3.3 Prove Lines are Parallel

Goal • Use angle relationships to prove that lines are parallel.

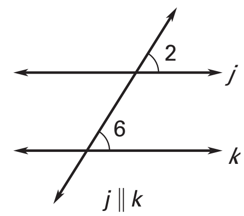
Your Notes

VOCABULARY

Paragraph proof A proof can be written in paragraph form, called a paragraph proof.

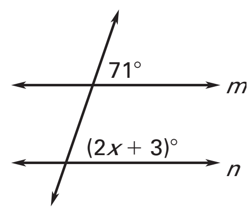
POSTULATE 16 CORRESPONDING ANGLES CONVERSE

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



Example 1 ***Apply the Corresponding Angles Converse***

Find the value of x that makes $m \parallel n$.



Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

$(2x + 3)° =$ 71° **Use Postulate 16 to write an equation.**

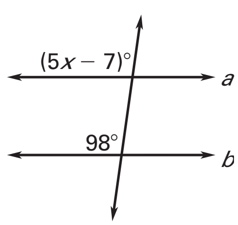
$2x =$ 68 **Subtract** 3 **from each side.**

$x =$ 34 **Divide each side by** 2**.**

The lines m and n are parallel when $x =$ 34.

Checkpoint **Find the value of x that makes $a \parallel b$.**

1.

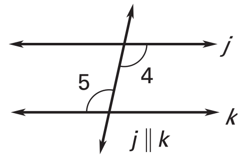


$x = 21$

Your Notes

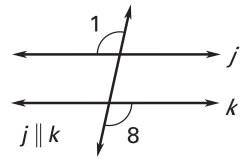
THEOREM 3.4 ALTERNATE INTERIOR ANGLES CONVERSE

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are _parallel_.



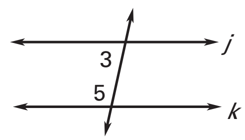
THEOREM 3.5 ALTERNATE EXTERIOR ANGLES CONVERSE

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are _parallel_.



THEOREM 3.6 CONSECUTIVE INTERIOR ANGLES CONVERSE

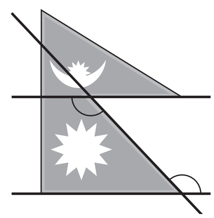
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are _parallel_.



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

Example 2 ***Solve a real-world problem***

Flags How can you tell whether the sides of the flag of Nepal are parallel?

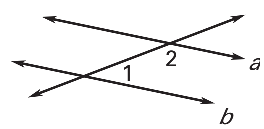


Solution

Because the _alternate interior angles_ are congruent, you know that the sides of the flag are _parallel_.

Checkpoint **Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.**

2. $m\angle 1 + m\angle 2 = 180°$

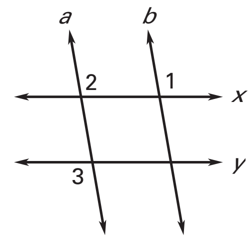


Yes, you can use the Consecutive Interior Angles Converse to prove $a \parallel b$.

Your Notes

Example 3 *Write a paragraph proof*

In the figure, $a \parallel b$ and $\angle 1$ is congruent to $\angle 3$. Prove $x \parallel y$.

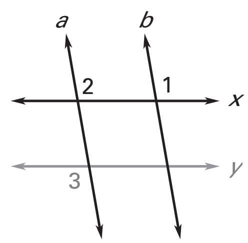


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

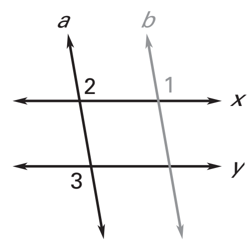
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



$\angle 1 \cong \angle 2$ because $a \parallel b$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$ then $x \parallel y$.

Plan in Action

In paragraph proofs, transitional words such as *so*, *then*, and *therefore* help to make the logic clear.

a. It is given that $a \parallel b$, so by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. Then $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. Therefore, by the Alternate Exterior Angles Converse, $x \parallel y$.

Checkpoint **Complete the following exercise.**

3. In Example 3, suppose it is given that $\angle 1 \cong \angle 3$ and $x \parallel y$. Complete the following paragraph proof showing that $a \parallel b$.

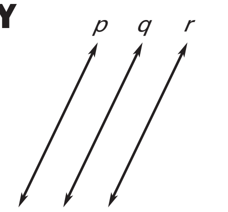
It is given that $x \parallel y$. By the Exterior Angles Postulate, $\angle 2 \cong \angle 3$.

It is also given that $\angle 1 \cong \angle 3$. Then $\angle 1 \cong \angle 2$ by the Transitive Property of Congruence for angles. Therefore, by the Corresponding Angles Converse, $a \parallel b$.

Your Notes

THEOREM 3.7 TRANSITIVE PROPERTY OF PARALLEL LINES

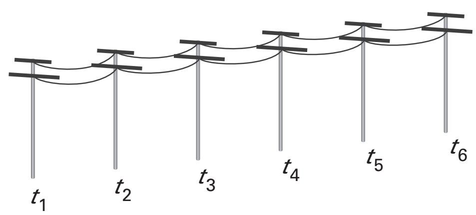
If two lines are parallel to the same line, then they are parallel to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Example 4 *Use the Transitive Property of Parallel Lines*

Utility poles Each utility pole shown is parallel to the pole immediately to its right. *Explain* why the leftmost pole is parallel to the rightmost pole.



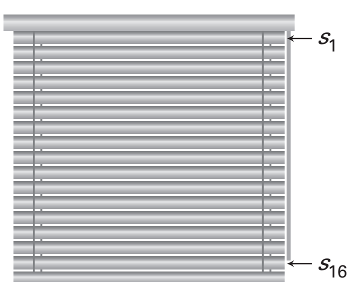
When you name several similar items, you can use one variable with subscripts to keep track of the items.

Solution

The poles from left to right can be named $t_1, t_2, t_3, \ldots, t_6$. Each pole is parallel to the one to its right, so $t_1 \parallel$ t_2, $t_2 \parallel$ t_3, and so on. Then $t_1 \parallel t_3$ by the Transitive Property of Parallel Lines. Similarly, because $t_3 \parallel t_4$, it follows that $t_1 \parallel$ t_4. By continuing this reasoning, $t_1 \parallel$ t_6. So, the leftmost pole is parallel to the rightmost pole.

Checkpoint **Complete the following exercise.**

4. Each horizontal piece of the window blinds shown is called a slat. Each slat is parallel to the slat immediately below it. *Explain* why the top slat is parallel to the bottom slat.



The slats from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{16}$. Each slat is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{16}$. So, the top slat is parallel to the bottom slat.

Homework