General Physics Lab 9

Archimedes' Principle

Objectives:

- To test Archimedes' Principle
- To experimentally determine the density of pennies

Equipment:

- Spring Balance
- Clear Container or Styrofoam Cup
- Water
- 100 Pennies
- Scotch Tape
- Thread

Physical Principles:

Pressure with Depth for a Static Fluid

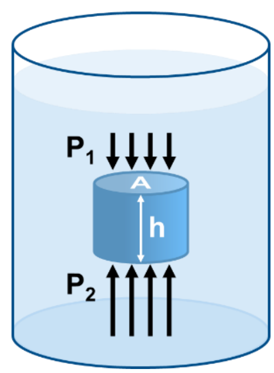

Fig. 1: Greater fluid pressure exerted on the bottom of a submerged object leads to a buoyant force.

The pressure, P, in a fluid increases with depth, h, according to the expression,

$$P_2 = P_1 + \rho g h \qquad (1)$$

or

$$\Delta P = \rho_{fluid} g h \, . \qquad (2)$$

An object immersed in fluid has more fluid pressure on the deeper bottom than on the shallower top (see Fig. 1). This leads to a buoyant force given by,

$$F_B = \Delta P A = \rho_{fluid} g h A = \rho_{fluid} g V \, . \qquad (3)$$

Eq. (3) is **Archimedes' Principle**.
In words, "The buoyant force, F_B, is equal to the weight, ρgV, of the displaced fluid."

We will use this principle to determine the density of an irregularly-shaped object. Since the object is totally submerged, the volume, V, of the displaced fluid is the same as the volume of the submerged object.

If the submerged object was a simple geometric solid (cylinder, cube, etc.), you could measure its volume directly. However, in this experiment, you will be using pennies, which have irregularities that prohibit accurate volume measurement. Instead, you will measure the weight.

We know that an object's volume and mass are directly related by the material's density ($\rho = m/V$). If we rearrange this equation to solve for volume,

$$V = \frac{m}{\rho} \tag{4}$$

we can substitute it into Eq. (3), yielding the following equation

$$F_B = \rho_{fluid}\, g\, \frac{m_{obj}}{\rho_{obj}} = \frac{\rho_{fluid}}{\rho_{obj}}\, m_{obj}\, g \tag{5}$$

where m_{obj} is the mass of the submerged object and ρ_{obj} is the density of the object.

You should now recognize that m_{obj} g is simply the object weight, W_{obj}. Substituting W_{obj} into Eq. (5) yields the following equation relating the buoyant force with the weight of the submerged object.

$$F_B = \frac{\rho_{fluid}}{\rho_{obj}}\, W_{obj} \tag{6}$$

This modified form of Archimedes' Principle is the equation you will test in this experiment. Using the known fluid density of water, varying the object weight, and measuring the buoyant force, you will be able to determine the material density, ρ_{obj}, from the slope of an F_B vs. W_{obj} graph.

Procedure:

Penny Density Prediction

In order to evaluate the experiment results, you need to predict the density of the pennies. Unfortunately, not all pennies have the same density but we do know how their composition has changed over the years.

Before 1982, pennies were composed of 95% copper and 5% zinc. Then part-way through 1982, the composition was changed to 2.5% copper and 97.5% zinc[1].

Using the known densities[2] of copper (8.940 g/cm^3) and zinc (7.135 g/cm^3), this translates to the following densities for pennies.

Table I: Penny Densities by Year

Before 1982	ρ_{penny} = 8.850 g/cm^3
During 1982	ρ_{penny} = 8.850 g/cm^3 or 7.180 g/cm^3
After 1982	ρ_{penny} = 7.180 g/cm^3

1. Separate all of your pennies from the taped bundles you used in the previous labs. Then sort them by year into categories of Pre-1982, Post-1982, and 1982 (see Fig. 2). Put away any pennies from 1982. We cannot use these for this experiment, because of their uncertain composition.
2. Determine which category (pre-1982 or post-1982) has the most coins. Since the different categories have different densities, you will only use the category with the most coins. Keep your selected category and put the rest away. Your penny density will be the density of the category you select (see Table I).

