General Physics Lab 5

RC Time Constant

Objectives:

- Observe the charging/discharging of a capacitor
- Explore the relationship between the time constant, resistance, and capacitance

Equipment:

- Multimeter with Probes (ideally able to measure capacitance)
- Breadboard
- Jumper Wire Kit
- 2 Alligator Clip Wires
- 9V Battery
- 9V Battery Connector
- 30 kΩ Resistor
- 1000 μF Capacitor
- Triangular Wood Block & 2 Rubber Bands
- Smartphone (camera) or Webcam

Physical Principles:

Capacitors

When a voltage, V, is applied to a capacitor with capacitance, C, the two conductors store an amount of charge, ±Q, given by

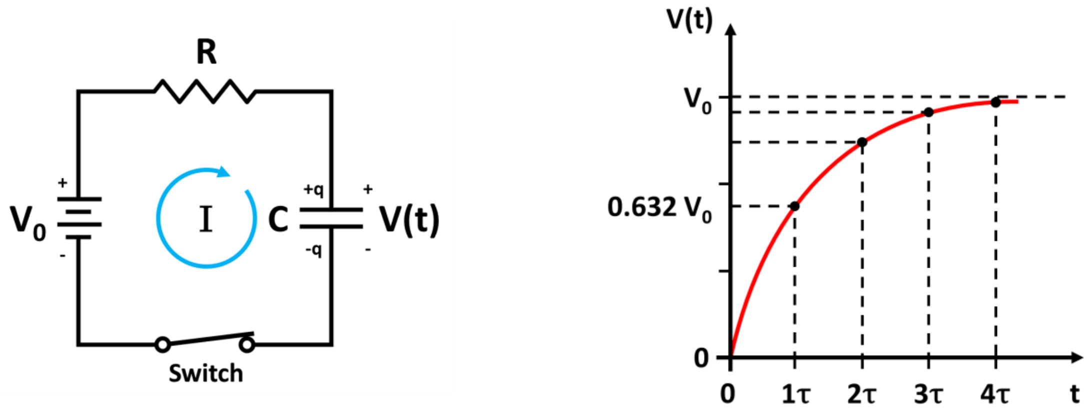
$$Q = CV \quad . \qquad (1)$$

Capacitors take time to either charge or discharge with the characteristic charging time, τ, given by the product of resistance, R, and capacitance, C.

$$\tau = RC \qquad (2)$$

The battery shown in Fig. 1a, supplies charge to the capacitor, C, when the switch is closed. The charge, q(t), on the capacitor increases as a function of time according to the relation,

$$q(t) = Q_0(1 - e^{-t/\tau}) \ , \qquad \text{(charging)} \qquad (3)$$



(a) (b)

Fig. 1: (a) Circuit for charging a capacitor. (b) Plot of voltage vs. time for a charging capacitor.

where $Q_0 = CV_0$ is the final charge accumulated on the capacitor as time approaches infinity and V_0 is the voltage applied by the battery.

Since the voltage, V(t), across the capacitor is directly proportional to the charge (see Eq. 1), we can also write the following.

$$V(t) = V_0(1 - e^{-t/\tau}) \qquad \text{(charging)} \qquad (4)$$

The graph of Eq. (4) appears as shown in Fig. 1b. As you can see, we would have to wait forever for the capacitor to fully charge, but the charge reaches nearly 2/3 (more precisely 0.632) of V_0 when the time is equal to the time constant, τ.

Note that when t = τ = RC, the voltage in Eq. (4) becomes,

$$V(\tau) = V_0(1 - e^{-1}) \approx 0.632\, V_0 \ . \qquad (5)$$

In the reverse process, a previously charged capacitor may be discharged through a resistor when the switch is closed and the battery is replaced with a wire (see Fig. 2a). The discharge process is described by the following equation.

