3.5 Write and Graph Equations of Lines

Goal • Find equations of lines.

Your Notes

VOCABULARY

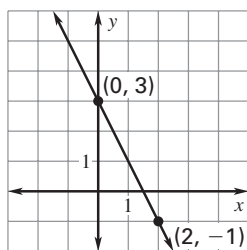
Slope-intercept form The general form of a linear equation in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

Standard form The general form of a linear equation in standard form is $Ax + By = C$, where A and B are not both zero.

Example 1 *Write an equation of a line from a graph*

Write an equation of the line in slope-intercept form.

Solution

Step 1 **Find** the slope. Choose two points on the graph of the line, $(0, 3)$ and $(2, -1)$.

$$m = \frac{3 - (-1)}{0 - 2} = \frac{4}{-2} = \underline{-2}$$

Step 2 **Find** the y-intercept. The line intersects the y-axis at the point $\underline{(0, 3)}$, so the y-intercept is $\underline{3}$.

Step 3 **Write** the equation.

$y = mx + b$ Use slope-intercept form.

$y = \underline{-2x + 3}$ Substitute $\underline{-2}$ for m and $\underline{3}$ for b.

Example 2   *Write an equation of a parallel line*

Write an equation of the line passing through the point (1, −1) that is parallel to the line with the equation y = 2x − 1.

Solution

Step 1 **Find** the slope *m*. The slope of a line parallel to y = 2x − 1 is the same as the given line, so the slope is __2__ .

Step 2 **Find** the *y*-intercept *b* by using *m* = __2__ and (*x*, *y*) = __(1, −1)__ .

$$y = mx + b \qquad \text{Use slope-intercept form.}$$

$$\underline{-1} = \underline{2} \, (\underline{1}) + b \qquad \text{Substitute for } x, y, \text{ and } m.$$

$$\underline{-3} = b \qquad \text{Solve for } b.$$

Because *m* = __2__ and *b* = __−3__ , an equation of the line is *y* = __2x − 3__ .

> The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

✔ *Checkpoint* **Complete the following exercises.**

1. Write an equation of the line in the graph at the right.

y = 3*x* − 5
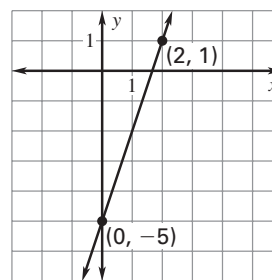
2. Write an equation of the line that passes through the point (−2, 5) and is parallel to the line with the equation *y* = −2*x* + 3.

y = −2*x* + 1

Example 3   *Write an equation of a perpendicular line*

Write an equation of the line *j* passing through the point (3, 2) that is perpendicular to the line *k* with the equation $y = -3x + 1$.

Solution

Step 1 Find the slope *m* of line *j*. The slope of *k* is __−3__ .

$$\underline{-3} \cdot m = \underline{-1}$$ The product of the slopes of perpendicular lines is __−1__ .

$$m = \frac{1}{3}$$ Divide each side by __−3__ .

Step 2 Find the *y*-intercept *b* by using $m = \frac{1}{3}$ and $(x, y) = \underline{(3, 2)}$.

$$y = mx + b$$ Use slope-intercept form.

$$\underline{2} = \frac{1}{3} (\underline{3}) + b$$ Substitute for *x*, *y*, and *m*.

$$\underline{1} = b$$ Solve for *b*.

Because $m = \frac{1}{3}$ and $b = \underline{1}$, an equation of line *j* is $y = \underline{\frac{1}{3}x + 1}$.

You can check that the lines *j* and *k* are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.
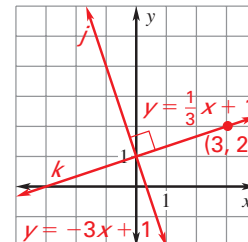


✔ *Checkpoint* **Complete the following exercise.**

3. Write an equation of the line passing through the point (−8, −2) that is perpendicular to the line with the equation $y = 4x - 3$.

$$y = -\frac{1}{4}x - 4$$

Example 4   *Write an equation of a line from a graph*

Rent The graph models the total cost of renting an apartment. Write an equation of the line. *Explain* the meaning of the slope and the *y*-intercept of the line.