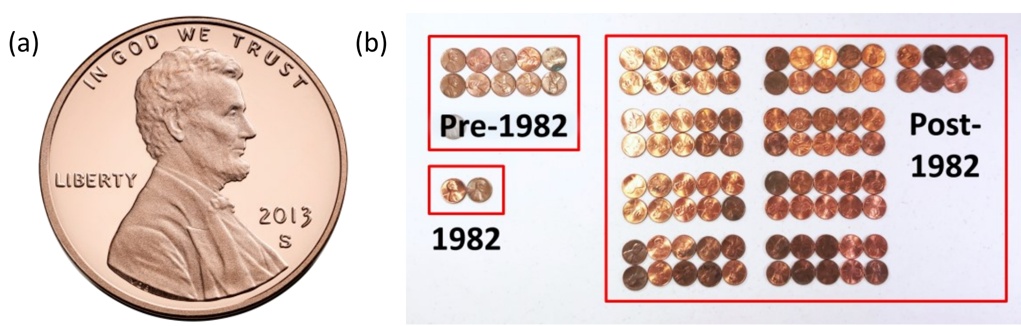

Fig. 2: (a) The year can be found on the front face of the penny. The penny in this example is 2013. (b) Sort your pennies by year, grouping them into pre-1982, post 1982, and 1982. In this example, there are 11 from before 1982, 2 from 1982, and 87 after 1982.

[1] Penny Year Compositions, Coin Collecting Enterprises
[2] Metals and Alloys – Densities, The Engineering Toolbox

Setup

1. If you have at least 80 pennies in your selected category, keep 80 and put the rest away. If you have less than 80 pennies, keep them all.
2. Divide your selected pennies into 4 roughly equal groups. It's ok if they don't all have exactly the same number.

Example 1: (Fig. 2b)
Suppose you have 11 pennies from before 1982, 2 from 1982, and 87 after 1982. The post-1982 category has the most, so you will use these. According to Table I, the density will be ρ = 7.180 g/cm^3. 87 is more than 80, so you will keep 80 of them and put the remaining 7 away. The 80 pennies are divided into 4 groups of 20 pennies per group.

Example 2: (no picture)
Suppose you have 62 pennies from before 1982, 1 from 1982, and 37 after 1982. The pre-1982 category has the most, so you will use these. According to Table I, the density will be ρ = 8.850 g/cm^3. 62 is less than 80, so you will keep all of them. The 62 pennies are divided into 4 groups with 15 in two of the groups and 16 in the other two.

3. Stack the pennies in each of your 4 groups, and tape the pennies in each stack together with Scotch tape. If there are more than 10-12 pennies per group, it may be easier to make 2 stacks per group and then tape the 2 stacks together. If you do 2 stacks per group, wait to join the two stacks until step 5 when you can attach the thread as well. This way, you will use less tape. Make sure to press the tape in tightly around the pennies to avoid air pockets getting trapped under the tape (see Fig. 3a).
4. Cut 4 lengths of thread, each about 20-25 cm long. Then tie loop knots at both ends of all four threads (see Fig. 3b). When you finish, you should have 4 threads (all about the same length) with loops on each end.
5. Place 1 knotted thread on top of each penny group and tape the thread to the pennies. (see Fig. 3c & d). After this, you should have 4 groups of pennies with thread taped to the top of each (see Fig. 3e). Again make sure to press the tape in tightly to avoid air pockets.
6. Somewhere on each penny weight, label the weights 1, 2, 3, and 4 (see Fig. 3f). This will help you keep track of the groups and it will be the order you use them in.

Example 1: (continued)
For each group of 20 pennies, they are divided into 2 stacks of 10 pennies each. Each stack is taped separately; then the 2 stacks and 1 knotted thread are taped together.

Example 2: (continued)
For each group of 15 or 16 pennies, they are divided into 2 stacks of 7 or 8 pennies each. Each stack is taped separately; then the 2 stacks and 1 knotted thread are taped together.