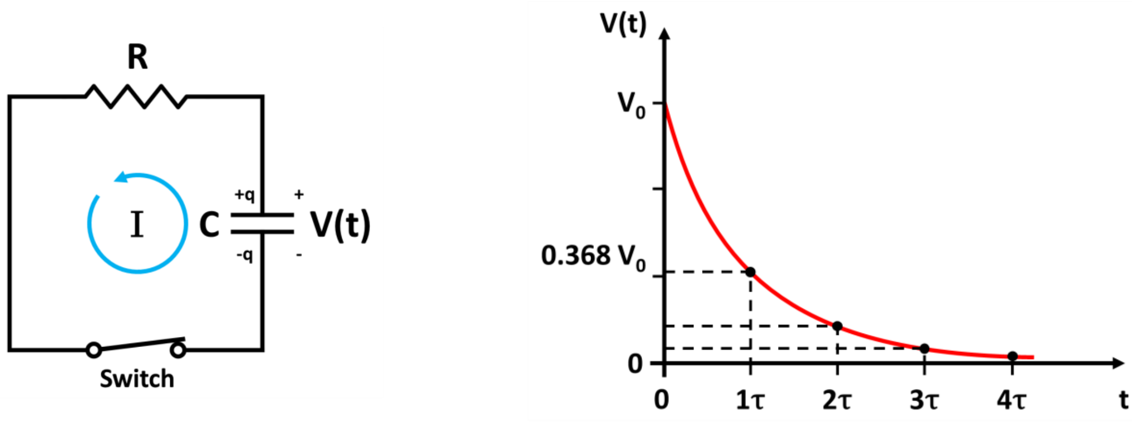
$$q(t) = Q_0 e^{-t/\tau} \qquad \text{(discharging)} \qquad (6)$$

where Q_0 is the initial charge on the capacitor.

Also, the voltage as a function of time becomes,

$$V(t) = V_0 e^{-t/\tau} \ , \qquad \text{(discharging)} \qquad (7)$$

where V_0 is the initial voltage on the capacitor before discharging.



(a) (b)

Fig. 2: (a) Circuit for discharging a capacitor. (b) Plot of voltage vs. time for a discharging capacitor.

The graph of Eq. (7) appears as shown in Fig. 2b and decays as an inverse exponential with time. The characteristic decay time is again given by $\tau = RC$. In one RC time constant, the voltage decreases to approximately 1/3 of its initial value since,

$$V(\tau) = V_0 e^{-1} \approx 0.368\, V_0 \ . \qquad (8)$$

Suggestion: Divide the work between you and your partner so that one of you can be analyzing the charging data while the other collects and analyzes the discharging data.

Procedure:

Measure Resistance, Capacitance, and Voltage

1. Locate the 30 kΩ resistor from your lab kit.

   You can tell the resistor's nominal value from the 4 or 5-band color code printed on the resistor. To interpret this code, look up a resistor color code table or use an online calculator such as this one: https://resistorcolorcodecalc.com/.

2. Turn the multimeter dial to measure ohms (Ω), set it to the 200k range, and insert the probes in the appropriate ports (black in COM, red in Ω).
3. Carefully (it is easy to bend the resistor leads and hard to re-straighten them) place the 30 kΩ resistor in the breadboard and measure the resistance in Ω (see Fig. 3).

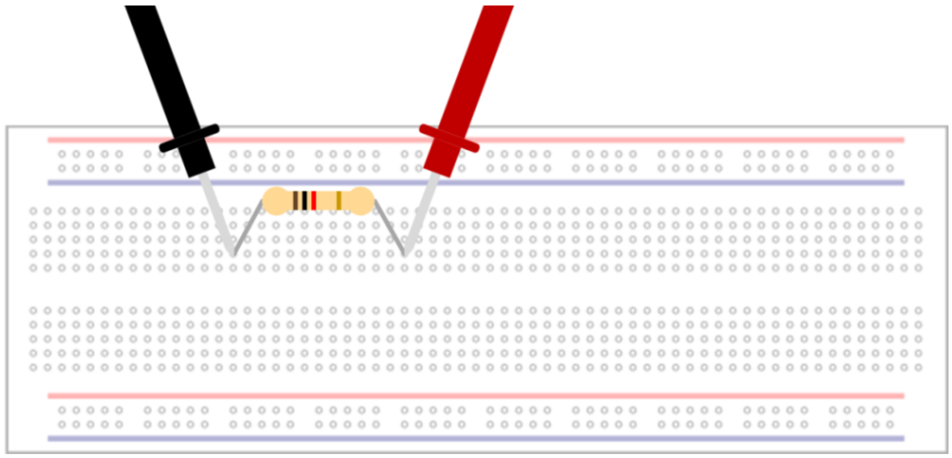


Fig. 3: Carefully place the resistor in the breadboard as shown and measure the resistance in Ω. The resistor in this figure is an example resistor only.

