6.5 Solve Absolute Value Equations

Goal • Solve absolute value equations.

Your Notes

VOCABULARY

Absolute value equation An equation that contains an absolute value expression

Absolute deviation The absolute deviation of a number x from a given value is the absolute value of the difference of x and the given value.

SOLVING AN ABSOLUTE VALUE EQUATION

The equation $|ax + b| = c$ where $c \geq 0$ is equivalent to the statement $ax + b = c$ *or* $ax + b = -c$.

Example 1 ***Solve an absolute value equation***

Solve $|x - 9| = 2$.

Solution

$|x - 9| = 2$ — **Write original equation.**

$x - 9 = 2$ *or* $x - 9 = -2$ — **Rewrite as two equations.**

$x =$ 11 *or* $x =$ 7 — **Add** 9 **to each side.**

The solutions are 11 and 7. Check your solution.

CHECK

$|x - 9| = 2$ $\quad$ $|x - 9| = 2$ — **Write original equation.**

$|$ 11 $- 9| = 2$ $\quad$ $|$ 7 $- 9| = 2$ — **Substitute for x.**

$|$ 2 $| = 2$ $\quad$ $|$ −2 $| = 2$ — **Subtract.**

2 = 2 ✓ $\quad$ 2 = 2 ✓ — **Simplify. Solution checks.**

Your Notes

Example 2 ***Rewrite an absolute value equation***

Solve $4|2x + 8| + 6 = 30$.

Solution

First, rewrite the equation in the form $|ax + b| = c$.

$4|2x + 8| + 6 = 30$ — **Write original equation.**

$4|2x + 8| = 24$ — **Subtract 6 from each side.**

$|2x + 8| = 6$ — **Divide each side by 4.**

Next, solve the absolute value equation.

$|2x + 8| = 6$ — **Write absolute value equation.**

$2x + 8 = 6$ *or* $2x + 8 = -6$ — **Rewrite as two equations.**

$2x = -2$ *or* $2x = -14$ — **Subtract 8 from each side.**

$x = -1$ *or* $x = -7$ — **Divide each side by 2.**

Remember to check your solutions in the original equation for accuracy.

✔ ***Checkpoint*** **Solve the equation.**

1. $|x + 6| = 11$

5 and -17

2. $3|5x - 10| + 6 = 21$

3 and 1

Your Notes

Example 3 ***Decide if an equation has no solutions***

Solve $|7x - 3| + 8 = 5$, if possible.

Solution

$|7x - 3| + 8 = 5$ Write original equation.

$|7x - 3| = \underline{-3}$ Subtract $\underline{8}$ from each side.

The absolute value of a number is never $\underline{\text{negative}}$. So, there are no solutions.

Example 4 ***Use absolute deviation***

The absolute deviation of x from 10 is 1.8. Find the values of x that satisfy this requirement.

Solution

Absolute deviation $= |x - \text{given value}|$

$\underline{1.8}$ $= |x - \underline{10}|$

$\underline{1.8 = |x - 10|}$ **Write original equation.**

$\underline{1.8} = x - \underline{10}$ *or* $\underline{-1.8} = x - \underline{10}$ **Rewrite as two equations.**

$\underline{11.8} = x$ *or* $\underline{8.2} = x$ **Add $\underline{10}$ to each side.**

So, x is $\underline{11.8}$ or $\underline{8.2}$.

Checkpoint **Complete the following exercise.**

3. Find the values of x that satisfy the definition of absolute value for a given value of -13.6 and an absolute deviation of 2.8.

-10.8 and -16.4

Homework