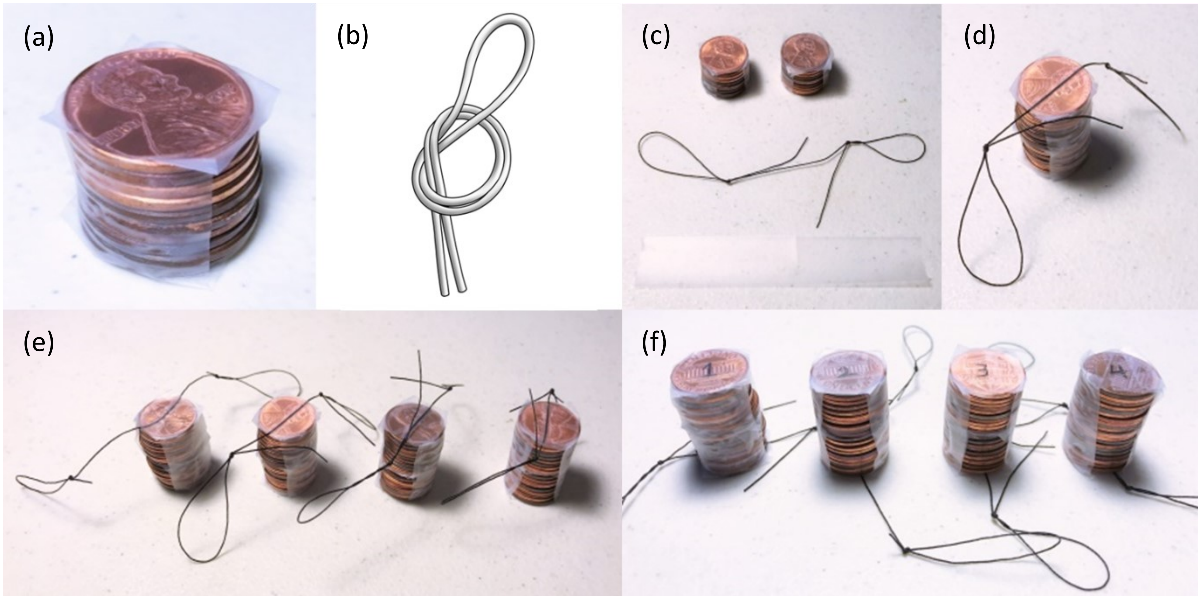

Fig. 3: (a) Stack the pennies in each of the 4 groups and tape them together with Scotch tape. If there are more than 10-12 pennies per group, it may be easier to make two separate stacks and then tape the stacks together. Press the tape in tightly to avoid trapped air pockets.
(b) Cut 4 lengths of thread, each 20-25 cm long and tie loop knots at both ends of each tread.
(c, d) Tape 1 knotted thread on top of the group.
(e) Repeat the previous step to make 4 weights with thread on top of each.
(f) Number the weights 1 – 4 so you can tell them apart.

Measure Weight in Air

1. Hang your spring balance vertically by the top loop and zero it.
2. Hang the #1 weight from the spring balance hook and record its weight in Newtons (see Fig. 4a & b).
3. Add the #2 weight to the spring balance hook and record the total weight (#1 and #2 together) in Newtons (see Fig. 4c).
4. Add the #3 weight to the hook and record the total weight in Newtons (see Fig. 4d).
5. Add the #4 weight to the hook and record the total weight in Newtons (see Fig. 4e).