4. If your multimeter can measure capacitance, turn the multimeter dial to the appropriate capacitance range, insert the probes in the appropriate ports (black in COM, red in F), and measure the capacitance in farads (F) the same as you did with the resistor.
5. If your multimeter cannot measure capacitance, there are two options.
    a. The capacitor in your lab kit may have been measured ahead of time. If so, record this value as the measured capacitance.
    b. If the capacitor was not measured ahead of time, record the nominal capacitance (1000 µF = 0.001 F) and use this instead of the measured value.
6. Turn the multimeter dial to measure DC voltage (V ⎓), set it to the 20V range, insert the probes in the appropriate ports (black in COM, red in V), and measure the battery voltage, $V_{battery}$, by touching one probe to each pole of the battery (see Fig. 4). If the battery voltage has dropped below 8.5V, replace it with a different battery.

WARNING: Never attempt to measure across the poles of a battery or other power source while the multimeter is in current mode (A). Doing so will blow the internal fuse or even destroy the multimeter.

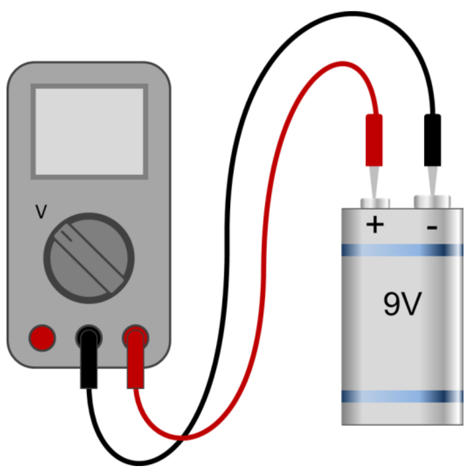


Fig. 4: Set multimeter dial to measure voltage (20V range), black probe in COM port, red probe in V port. Touch red probe to positive battery pole and black probe to negative battery pole.

Build the Circuit

1. Build the circuit as shown in Fig. 5.
   Do NOT connect the battery to the circuit until you are ready to charge the capacitor.

   WARNING: The capacitor is directional. Take care to connect the positive lead (long) and negative lead (short) as shown in the diagram. Reversing the polarity could damage or destroy the capacitor.

2. Set the multimeter to measure voltage (20V range) and connect the probes (black in COM, red in V).
3. Use alligator clip wires to connect the red multimeter probe to the positive capacitor lead and the black multimeter probe to the negative capacitor lead (see Fig. 5b).

