fig. 2: Simple smartphone stand

10. Set the smartphone or webcam on the table facing the bottom of the track where you can capture the final velocity, v_f, of the rolling objects. Set it up so that the ball rolls into view of the camera from the left side.
11. Move the camera out far enough to see both ends of the ruler and at least the last half of the track.
12. Make sure the camera is not angled relative to the track or the motion data will be inaccurate. The camera and the track should be parallel to each other (see Fig. 3).

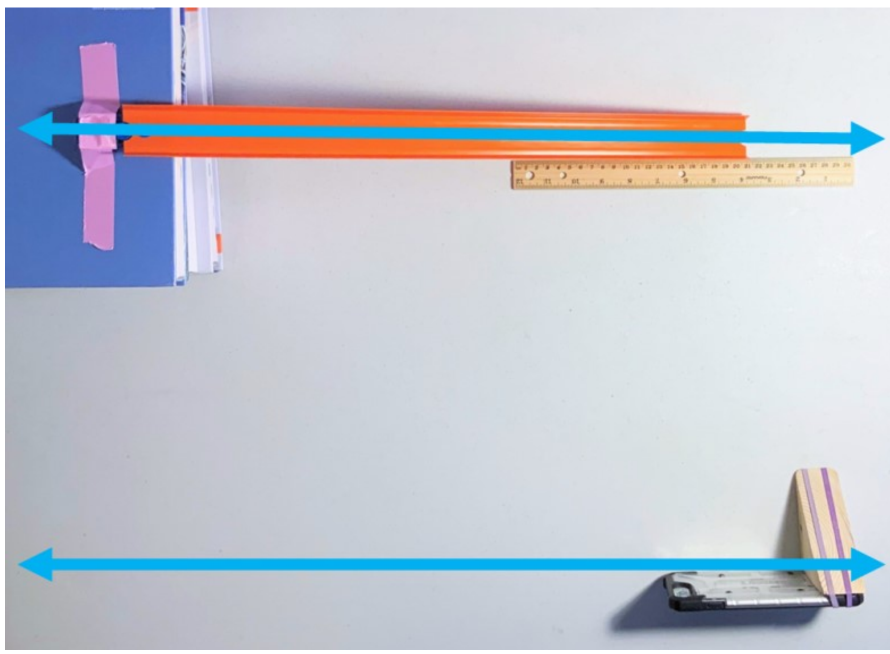


Fig. 3: Place the camera on the table facing the bottom of the track.
The camera, the track, and the ruler must all be parallel to each other.

Video Capture

1. Start recording a video, place an object at the top of the track with the center of the object directly above the point where you measured the height, and let the object roll down the track and onto the table. When you release the object, simply let go, without giving it any initial velocity.
2. Either in the same (short) video or in a second video, repeat the previous step with your second object, making sure to start it above the point where you measured the height.
3. Transfer the video(s) to your computer via some means (email, Google Drive, iCloud, Dropbox, etc.) for analysis using Tracker Video Analysis software.

Tracking the Motion

Complete the following steps in Tracker. You will need to do this for both of your objects. If you filmed both objects in the same video, you can create separate point masses for each and do everything in the same Tracker file. If you filmed them in separate clips, then just follow the normal procedure and analyze each video separately.

1. Open the video file in Tracker.
2. Save the Tracker file.
3. Using the Calibration Tools, add a new Calibration Stick, and align the calibration stick to the ruler (Shift+Click on the end points). Since the front of the ruler may appear larger than the back edge, try to place the end points on the back corners of the ruler. Set the length to the length of your ruler (see Fig. 4).
4. Click on the Coordinate Axes and place them on the video. Line up the origin on the edge of the track at the left side of the screen. Then rotate the x-axis down until it is aligned with the edge of the track (see Fig. 4).

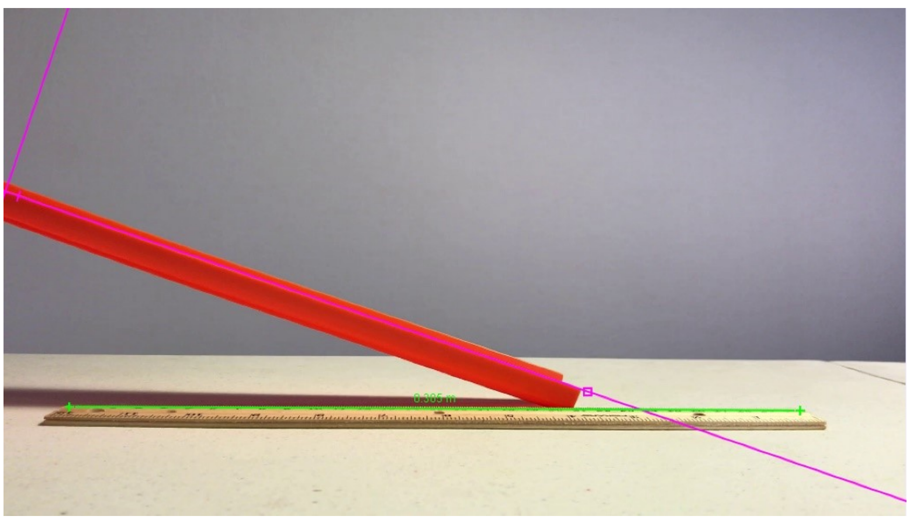


Fig. 4: Align the calibration stick with the back edge of the ruler.
Align the coordinate axes with the x-axis parallel to the track.

