8.6 Write and Graph Exponential Decay Functions

Goal • Write and graph exponential decay functions.

Your Notes

VOCABULARY

Exponential decay A quantity that decreases by the same percent over equal time periods

Example 1 ***Graph an exponential function***

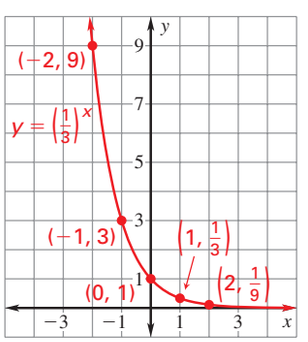
Graph the function $y = \left(\frac{1}{3}\right)^x$ and identify its domain and range.

Solution

Step 1 **Make** a table of values.
The domain is all real numbers.

x	-2	-1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

Step 2 **Plot** the points.



Step 3 **Draw** a smooth curve through the points. From either the table or the graph, you can see that the range is all positive real numbers.

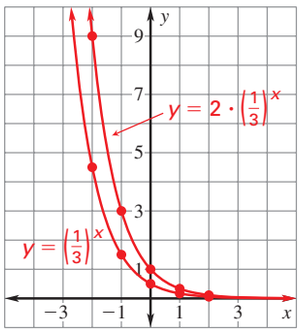
Your Notes

Example 2 ***Compare graphs of exponential functions***

Graph $y = 2 \cdot \left(\frac{1}{3}\right)^x$. Compare the graph with the graph of $y = \left(\frac{1}{3}\right)^x$.

Solution

x	$y = \left(\frac{1}{3}\right)^x$	$y = 2 \cdot \left(\frac{1}{3}\right)^x$
-2	9	18
-1	3	6
0	1	2
1	$\frac{1}{3}$	$\frac{2}{3}$
2	$\frac{1}{9}$	$\frac{2}{9}$

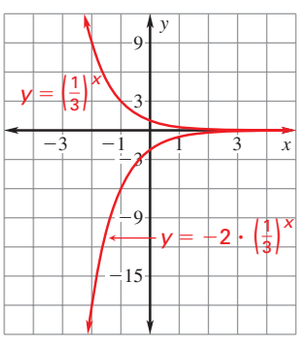


Because the y-values for $y = 2 \cdot \left(\frac{1}{3}\right)^x$ are 2 times the corresponding y-values for $y = \left(\frac{1}{3}\right)^x$, the graph of $y = 2 \cdot \left(\frac{1}{3}\right)^x$ is a vertical stretch of the graph of $y = \left(\frac{1}{3}\right)^x$.

Checkpoint **Complete the following exercise.**

1. Graph $y = -2 \cdot \left(\frac{1}{3}\right)^x$. Compare the graph with the graph of $\left(\frac{1}{3}\right)^x$.

The graph of $y = -2 \cdot \left(\frac{1}{3}\right)^x$ is a vertical stretch and a reflection in the x-axis of the graph of $y = \left(\frac{1}{3}\right)^x$.



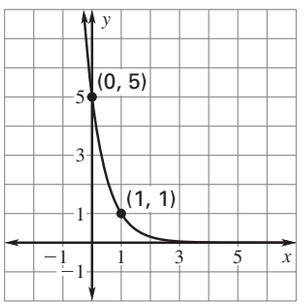
Your Notes

Example 3 ***Classify and write rules for functions***

Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.

Solution

The graph represents exponential decay ($y = ab^x$ where $0 < b < 1$). The y-intercept is 5, so $a =$ 5. Find the value of b by using the point (1, 1) and $a =$ 5.



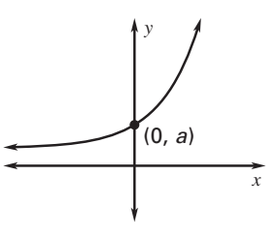
$y = ab^x$	**Write function.**
$1 = 5 \cdot b^1$	**Substitute.**
$0.2 = b$	**Solve.**

A function rule is $y = 5(0.2)^x$.

EXPONENTIAL GROWTH AND DECAY

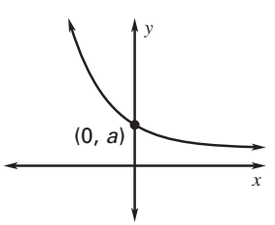
Exponential Growth

$y = ab^x$, $a > 0$ and $b > 1$



Exponential Decay

$y = ab^x$, $a > 0$ and $0 < b < 1$



EXPONENTIAL DECAY MODEL

$y = a(1 + r)^t$

a is the initial amount. r is the decay rate.

$1 - r$ is the decay factor. t is the time period.

Your Notes

Example 4 ***Use the exponential decay model***

Population The population of a city decreased from 1995 to 2003 by 1.5% annually. In 1995 there were about 357,000 people living in the city. Write a function that models the city's population since 1995. Then find the population in 2003.

Solution

Let P be the population of the city (in thousands), and let t be the time (in years) since 1995. The initial value is 357, and the decay rate is 0.015.

$P = a(1 - r)^t$ **Write exponential decay model.**

$= 357(1 - 0.015)^t$ **Substitute 357 for *a*, and 0.015 for *r*.**

$= 357(0.985)^t$ **Simplify.**

To find the population in 2003, 8 years after 1995, substitute 8 for t.

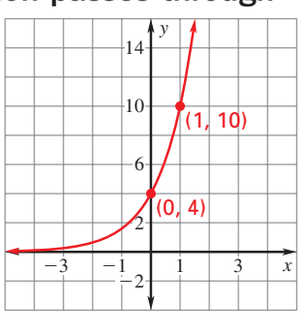
$P = 357(0.985)^8$ **Substitute 8 for *t*.**

≈ 316.3 **Use a calculator.**

The city's population was about 316,300 in 2003.

Checkpoint **Complete the following exercises.**

2. The graph of an exponential function passes through the points (0, 4) and (1, 10). Graph the function. Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.



Exponential growth;
$y = 4(2.5)^x$

3. In Example 4, suppose that the decay rate of the city's population remains the same beyond 2003. What will be the population in 2020?

about 244,700

Homework