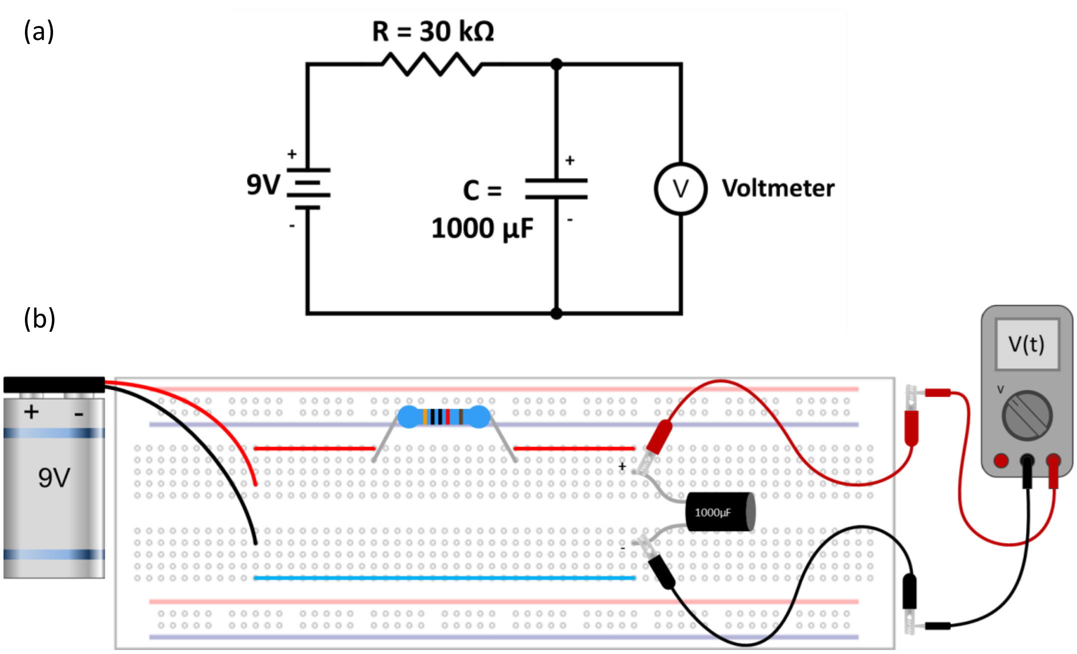


Fig. 5: RC Charging Circuit - (a) Circuit Diagram and (b) Breadboard wiring diagram with multimeter (acting as a voltmeter) connected in parallel to the capacitor using alligator clip wires. Make sure any wires/leads that should be connected are placed in the same row of 5 holes. Pay attention to the capacitor polarity (+/-). Remember that the longer lead is positive.

4. Use the built-in stand and/or other objects to prop up the multimeter at an angle.
5. Prop up/position your smartphone or webcam (see Fig. 6) so that the camera is pointed at the multimeter screen.
   If you choose to arrange things differently, that is fine. The important thing is that the camera can see the multimeter display clearly. Alternatively, you could hold your phone while filming the multimeter but having it hands-free will make the experiment easier.

