4.4 Prove Triangles Congruent by SAS and HL

Goal • Use sides and angles to prove congruence.

Your Notes

VOCABULARY

Leg of a right triangle In a right triangle, a side adjacent to the right angle is called a leg.

Hypotenuse In a right triangle, the side opposite the right angle is called the hypotenuse.

POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE

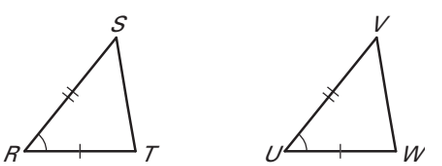
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong$ $\overline{UV}$,

Angle $\angle R \cong$ $\angle U$, and

Side $\overline{RT} \cong$ $\overline{UW}$,

then $\triangle RST \cong$ $\triangle UVW$.

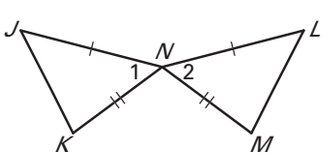


Example 1 ***Use the SAS Congruence Postulate***

Write a proof.

Given $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$

Prove $\triangle JKN \cong \triangle LMN$

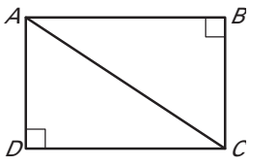


Statements	Reasons
1. $\overline{JN} \cong$ $\overline{LN}$, $\overline{KN} \cong$ $\overline{MN}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Theorem
3. $\triangle JKN \cong \triangle LMN$	3. SAS Congruence Postulate

Your Notes

Example 2 *Use SAS and properties of shapes*

In the diagram, *ABCD* is a rectangle. What can you conclude about △*ABC* and △*CDA*?

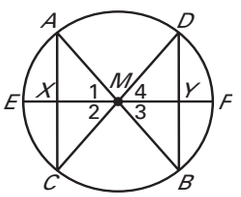


Solution

By the Right Angles Congruence Theorem, ∠*B* ≅ ∠*D*. Opposite sides of a rectangle are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$.

△*ABC* and △*CDA* are congruent by the SAS Congruence Postulate.

✔ ***Checkpoint*** **In the diagram, $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ pass through the center *M* of the circle. Also, ∠1 ≅ ∠2 ≅ ∠3 ≅ ∠4.**



1. Prove that △*DMY* ≅ △*BMY*.

Statements	Reasons
1. ∠3 ≅ ∠4	1. Given
2. $\overline{DM} \cong \overline{BM}$	2. Definition of a circle
3. $\overline{MY} \cong \overline{MY}$	3. Reflexive Property of Congruence
4. △*DMY* ≅ △*BMY*	4. SAS Congruence Postulate

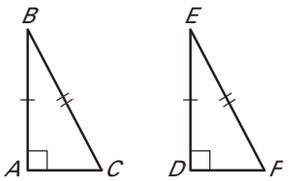
2. What can you conclude about $\overline{AC}$ and $\overline{BD}$?

Because they are vertical angles, ∠*AMC* ≅ ∠*BMD*. All points on a circle are the same distance from the center, so *AM* = *BM* = *CM* = *DM*. By the SAS Congruence Postulate, △*AMC* ≅ △*BMD*. Corresponding parts of congruent triangles are congruent, so you know $\overline{AC} \cong \overline{BD}$.

Your Notes

THEOREM 4.5: HYPOTENUSE-LEG CONGRUENCE THEOREM

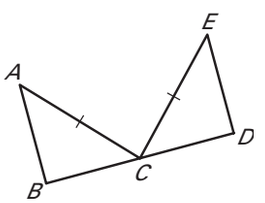
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are congruent.



Example 3 *Use the Hypotenuse-Leg Theorem*

Write a proof.

Given $\overline{AC} \cong \overline{EC}$, $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$, $\overline{AC}$ is a bisector of $\overline{BD}$.



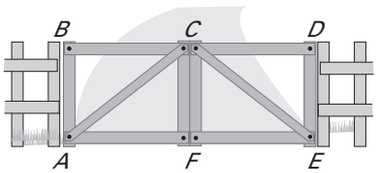
Prove $\triangle ABC \cong \triangle EDC$

	Statements	Reasons
H	1. $\overline{AC} \cong \overline{EC}$	1. Given
	2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	2. Given
	3. $\angle B$ and $\angle D$ are right angles.	3. Definition of $\perp$ lines
	4. $\triangle ABC$ and $\triangle EDC$ are right triangles.	4. Definition of a right triangle
	5. $\overline{AC}$ is a bisector of $\overline{BD}$.	5. Given
L	6. $\overline{BC} \cong \overline{DC}$	6. Definition of segment bisector
	7. $\triangle ABC \cong \triangle EDC$	7. HL Congruence Theorem

Your Notes

Example 4 ***Choose a postulate or theorem***

Gate The entrance to a ranch has a rectangular gate as shown in the diagram. You know that $\triangle AFC \cong \triangle EFC$. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$?



Solution

You are given that *ABDE* is a rectangle, so $\angle B$ and $\angle D$ are right angles. Because opposite sides of a rectangle are congruent, $\overline{AB} \cong \overline{DE}$. You are also given that $\triangle AFC \cong \triangle EFC$, so $\overline{AC} \cong \overline{EC}$. The hypotenuse and a leg of each triangle is congruent.

You can use the HL Congruence Theorem to conclude that $\triangle ABC \cong \triangle EDC$.

Checkpoint **Complete the following exercises.**

3. *Explain* why a diagonal of a rectangle forms a pair of congruent triangles.

A diagonal of a rectangle will be the hypotenuse of each triangle formed. Because the hypotenuse is congruent to itself, and because opposite sides of a rectangle are congruent, you can use the HL Congruence Theorem to conclude the triangles are congruent.

4. In Example 4, suppose it is given that *ABCF* and *EDCF* are squares. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$? *Explain*.

It is given that *ABCF* and *EDCF* are squares, so $\angle B$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{DE}$, and $\overline{BC} \cong \overline{DC}$. You can use the SAS Congruence Postulate to conclude that $\triangle ABC \cong \triangle EDC$.

Homework