

4.1

Apply Triangle Sum Properties

- Goal** • Classify triangles and find measures of their angles.

Your Notes

VOCABULARY

Triangle A triangle is a polygon with three sides.

Interior angles When the sides of a polygon are extended, the original angles are the interior angles.

Exterior angles When the sides of a polygon are extended, the angles that form linear pairs with the interior angles are the exterior angles.

Corollary to a theorem A corollary to a theorem is a statement that can be proved easily using the theorem.

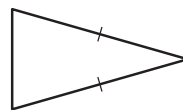
CLASSIFYING TRIANGLES BY SIDES

Scalene Triangle



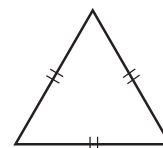
No congruent sides

Isosceles Triangle



At least 2 congruent sides

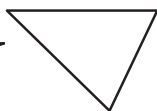
Equilateral Triangle



3 congruent sides

CLASSIFYING TRIANGLES BY ANGLES

Acute Triangle



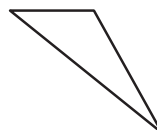
3 acute angles

Right Triangle



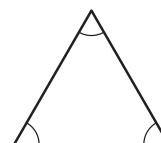
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



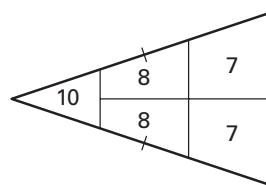
3 congruent angles

Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

Your Notes

Example 1 Classify triangles by sides and by angles

Shuffleboard Classify the triangular shape of the shuffleboard scoring area in the diagram by its sides and by measuring its angles.



Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are about 72°, 72°, and 36°. It is an acute isosceles triangle.

✓ **Checkpoint** Complete the following exercise.

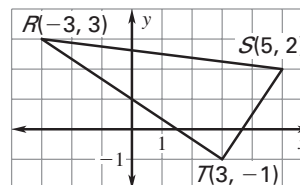
1. Draw an isosceles right triangle and an obtuse scalene triangle.

Sample Drawings:



Example 2 Classify a triangle in a coordinate plane

Classify $\triangle RST$ by its sides. Then determine if the triangle is a right triangle.



Solution

Step 1 Use the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the side lengths.

$$RT = \sqrt{(3 - (-3))^2 + (-1 - 3)^2} = \sqrt{52}$$

$$RS = \sqrt{(5 - (-3))^2 + (2 - 3)^2} = \sqrt{65}$$

$$ST = \sqrt{(3 - 5)^2 + (-1 - 2)^2} = \sqrt{13}$$

Step 2 Check for right angles. The slope of \overline{RT} is

$$\frac{-1 - 3}{3 - (-3)} = -\frac{2}{3}. \text{ The slope of } \overline{ST} \text{ is}$$

$$\frac{-1 - 2}{3 - 5} = \frac{3}{2}. \text{ The product of the slopes is}$$

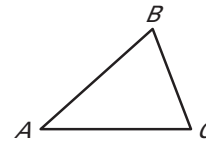
$$-1, \text{ so } \overline{RT} \perp \overline{ST} \text{ and } \angle RTS \text{ is a } \underline{\text{right}} \text{ angle.}$$

Therefore, $\triangle RST$ is a right scalene triangle.

Your Notes

THEOREM 4.1: TRIANGLE SUM THEOREM

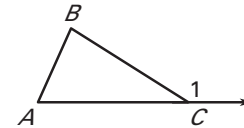
The sum of the measures of the interior angles of a triangle is 180° .



$$m\angle A + m\angle B + m\angle C = \underline{180^\circ}$$

THEOREM 4.2: EXTERIOR ANGLE THEOREM

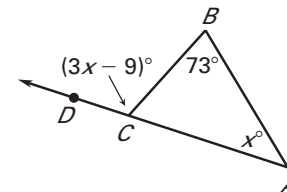
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.



$$m\angle 1 = m\angle \underline{A} + m\angle \underline{B}$$

Example 3 Find angle measure

Use the diagram at the right to find the measure of $\angle DCB$.



Solution

Step 1 Write and solve an equation to find the value of x .

$$(3x - 9)^\circ = \underline{73^\circ + x^\circ}$$

$$x = \underline{41^\circ}$$

Exterior Angle Theorem

Solve for x .

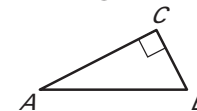
Step 2 Substitute 41 for x in $3x - 9$ to find $m\angle DCB$.

$$3x - 9 = 3 \cdot \underline{41} - 9 = \underline{114}$$

The measure of $\angle DCB$ is 114° .

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are complementary.

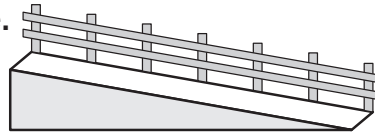


$$m\angle A + m\angle B = \underline{90^\circ}$$

Your Notes

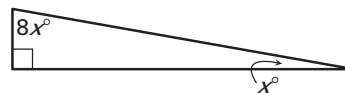
Example 4 Find angle measures from a verbal description

Ramps The front face of the wheelchair ramp shown forms a right triangle. The measure of one acute angle in the triangle is eight times the measure of the other. Find the measure of each acute angle.



Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be x° . Then the measure of the larger acute angle is $8x^\circ$.



Use the Corollary to the Triangle Sum Theorem to set up and solve an equation.

$$x^\circ + 8x^\circ = 90^\circ \quad \text{Corollary to the Triangle Sum Theorem}$$

$$x = 10 \quad \text{Solve for } x.$$

So, the measures of the acute angles are 10° and 80° .

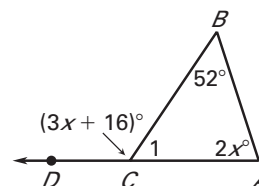
✓ Checkpoint Complete the following exercises.

2. Triangle JKL has vertices $J(-2, -1)$, $K(1, 3)$, and $L(5, 0)$. Classify it by its sides. Then determine if it is a right triangle.

isosceles triangle; right triangle

3. Find the measure of $\angle 1$ in the diagram shown.

56°



4. In Example 4, what is the measure of the obtuse angle formed between the ramp and a segment extending from the horizontal leg?

170°

Homework