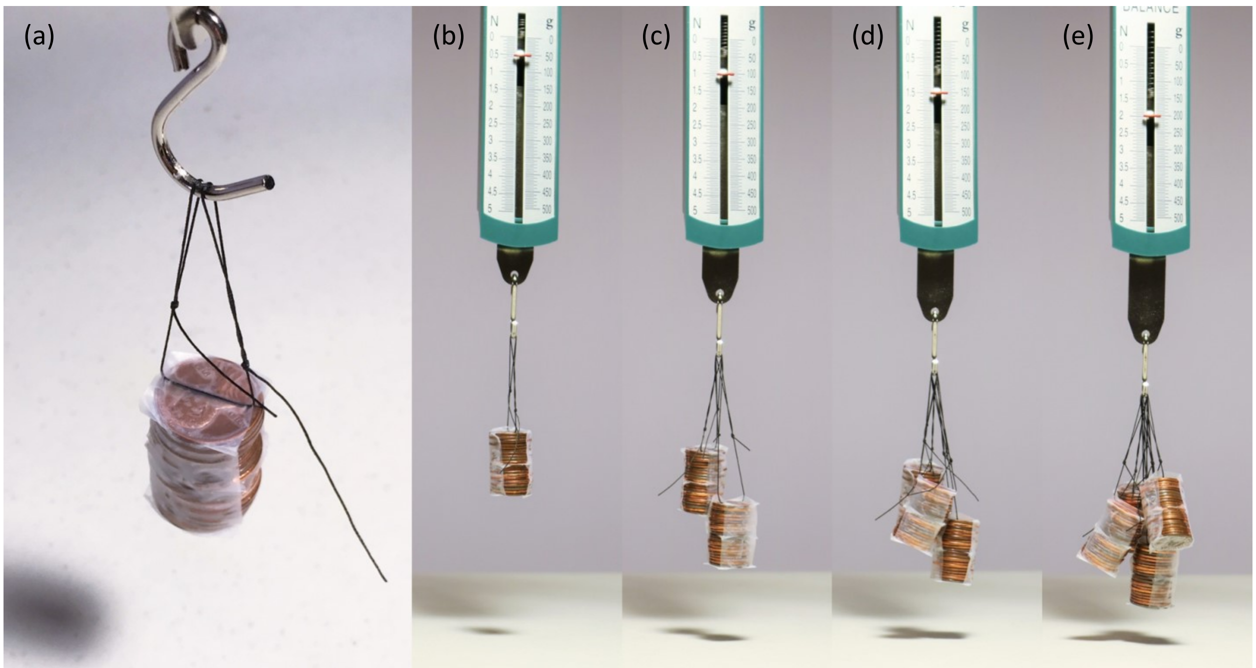

Fig. 4: (a) Hang the pennies from the spring balance hook using both thread loops.
(b) Weigh the #1 weight on the spring balance. (c) Add the #2 weight and record the total weight.
(d) Add the #3 weight and record the total weight. (e) Add the #4 weight and record the total weight.

Measure Apparent Weight in Water

1. Fill a small container (no shorter than 9-10 cm) with water. If you can find a clear container (empty jar, drinking glass, etc.), the following steps may be easier. If you cannot find anything like that, use an opaque container or the Styrofoam cup from your lab kit instead. Do NOT damage the Styrofoam cup since you will need it for lab 10.
2. With all 4 weights hanging on the spring balance, carefully submerge them below the surface of the water until they rest on the bottom of the container and are completely covered by water.
3. Gently tap/jostle/poke the pennies to encourage any trapped air bubbles (see Fig. 5) to dislodge and rise to the surface. It may take a minute or two of doing this for the air to fully escape from the gaps between the pennies. When it looks like all/most of the air has escaped, you may continue.

WARNING: Be careful when jostling the pennies that you don't un-zero the spring balance. After the underwater measurements, remove the spring balance and verify that it is still zeroed. If it is not, you will likely need to zero it again and redo the water measurements.

Fig. 5: Notice how bubbles trapped between the coins and the tape start to escape when submerged in water. Encourage all/most of the bubbles to dislodge and rise to the surface before weighing the pennies in water. If you do not, the air will increase the buoyancy of the pennies and introduce extra error into the experiment.

For the next steps, you will weigh the groups of pennies in the reverse order as you did in air. First you will weigh all of the them, then only #1,2,3, then #1 & 2, and finally #1 (see Fig. 6).

Make sure to keep the pennies submerged for the rest of the experiment or more air may become trapped and you will have to repeat the previous step.

4. Lift the pennies up a small amount until they are suspended in the water (not touching the bottom or sides of the container or extending above the surface). Record the apparent weight (force) on the spring balance (see Fig. 6a).

5. Without removing the pennies from the water, remove the #4 weight and set it aside. Then suspend the remaining pennies (#1,2,3) as in the previous step, and record the new force (see Fig. 6b).
6. Remove the #3 weight and weigh the remaining pennies (#1 & 2) as before (see Fig. 6c).
7. Remove the #2 weight and weigh the remaining pennies (#1) as before (see Fig. 6d).