5. Click Create > Point Mass and name it (ex. Solid Disk, Hollow Cylinder, Hollow Sphere). You can rename it by right clicking on the default name to bring up an option menu.
6. Slide through the video to find the frame where the object first comes into view.
7. Add points (Shift+Click) on the object for each frame starting with this first frame and stopping on the last frame where the ball is still on the track. Technically, you only need the last two positions, but if you can, track the entire path as explained above.
8. You will notice that the object is blurred in front and behind in the direction of motion. This makes it difficult to track a single point on the object. One way you might do it is to track the top edge of the object. This works well for rolling objects as the blurred image in front and behind creates an intersection point that is often visible (see Fig. 5).

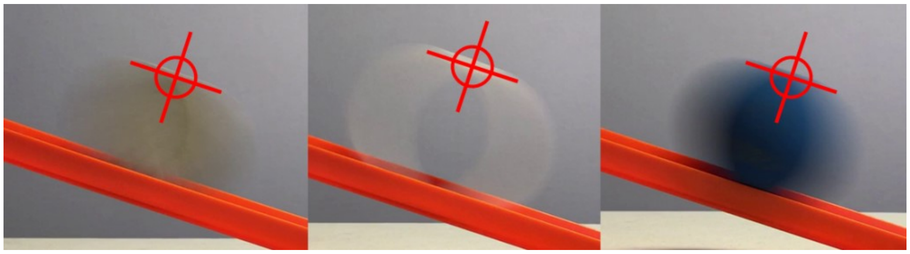


Fig. 5: Track the top edge of the rolling objects. Look for an intersection point where the blurred images overlap.

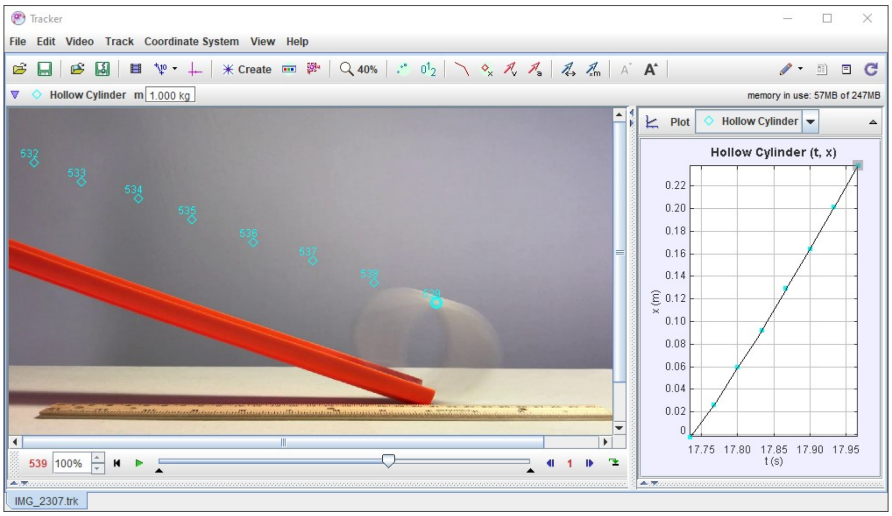


Fig. 6: Tracking the motion of a hollow cylinder rolling without slipping down an inclined plane.

Export the Data

After tracking the positions of each rolling object, complete the following steps to export the data. If you tracked both objects in the same video, you can select which point mass is shown in the table using the dropdown next to the Table button.

1. Click Table and select only x.
2. Option 1 – Copy and paste the data
    a. Select all the data in the table
    b. Right click > Copy Selected Cells > Full Precision
    c. Paste the data into Google Sheets or Excel

   Option 2 – Export a CSV file with the velocity data
    a. Click File > Export > Data File and set the following parameters:
       Cells = All Cells, Number Format = Full Precision, Delimiter = Comma
    b. Click Save As and name the file with the extension ".csv" (ex. "filename.csv")
    c. Using Google Sheets or Excel, import each CSV file.

Analysis:

1. Examine the data for your rolling objects and record the position, x, and time, t, for the last two points as the object is about to exit the incline.
2. Calculate the speed of each object at the bottom of the incline by taking the difference in position for the last two points and dividing by the corresponding difference in time. Record these times, positions, and velocities in your eJournal.

$$v_{f\ measured} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{13}$$

3. Calculate the predicted speed of each object at the bottom of the incline according to conservation of energy (Eq. 12).
4. Compare the measured speed to the predicted speed by calculating the percent difference.
5. Which object is fastest at the bottom?
6. Compare the speeds of each object with the theoretical speed of a block sliding down a frictionless incline or an object in freefall,

$$v_f = \sqrt{2gh} \tag{14}$$

where the change in height, h, is identical to your experiment.

Note that the speed of the rolling object (Eq. 12), will be less than the speed of a sliding/falling object (Eq. 14).

7. Calculate the initial potential energy of each object at the top of the incline and record this in your eJournal.
8. Calculate the translational, rotational, and total kinetic energies at the bottom of the incline and record them in your eJournal.
9. Calculate the percentage of the total kinetic energy that is translational and the percentage that is rotational for each object at the bottom of the incline. Record these in your eJournal.
10. Use a percent difference for each object to compare the total kinetic energy at the bottom of the incline with the potential energy at the top of the incline. Record the percent differences in your eJournal.