4.2 Apply Congruence and Triangles

Goal • Identify congruent figures.

Your Notes

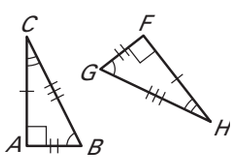
VOCABULARY

Congruent figures In two congruent figures, all the parts of one figure are congruent to the corresponding parts of the other figure.

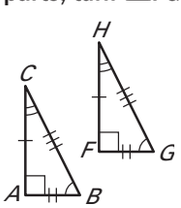
Corresponding parts In congruent polygons, the corresponding parts are the corresponding sides and the corresponding angles.

Example 1 *Identify congruent parts*

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



To help you identify corresponding parts, turn $\triangle FGH$.



Solution

The diagram indicates that $\triangle ABC \cong \triangle$ FGH .

Corresponding angles $\angle A \cong$ $\angle F$, $\angle B \cong$ $\angle G$, $\angle C \cong$ $\angle H$

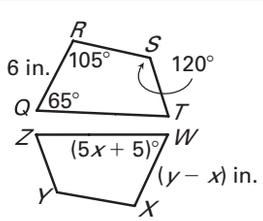
Corresponding sides $\overline{AB} \cong$ $\overline{FG}$, $\overline{BC} \cong$ $\overline{GH}$, $\overline{CA} \cong$ $\overline{HF}$

Example 2 *Use properties of congruent figures*

In the diagram, $QRST \cong WXYZ$.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know $\angle Q \cong \angle W$.

$m\angle Q =$ $m\angle W$

$65° =$ $(5x + 5)°$

$60 =$ $5x$

$12 = x$

b. You know $\overline{QR} \cong \overline{WX}$.

$QR =$ WX

$6 =$ $y - x$

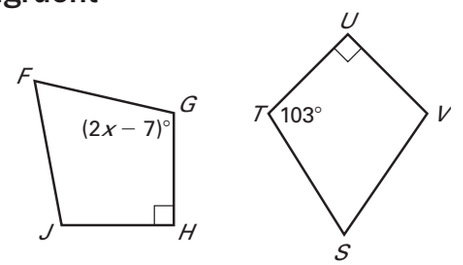
$6 =$ $y - 12$

$18 = y$

Your Notes

✔ ***Checkpoint*** **In Exercises 1 and 2, use the diagram shown in which *FGHJ* ≅ *STUV*.**

1. Identify all pairs of congruent corresponding parts.



Corresponding angles: $\angle F \cong \angle S$, $\angle G \cong \angle T$, $\angle H \cong \angle U$, $\angle J \cong \angle V$

Corresponding sides: $\overline{FG} \cong \overline{ST}$, $\overline{GH} \cong \overline{TU}$, $\overline{HJ} \cong \overline{UV}$, $\overline{JF} \cong \overline{VS}$

2. Find the value of x and find $m\angle G$.

$x = 55$; $m\angle G = 103°$

Example 3 ***Show that figures are congruent***

Maps If you cut the map in half along $\overline{PR}$, will the sections of the map be the same size and shape? *Explain*.

Solution

From the diagram, $\angle S \cong$ ∠Q because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{PQ} \parallel$ $\overline{RS}$. Then $\angle 1 \cong$ ∠4 and $\angle 2 \cong$ ∠3 by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

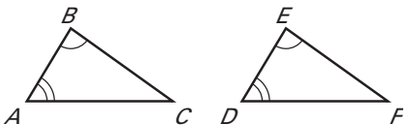
The diagram shows $\overline{PQ} \cong$ $\overline{RS}$ and $\overline{QR} \cong$ $\overline{SP}$. By the Reflexive Property, $\overline{PR} \cong \overline{RP}$. All corresponding parts are congruent, so $\triangle PQR \cong$ $\triangle RSP$.

Yes, the two sections will be the same size and shape.

Your Notes

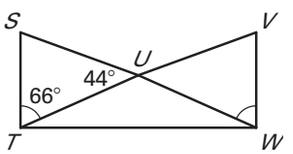
THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.



Example 4 *Use the Third Angles Theorem*

Find $m\angle V$.



Solution

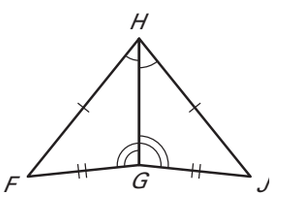
$\angle SUT \cong \angle VUW$ by the Vertical Angles Theorem. The diagram shows that $\angle STU \cong$ $\angle VWU$, so by the Third Angles Theorem, $\angle S \cong$ $\angle V$. By the Triangle Sum Theorem, $m\angle S =$ $180° - 66° - 44°$ $=$ $70°$. So, $m\angle S = m\angle V =$ $70°$ by the definition of congruent angles.

Example 5 *Prove that triangles are congruent*

Write a proof.

Given $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$, $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$

Prove $\triangle FGH \cong \triangle JGH$



Plan for Proof

a. Use the Reflexive Property to show $\overline{HG} \cong \overline{HG}$.

b. Use the Third Angles Theorem to show $\angle F \cong \angle J$.

Plan in Action

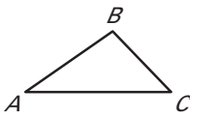
Statements	Reasons
1. $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$	1. Given
a. 2. $\overline{HG} \cong \overline{HG}$	2. Reflexive Property of Congruence
3. $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$	3. Given
b. 4. $\angle F \cong \angle J$	4. Third Angles Theorem
5. $\triangle FGH \cong \triangle JGH$	5. Definition of $\cong \triangle$s

Your Notes

THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES

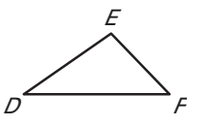
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong$ $\triangle ABC$.



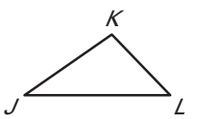
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



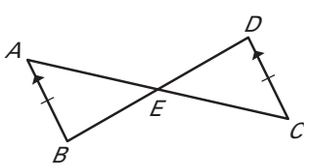
Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



✔ ***Checkpoint*** **Complete the following exercises.**

3. In the diagram at the right, E is the midpoint of $\overline{AC}$ and $\overline{BD}$. Show that $\triangle ABE \cong \triangle CDE$.

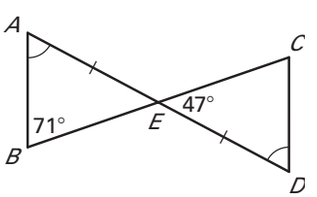


From the diagram, $\overline{AB} \cong \overline{CD}$. Point E is the midpoint of $\overline{AC}$ and $\overline{BD}$, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$ by the definition of midpoint. So all pairs of corresponding sides are congruent.

The diagram shows $\overline{AB} \parallel \overline{CD}$, so $\angle A \cong \angle C$ and $\angle B \cong \angle D$ by the Alternate Interior Angles Theorem. Also, $\angle AEB \cong \angle CED$ by the Vertical Angles Theorem. All corresponding parts are congruent, so $\triangle ABE \cong \triangle CDE$.

4. In the diagram, what is the measure of $\angle D$?

62°



Homework

5. By the definition of congruence, what additional information is needed to know that $\triangle ABE \cong \triangle DCE$ in Exercise 4?

You must know that $\overline{AB} \cong \overline{DC}$ and $\overline{BE} \cong \overline{CE}$ to conclude that $\triangle ABE \cong \triangle DCE$. The remaining information can be inferred from the graph.