Renting Costs
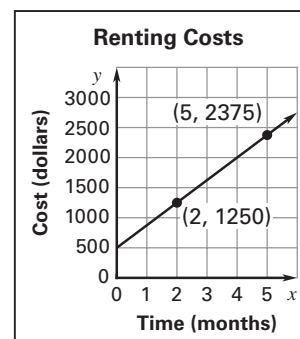
Step 1 Find the slope.

$$m = \frac{2375 - 1250}{5 - 2}$$

$$= \frac{1125}{3} = \underline{375}$$

Step 2 Find the *y*-intercept. Use a point on the graph.

$y = mx + b$ **Use slope-intercept form.**

$\underline{1250} = \underline{375} \cdot \underline{2} + b$ **Substitute.**

$\underline{500} = b$ **Simplify.**

Step 3 Write the equation. Because $m = \underline{375}$ and $b = \underline{500}$, an equation is $y = \underline{375x + 500}$.

The equation $y = \underline{375x + 500}$ models the cost. The slope is the __monthly rent__, and the __*y*-intercept__ is the initial cost to rent the apartment.

Example 5   *Graph a line with equation in standard form*

Graph $2x + 3y = 6$.

The equation is in standard form, so use the __intercepts__.

Step 1 Find the intercepts.

To find the *x*-intercept, let $y = \underline{0}$.

$2x + 3y = 6$

$2x + 3(\underline{0}) = 6$

$x = \underline{3}$

To find the *y*-intercept, let $x = \underline{0}$.

$2x + 3y = 6$

$2(\underline{0}) + 3y = 6$

$y = \underline{2}$

Step 2 Graph the line.

The intercepts are __(3, 0)__ and __(0, 2)__. Graph these points, then draw a line through the points.
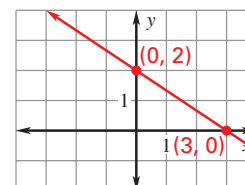
Example 6   *Solve a real-world problem*

Subscriptions You can buy a magazine at a store for $3. You can subscribe yearly to the magazine for a flat fee of $18. After how many magazines is the subscription a better buy?

**Solution**

Step 1 Model each purchase with an equation.

   Cost of yearly subscription: $y = $ __18__

   Cost of one magazine: $y = $ __3__ x, where x represents
   the number of magazines

Step 2 Graph each equation.

The point of intersection is __(6, 18)__ . Using the graph, you can see that it is cheaper to buy magazines individually if you buy less than __6__ magazines per year. If you buy more than __6__ magazines per year, it is cheaper to buy a subscription.

> The point at which the costs are the same is sometimes called the *break-even point*.

Magazine Purchases

Cost (dollars) — Number of magazines

$y = 18$
(6, 18)
$y = 3x$
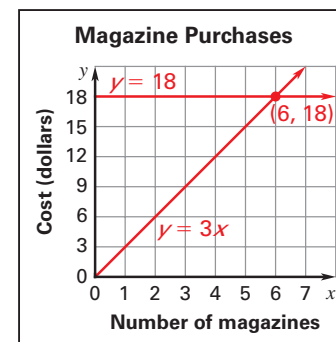
✔ *Checkpoint* **Complete the following exercises.**

4. The equation $y = 650x + 425$ models the total cost of joining a health club for x years. What are the meaning of the slope and y-intercept of the line?

The slope is the cost per year, $650, and the y-intercept is the initiation fee, $425.

5. Graph $y = 3$ and $x = 3$.

(0, 3) $y = 3$
1
(3, 0)
$x = 3$
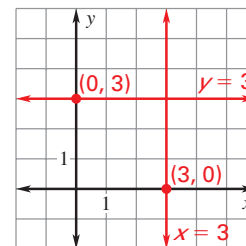
6. In Example 6, suppose you can buy the magazine at a different store for $2.50. After how many magazines is the subscription the better buy?

8 magazines

Homework