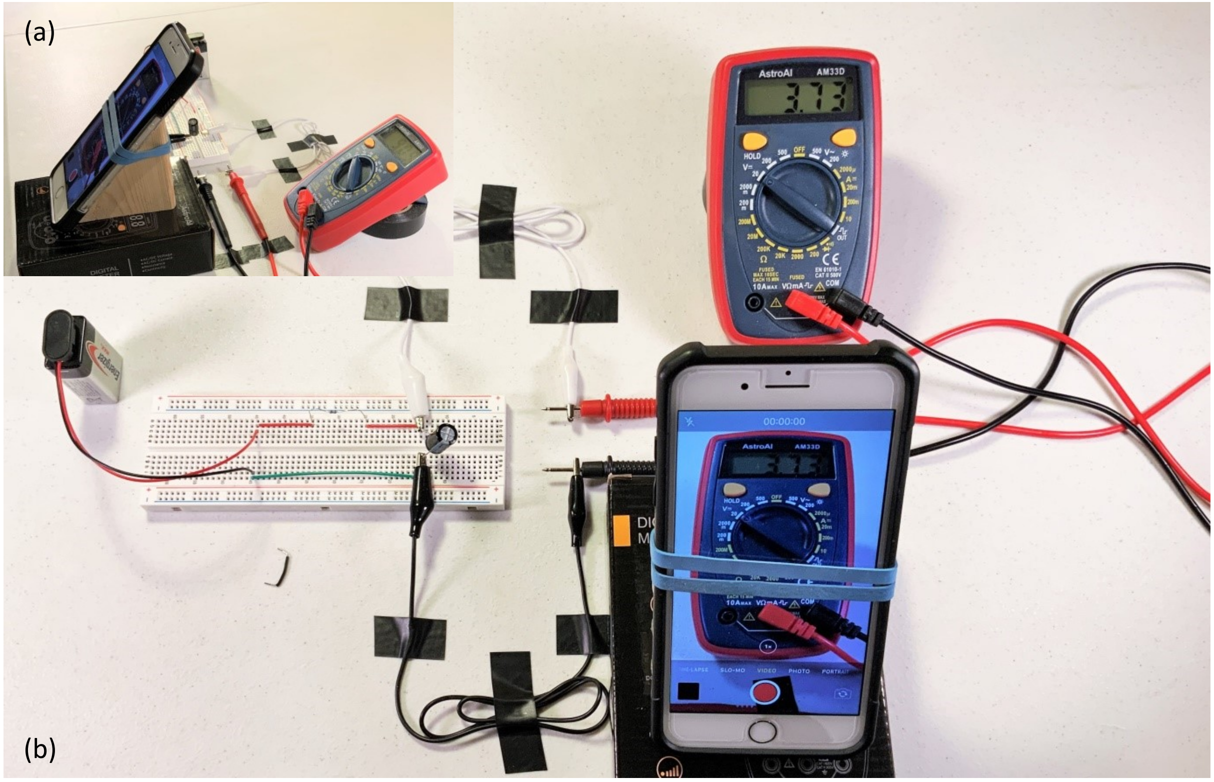


Fig. 6: Smartphone and multimeter arranged to record voltages as the capacitor charges.

Charging the Capacitor

1. Start recording a video on your phone or webcam.
2. Plug the battery into the circuit as shown in Fig. 5 and Fig. 6.
3. Continue recording the video for 2-3 minutes, during which time the voltage across the capacitor should climb from 0V to nearly 9V. It will rise quickly at first and then slow down over time (see Fig. 1b).

   Note: If you plugged in the battery before you were ready to record the video, the voltage will not change because the capacitor will already be charged. If you did this by mistake, look ahead and see how to discharge the capacitor. Then once it is discharged, come back to this section.

4. Stop the video, but leave the battery plugged into the circuit (keep it charged until you are ready to discharge it).
5. Export the video or share it with your partner so they can start the analysis.

Discharging the Capacitor

Now that the capacitor is fully charged, we will replace the battery in the previous charging experiment with a single jumper wire for discharging. This change needs to be performed quickly (within a second or two), so plan the procedures in advance.

1. In preparation, find a jumper wire that is long enough to bridge the gap across the center channel of the breadboard (see Fig. 7b).
2. Either have the wire ready or insert one end of the wire in the same row of 5 holes as the negative battery wire (leave the other end of the jumper disconnected).
3. Start recording a video on your phone or webcam.
4. Unplug the battery from the circuit and quickly bridge the gap with the jumper wire (see Fig. 7b). This must be done quickly or the capacitor voltage will begin to "leak".

   WARNING: Bridge the gap with the jumper wire after you remove the battery. If you bridge the gap first, you will be creating a short circuit across the battery.

5. Continue recording the video for 2-3 minutes, during which time the voltage across the capacitor should drop from about 9V to nearly 0V. It will drop quickly at first and then slow down over time (see Fig. 2b). After the 2-3 minutes, stop the video.

