5.5 Write Equations of Parallel and Perpendicular Lines

Goal • Write equations of parallel and perpendicular lines.

Your Notes

VOCABULARY

Converse A statement in which the hypothesis and conclusion of a conditional statement are interchanged

Perpendicular lines Two lines in a plane that intersect each other and form a right angle

PARALLEL LINES

If two nonvertical lines have the same slope, then they are parallel.

If two nonvertical lines are parallel, then they have the same slope.

Example 1 ***Write an equation of a parallel line***

Write an equation of the line that passes through (2, 4) and is parallel to the line $y = 4x + 1$.

Solution

Step 1 **Identify** the slope. The graph of the given equation has a slope of 4. So, the parallel line through (2, 4) has a slope of 4.

Step 2 **Find** the *y*-intercept. Use the slope and the given point.

$y = mx + b$ **Write slope-intercept form.**

$4 = 4(2) + b$ **Substitute 4 for *m*, 2 for *x*, and 4 for *y*.**

$-4 = b$ **Solve for *b*.**

Step 3 **Write** an equation. Use $y = mx + b$.

$y = 4x - 4$ **Substitute 4 for *m* and −4 for *b*.**

Your Notes

PERPENDICULAR LINES

If two nonvertical lines have the slopes that are negative reciprocals, then the lines are perpendicular.

If two nonvertical lines are perpendicular, then their slopes are negative reciprocals.

Example 2 ***Determine parallel or perpendicular lines***

Determine which of the following lines, if any, are parallel or perpendicular:

Line *a:* $12x - 3y = 3$

Line *b:* $y = 4x + 2$

Line *c:* $4y + x = 8$

Solution

Find the slopes of the lines.

Line *b*: The equation is in slope-intercept form. The slope is 4.

Write the equations for lines *a* and *c* in slope-intercept form.

Line *a*: $12x - 3y = 3$

$-3y = -12x + 3$

$y = 4x - 1$

Line *c*: $4y + x = 8$

$4y = -x + 8$

$y = -\frac{1}{4}x + 2$

Lines *a* and *b* have a slope of 4, so they are parallel.

Line *c* has a slope of $-\frac{1}{4}$, the negative reciprocal of 4, so it is perpendicular to lines *a* and *b*.

Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

1. Write an equation of the line that passes through $(-4, 6)$ and is parallel to the line $y = -3x + 2$.

$y = -3x - 6$

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2. Determine which of the following lines, if any, are parallel or perpendicular.

Line *a:* $4x + y = 2$

Line *b:* $5y + 20x = 10$

Line *c:* $8y = 2x + 8$

Lines *a* and *b* are parallel with a slope of -4. Line *c* is perpendicular to lines *a* and *b* with a slope of $\frac{1}{4}$.

Example 3 ***Determine whether lines are perpendicular***

Determine if the following lines are perpendicular.

Line *a:* $6y = 5x + 8$

Line *b:* $-10y = 12x + 10$

Solution

Find the slopes of the lines. Write the equations in slope-intercept form.

Line *a*: $6y = 5x + 8$

$y = \frac{5}{6}x + \frac{4}{3}$

Line *b*: $-10y = 12x + 10$

$y = -\frac{6}{5}x - 1$

The slope of line *a* is $\frac{5}{6}$. The slope of line *b* is $-\frac{6}{5}$.

The two slopes are negative reciprocals, so lines *a* and *b* are perpendicular.

Your Notes

Example 4 ***Write an equation of a perpendicular line***

Write an equation of the line that passes through $(-3, 4)$ and is perpendicular to the line $y = \frac{1}{3}x + 2$.

Solution

Step 1 **Identify** the slope. The graph of the given equation has a slope of $\frac{1}{3}$. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line through $(-3, 4)$ is -3.

Step 2 **Find** the y-intercept. Use the slope and the given point.

$y = mx + b$ — **Write slope-intercept form.**

$4 = -3(-3) + b$ — **Substitute** -3 **for** m, -3 **for** x, **and** 4 **for** y.

$-5 = b$ — **Solve for** b.

Step 3 **Write** an equation.

$y = mx + b$ — **Write slope-intercept form.**

$y = -3x - 5$ — **Substitute** -3 **for** m **and** -5 **for** b.

Checkpoint **Complete the following exercises.**

3. Determine whether line a through $(1, 3)$ and $(3, 4)$ is perpendicular to line b through $(1, -3)$ and $(2, -5)$. Justify your answer using slopes.

Line a: $m = \frac{1}{2}$; Line b: $m = -2$; perpendicular

4. Write an equation of the line that passes through $(4, -2)$ and is perpendicular to the line $y = 5x + 2$.

$y = -\frac{1}{5}x - \frac{6}{5}$

Homework