8.5 Write and Graph Exponential Growth Functions

Goal • Write and graph exponential growth models.

Your Notes

VOCABULARY

Exponential function A function of the form $y = ab^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$

Exponential growth A quantity that increases by the same percent over equal time periods

Compound interest Interest earned on both an initial investment and on previously earned interest

Example 1 *Write a function rule*

Write a rule for the function.

x	−2	−1	0	1	2
y	$\frac{2}{9}$	$\frac{2}{3}$	2	6	8

Solution

Step 1 Tell whether the function is exponential. Here the *y*-values are multiplied by __3__ for each increase of **1** in *x*, so the table represents an exponential function of the form __$y = ab^x$__ where __$b = 3$__.

Step 2 Find the value of *a* by finding the value of *y* when *x* = 0. When *x* = 0, *y* = __ab^0__ = __$a \cdot 1$__ = __a__. The value of *y* when *x* = 0 is __2__, so __$a = 2$__.

Step 3 Write the function rule. A rule for the function is *y* = __$2 \cdot 3^x$__.
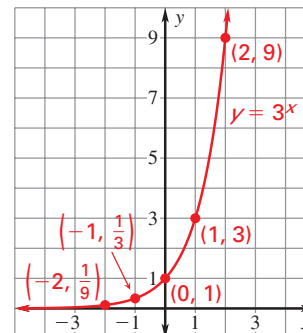
Example 2   *Graph an exponential function*

Graph the function $y = 3^x$. Identify its domain and range.

Solution

Step 1 Make a table by choosing a few values for x and finding the values of y. The domain is ___all real numbers___ .

| x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

Step 2 Plot the points.

Step 3 Draw a smooth curve through the points. From either the table or the graph, you can see that the range is ___all positive real numbers___ .

Example 3   *Compare graphs of exponential functions*

Graph $y = 2 \cdot 3^x$. Compare the graph with the graph of $y = 3^x$.

Solution

To graph each function, make a table of values, plot the points, and draw a smooth curve through the points.
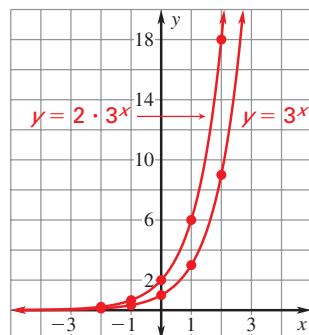
x	$y = 3^x$	$y = 2 \cdot 3^x$
−2	$\frac{1}{9}$	$\frac{2}{9}$
−1	$\frac{1}{3}$	$\frac{2}{3}$
0	1	2
1	3	6
2	9	18

Because the y-values for $y = 2 \cdot 3^x$ are ___2 times___ the corresponding y-values for $y = 3^x$, the graph of $y = 2 \cdot 3^x$ is a ___vertical stretch___ of the graph of $y = 3^x$.

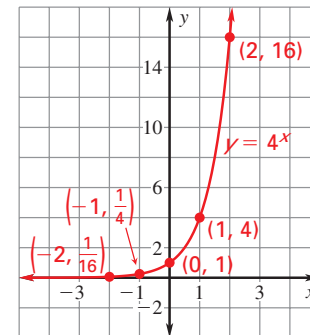✔ *Checkpoint* **Complete the following exercises.**

1. Write a rule for the function.

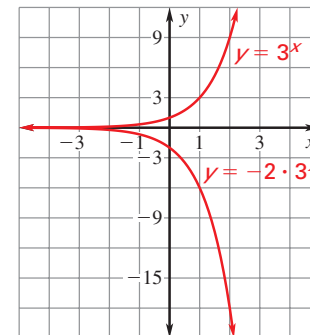| x	−2	−1	0	1	2
y	$-\dfrac{1}{16}$	$-\dfrac{1}{4}$	−1	−4	−16

$y = -1 \cdot 4x$

2. Graph $y = 4^x$. Identify its domain and range.

The domain is all real numbers. The range is all positive real numbers.

3. Graph $y = -2 \cdot 3^x$. Compare the graph with the graph of $y = 3^x$.

The graph of $y = -2 \cdot 3^x$ is a vertical stretch and a reflection in the *x*-axis of the graph of $y = 3^x$.

EXPONENTIAL GROWTH MODEL

$y = a(1 + r)^t$

a is the __initial amount__ .   *r* is the __growth rate__ .

$1 + r$ is the __growth factor__ .   *t* is the __time period__ .

Example 4   *Solve a compound interest problem*

Investment You put $250 in a savings account that earns 4% annual interest compounded yearly. You do not make any deposits or withdrawals. How much will your investment be worth in 10 years?

Solution

The initial amount is __$250__ , the interest rate is __4%__ , or __0.04__ , and the time period is __10 years__ .

$y = a(1 + r)t$ Write exponential growth model.

$= \underline{250}(1 + \underline{0.04})^{\underline{10}}$ Substitute __250__ for *a*, __0.04__ for *r*, and __10__ for *t*.

$= 250(\underline{1.04})^{10}$ Simplify.

$\approx \underline{370.06}$ Use a calculator.

You will have __$370.06__ in 10 years.

✔ *Checkpoint* **Complete the following exercise.**

4. In Example 4, suppose the annual interest rate is 5%. How much will your investment be worth in 10 years?

about $407.22

Homework