

3.6

Prove Theorems About Perpendicular Lines

Goal • Find the distance between a point and a line.

Your Notes

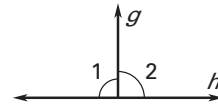
VOCABULARY

Distance from a point to a line **The distance from a point to a line is the length of the perpendicular segment from the point to the line.**

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

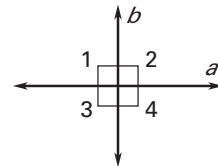
If $\angle 1 \cong \angle 2$, then $g \perp h$.



THEOREM 3.9

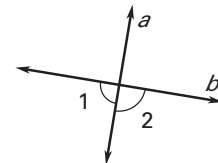
If two lines are perpendicular, then they intersect to form four right angles.

If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.



Example 1 Draw conclusions

In the diagram at the right, $\angle 1 \cong \angle 2$. What can you conclude about a and b ?



Solution

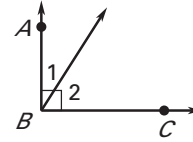
Lines a and b intersect to form a linear pair of congruent angles, $\angle 1$ and $\angle 2$. So, by Theorem 3.8, $a \perp b$.

Your Notes

THEOREM 3.10

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.

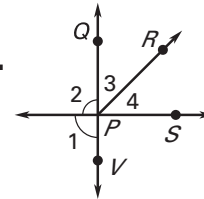


Example 2 Write a proof

In the diagram at the right, $\angle 1 \cong \angle 2$.
Prove that $\angle 3$ and $\angle 4$ are complementary.

Given $\angle 1 \cong \angle 2$

Prove $\angle 3$ and $\angle 4$ are complementary.



Statements

- $\angle 1 \cong \angle 2$
- $\overrightarrow{PS} \perp \overrightarrow{PQ}$
- $\angle 3$ and $\angle 4$ are complementary.

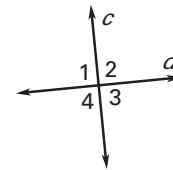
Reasons

- Given
- Theorem 3.8
- Theorem 3.10

Checkpoint Complete the following exercises.

- If $c \perp d$, what do you know about the sum of the measures of $\angle 3$ and $\angle 4$?
Explain.

Because $c \perp d$, angles 1, 2, 3, and 4 are right angles by Theorem 3.9.
So, $m\angle 3 + m\angle 4 = 180^\circ$.



- Using the diagram in Example 2, complete the following proof that $\angle QPS$ and $\angle 1$ are right angles.

Statements

- $\angle 1 \cong \angle 2$
- $\overrightarrow{PS} \perp \overrightarrow{PQ}$
- $\angle QPS$ and $\angle 1$ are right angles.

Reasons

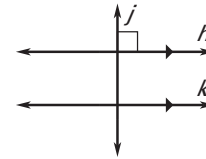
- Given
- Theorem 3.8
- Theorem 3.9

Your Notes

THEOREM 3.11 PERPENDICULAR TRANSVERSAL THEOREM

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

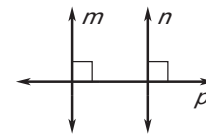
If $h \parallel k$ and $j \perp h$, then $j \perp k$.



THEOREM 3.12 LINES PERPENDICULAR TO A TRANSVERSAL THEOREM

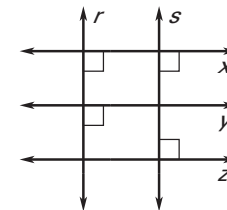
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If $m \perp p$ and $n \perp p$, then $m \parallel n$.



Example 3 Draw conclusions

Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.



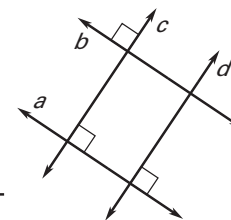
Solution

Lines r and s are both perpendicular to x , so by Theorem 3.12, $r \parallel s$. Similarly, lines x and y are both perpendicular to r , so $x \parallel y$. Also, lines x and z are both perpendicular to s , so $x \parallel z$. Finally, because y and z are both parallel to x , you know that $y \parallel z$ by the Transitive Property of Parallel Lines.

✓ **Checkpoint** Use the diagram to complete the following exercises.

3. Is $c \parallel d$? Explain.

Yes, because c and d are both perpendicular to a , $c \parallel d$ by Theorem 3.12.



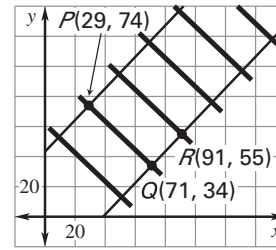
4. Is $b \perp d$? Explain.

Yes, because $b \perp c$, and $c \parallel d$ as explained in Exercise 3, then $b \perp d$ by Theorem 3.11.

Your Notes

Example 4 Find the distance between two parallel lines

Railroads The section of broad gauge railroad track at the right are drawn on a graph where units are measured in inches. What is the width of the track?



Solution

You need to find the length of a perpendicular segment from one side of the track to the other.

Using $Q(71, 34)$ and $R(91, 55)$, the slope of each rail is

$$\frac{55 - \boxed{34}}{91 - \boxed{71}} = \frac{21}{20}$$

The segment PQ has a slope of

$$\frac{74 - \boxed{34}}{29 - \boxed{71}} = \frac{40}{-42} = \frac{-20}{21}$$

The segment PQ is perpendicular to the rail so PQ is

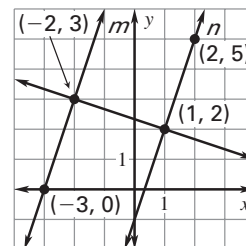
$$d = \sqrt{(\boxed{29 - 71})^2 + (\boxed{74 - 34})^2} = \underline{58}.$$

The width of the track is 58 inches.

✓ Checkpoint Complete the following exercise.

5. What is the approximate distance from line m to line n ?

about 3.2 units



Homework