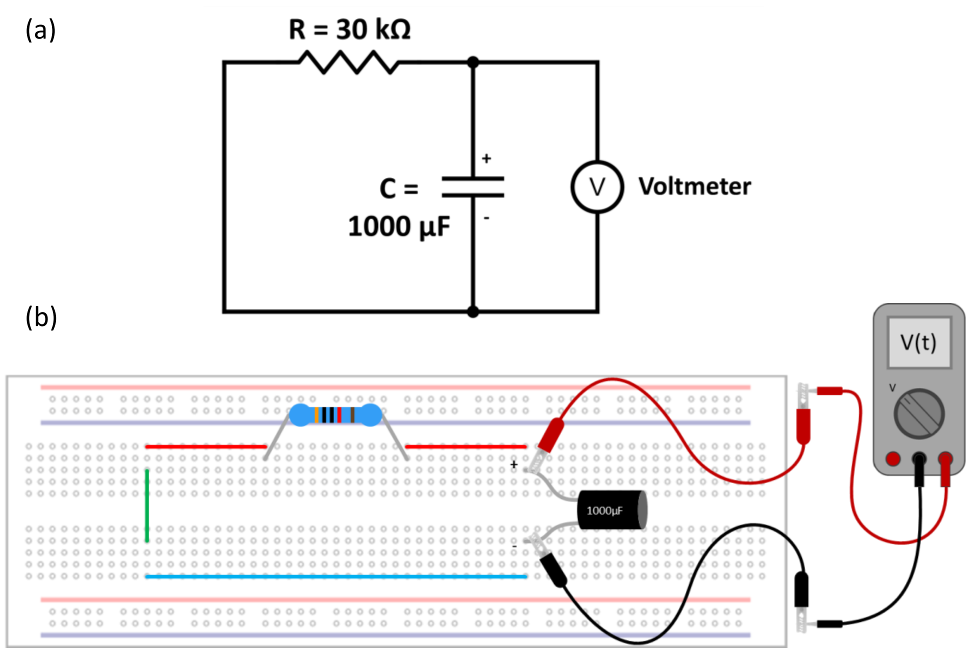


Fig. 7: RC Discharging Circuit - (a) Circuit Diagram and (b) Breadboard wiring diagram with multimeter (acting as a voltmeter) connected in parallel to the capacitor using alligator clip wires. In this diagram, a green jumper wire is bridging the gap where the battery was. Note that it does not need to be in the same holes that the battery wires were, but each end needs to be in the same row of 5 holes as the top and bottom jumper wires that it is connecting.

IMPORTANT: When you finish the experiment, turn off the multimeter to save the battery. The simpler inexpensive multimeters often do not shut off automatically.

Analysis:

Extract Data from Videos

This process will be identical for both videos.

1. Open the video, either on your phone or in a media player on your computer.
2. Download/access the provided spreadsheet ("RC Circuit Data").
    a. In the spreadsheet, there are tabs for Charging, Discharging, and an example data set.
    b. In each of these tabs, you should see columns for "Minutes", "Seconds", "t_video (s)", and "t (s)".
    c. The "Minutes" and "Seconds" columns represent the time (mm:ss) as displayed in the video player.
    d. The "t_video (s)" column converts the time from minutes and seconds into seconds (t_{video} = 60*Minutes + Seconds).
    e. The "t (s)" column adjusts the time so that the first measurement is at t = 0. This corrects for any delay in starting the charging/discharging process ($t = t_{video} – t_0$).
    f. Next there are columns for "V", and in the discharge tab, also "ln(V)". V is the voltage you will record at each timestamp and ln(V) calculates the natural log of V. You will need this later in the analysis.
3. Slide through the video until you find the time just before the voltage display begins to change. Record this time (minutes and seconds) and the voltage in the spreadsheet.
4. Continue going through the video (see Fig. 8) and record the time and voltage at roughly 0.5V intervals until you are within 0.5V of the final value.

    For example, you might have started at 7 seconds with 0V. Then you might get to 0.44V at 9 seconds. Then 1.03V at 10 seconds and so on.

5. Once you get to 0.5V away from the final value, continue recording the time and voltage for every 0.1V change until you are within 0.1V of the final value, then stop collecting data.

Fig. 8: Slide through the video and record the time (minutes and seconds) and voltage at 0.5V intervals.

Note: As you go through the video, you will notice that sometimes the multimeter display shows partial numbers or combinations of numbers (due to the display's slow refresh rate). Just find a different frame where the number is easy to read and use that instead.

Plot of Charging Capacitor

For the charging analysis, you will use the battery voltage, $V_{battery}$, as your source voltage, V_0.

1. According to Eq. (5) and Fig. 1b, the characteristic charging time for a capacitor, τ, is the amount of time required for the capacitor to charge to 63.2% of its final value, V_0 (battery voltage).
2. Scroll through your data of voltage vs. time for the charging capacitor and determine the time at which $V(t) = 0.632V_0$. If you do not have a data point at that exact time, just use the closest point or interpolate the time between the points on either side. Record this time as the estimated time constant, τ_{est}.
3. Calculate the predicted time constant, τ_{pred}, using Eq. (2).
4. Calculate the percent difference between τ_{pred} and τ_{est}.

