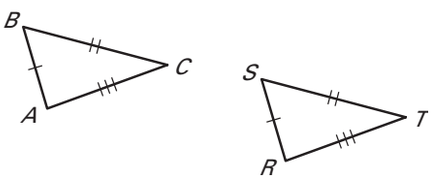
4.3 Prove Triangles Congruent by SSS

Goal • Use side lengths to prove triangles are congruent.

Your Notes

POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE

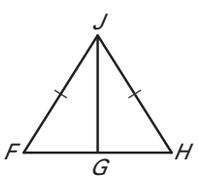
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.



If Side $\overline{AB} \cong$ $\overline{RS}$,
Side $\overline{BC} \cong$ $\overline{ST}$, and
Side $\overline{CA} \cong$ $\overline{TR}$,
then $\triangle ABC \cong$ $\triangle RST$.

Example 1 ***Use the SSS Congruence Postulate***

Write a proof.



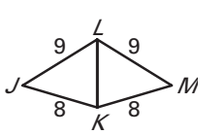
Given $\overline{FJ} \cong \overline{HJ}$,
G is the midpoint of $\overline{FH}$.

Prove $\triangle FGJ \cong \triangle HGJ$

Proof It is given that $\overline{FJ} \cong$ $\overline{HJ}$. Point G is the midpoint of $\overline{FH}$, so $\overline{FG} \cong \overline{HG}$. By the Reflexive Property, $\overline{GJ} \cong \overline{JG}$. So, by the SSS Congruence Postulate, $\triangle FGJ \cong \triangle HGJ$.

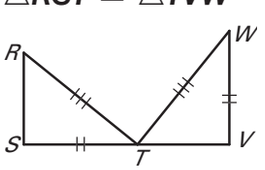
✔ ***Checkpoint*** **Decide whether the congruence statement is true. *Explain* your reasoning.**

1. $\triangle JKL \cong \triangle MKL$



True; all corresponding sides are congruent.

2. $\triangle RST \cong \triangle TVW$

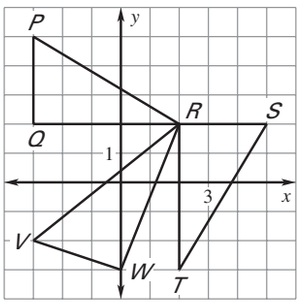


False; $\overline{RS} \ncong \overline{TV}$

Your Notes

Example 2 Congruence in the coordinate plane

Determine whether $\triangle PQR$ is congruent to the other triangles shown at the right.



Solution

By counting, $PQ = 3$ and $QR = 5$.
Use the distance formula to find PR.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

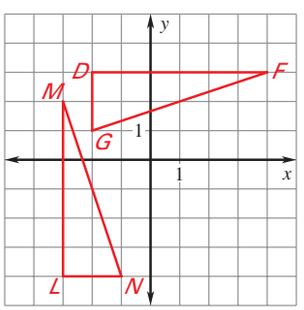
$PR = \sqrt{(2 - (-3))^2 + (2 - 5)^2} = \sqrt{34}$

By the SSS Congruence Postulate, any triangle with side lengths 3, 5, and $\sqrt{34}$ will be congruent to $\triangle PQR$. The distance from R to S is 3. The distance from R to T is 5. The distance from S to T is $\sqrt{(2 - 5)^2 + (-3 - 2)^2} = \sqrt{34}$. So, $\triangle PQR \cong \triangle SRT$.

The distance from W to V is $\sqrt{(-3 - 0)^2 + (-2 - (-3))^2} = \sqrt{10}$. No side of $\triangle PQR$ has a length of $\sqrt{10}$, so $\triangle PQR \ncong \triangle VWR$.

Checkpoint **Complete the following exercise.**

3. $\triangle DFG$ has vertices $D(-2, 4)$, $F(4, 4)$, and $G(-2, 2)$. $\triangle LMN$ has vertices $L(-3, -3)$, $M(-3, 3)$, and $N(-1, -3)$. Graph the triangles in the same coordinate plane and show that they are congruent.

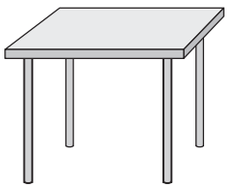


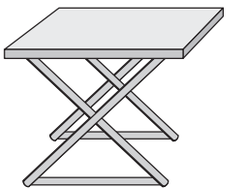
$DG = LN = 2$, $DF = LM = 6$, and $FG = MN = \sqrt{40}$, so $\triangle DFG \cong \triangle LMN$ by the SSS Congruence Postulate.

Your Notes

Example 3 ***Solve a real-world problem***

Stability *Explain* why the table with the diagonal legs is stable, while the one without the diagonal legs can collapse.



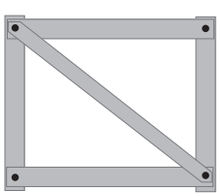


Solution

The table with the diagonal legs forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the table is stable. The table without the diagonal legs is not stable because there are many possible quadrilaterals with the given side lengths.

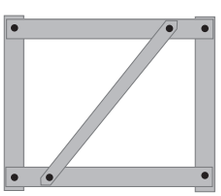
Checkpoint **Determine whether the figure is stable. *Explain* your reasoning.**

4.



Yes, the figure is stable. By the SSS Congruence Postulate, the triangles formed cannot change shape, so it is stable.

5.



No, the figure is not stable. There are many possible quadrilaterals with the given side lengths.

Homework