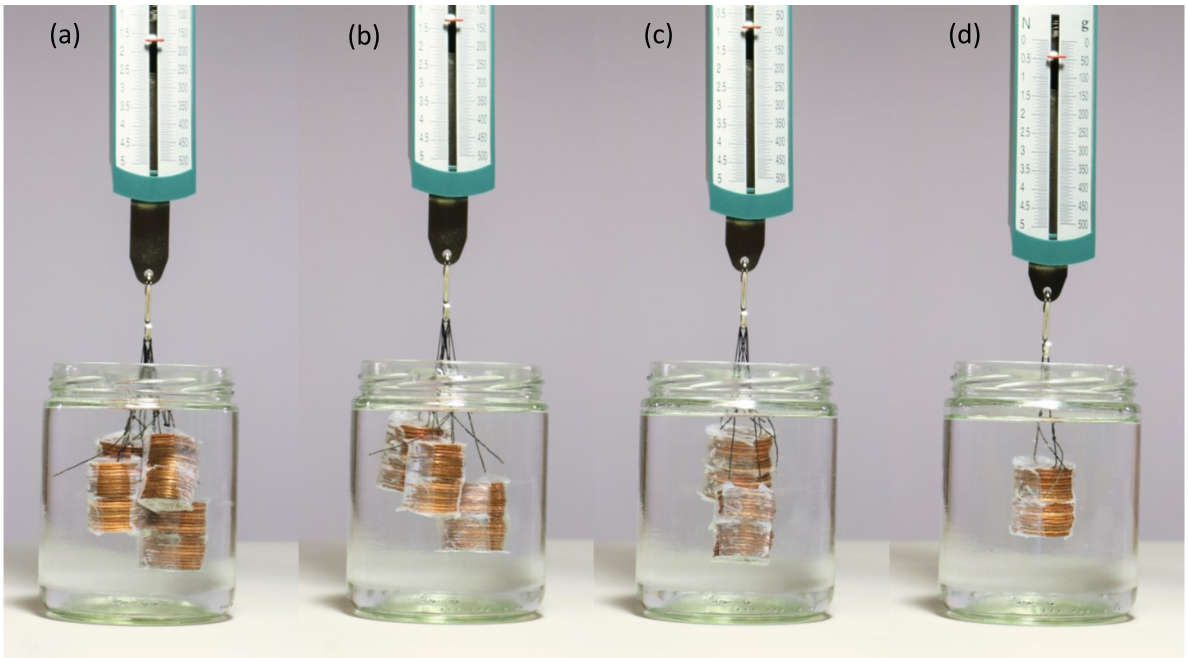

Fig. 6: After releasing the bubbles clinging to the pennies, record the weight of the pennies in water. (a) Weigh all the pennies together. (b) Remove the #4 weight and weigh the rest. (c) Remove the #3 weight and weigh the rest. (d) Remove the #2 weight and weigh the last one.

When you finish the experiment, separate the pennies from the tape and dry them off completely. If you need them in bundles for future labs, you will need to re-tape them.

Analysis:

1. Calculate the buoyant forces, F_B, by subtracting the weights in water from the weights in air.

$$F_B = W_{in\ air} - W_{in\ water} \quad (7)$$

2. From Eq. (6), we can see that a plot of buoyant force, F_B, vs. the weight of the pennies in air, $W_{pennies}$, should produce a linear graph with a slope equal to $\rho_{water}/\rho_{penny}$. Generate a plot of F_B vs. $W_{pennies}$, fit it with a linear trendline, and record the slope, m, and the correlation coefficient, R. Use the point (0,0) in your plot as this represents a zero buoyant force when the volume of the submerged object is zero.
3. Calculate the experimental penny density by dividing the known density of water at room temperature (0.998 g/cm^3) by the slope of your graph.

$$\rho_{penny\ exp} = \frac{\rho_{water}}{slope} \quad (8)$$

Note: Because the slope represents the ratio of the densities, we can choose whichever density units are most convenient. In order to compare with the predicted density in units of g/cm^3, those are the units we will use here.

4. Use a percent difference to compare your experimental penny density with your predicted density.

$$\%Diff = \frac{|\rho_{pred} - \rho_{exp}|}{\rho_{pred}} \times 100\%$$

5. Note that your experimental density is likely to be lower than the predicted density, although there may be different factors causing this. If the pennies are very corroded, overall penny density may be decreased slightly. Another likely factor is air bubbles trapped under the tape and between the pennies. This trapped air increases the buoyancy of the pennies leading to an apparently lower density. Comparing your experimental density to your predicted density, was your experimental density lower or higher than predicted? What do you think was the mostly likely cause of error for your experiment?
6. Interpret the correlation coefficient, R, from your linear trendline. Remember that R = +1 is a perfect positive correlation, R = -1 is a perfect negative correlation, and R = 0 is no correlation. Based on your R value, how linear were your experimental results? What might this indicate about the precision of your experiment?