$$\%Diff = \frac{|\tau_{pred} - \tau_{est}|}{\tau_{pred}} \times 100\%$$

5. In <u>Graphical Analysis</u>, plot the voltage, V(t), across the capacitor vs. time for the charging capacitor. Expect a plot as in Fig. 1b.
6. Fit a Natural Exponential curve to the charging capacitor data and record the fit parameters “a” and “c”.
   Note that you <u>must</u> plot the data in Graphical Analysis because Google Sheets and Excel do not handle exponential curve fits very well.
7. The parameters in the fit will be defined by the following equation.

$$y = a\exp(-cx) + b \qquad (9)$$

8. A comparison of Eq. (9) with Eq. (4) suggests that the parameter “a” corresponds to $-V_0$ in the circuit and the parameter “c” corresponds to 1/τ.
9. Calculate the percent difference between “-a” and V_0 (battery voltage).

$$\%Diff = \frac{|V_0 - (-a)|}{V_0} \times 100\%$$

10. Calculate the percent difference between 1/“c” and the predicted time constant, τ_{pred}, from Eq. (2).

$$\%Diff = \frac{|\tau_{pred} - (1/c)|}{\tau_{pred}} \times 100\%$$

Plot of Discharging Capacitor

Note: For the discharging analysis, you will need to use the initial capacitor voltage as V_0. When charging the capacitor, V_0 represents the source voltage (battery). Given enough time, the capacitor voltage will eventually equal the battery voltage, but if you perform the discharge experiment before the capacitor is fully charged, its initial voltage will be somewhat lower than the battery voltage. For this reason, you will need to use the initial capacitor voltage for V_0, rather than the battery voltage.

1. Look at the discharging data and record the initial capacitor voltage, V_0. This is the voltage just before you began discharging the capacitor.
2. According to Eq. (8) and Fig. 2b, the characteristic discharge time for a capacitor, τ, is the amount of time required for the capacitor to drain to 36.8% of its initial value.
3. Scroll through your data of voltage vs. time for the discharging capacitor and determine the time at which $V(t) = 0.368V_0$. If you do not have a data point at that exact time, just use the closest point or interpolate the time between the points on either side. Record this time as the estimated time constant, τ_{est}.
4. Calculate the predicted time constant, τ_{pred}, using Eq. (2).
5. Calculate the percent difference between τ_{pred} and τ_{est}.

$$\%Diff = \frac{\left|\tau_{pred} - \tau_{est}\right|}{\tau_{pred}} \times 100\%$$

6. In <u>Graphical Analysis</u>, plot the voltage, V(t), across the capacitor vs. time for the discharging capacitor. Expect a decaying exponential graph as in Fig. 2b.
7. Fit a Natural Exponential curve to the discharging capacitor data and record the fit parameters “a” and “c”.
   Note that you <u>must</u> plot the data in Graphical Analysis because Google Sheets and Excel do not handle exponential curve fits very well.
8. A comparison of Eq. (9) with Eq. (7) suggests that the parameter “a” corresponds to V_0 and the parameter “c” corresponds to 1/τ.
9. Calculate the percent difference between “a” and V_0 (initial capacitor voltage).

$$\%Diff = \frac{|V_0 - a|}{V_0} \times 100\%$$

10. Calculate the percent difference between 1/“c” and the predicted time constant, τ_{pred}, from Eq. (2).

$$\%Diff = \frac{\left|\tau_{pred} - \left(\frac{1}{c}\right)\right|}{\tau_{pred}} \times 100\%$$

11. The voltage vs. time curve may be linearized by taking the natural log (ln) of both sides of Eq. (7). This leads to the following equation.

$$\ln\big(V(t)\big) = -\frac{1}{\tau}t + \ln(V_0) \tag{10}$$

12. The plot of ln(V) vs. t is expected to be linear with a slope = -1/τ.
13. Use the calculated column in the Discharging Data tab of the provided spreadsheet to obtain the values of ln(V). Otherwise, you can make your own calculated column using the formula "=LN(cell)", where "cell" is the cell containing the voltage value you want.
14. Make a plot of ln(V) vs. t, fit a linear trendline to the data, and record the slope.
15. Calculate the percent difference between (-1/slope) and the predicted time constant, τ_{pred}.

$$\%Diff = \frac{\left|\tau_{pred} - \left(-\frac{1}{slope}\right)\right|}{\tau_{pred}} \times 100\%$$