4.5 Prove Triangles Congruent by ASA and AAS

Goal • Use two more methods to prove congruences.

Your Notes

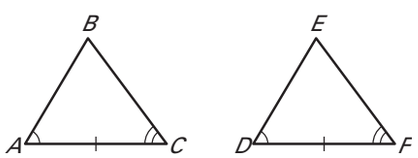
VOCABULARY

Flow proof A flow proof uses arrows to show the flow of a logical argument.

POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

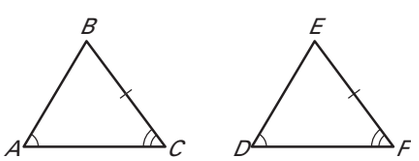
If **Angle** $\angle A \cong$ $\angle D$,
Side $\overline{AC} \cong$ $\overline{DF}$, and
Angle $\angle C \cong$ $\angle F$,
then $\triangle ABC \cong$ $\triangle DEF$.



THEOREM 4.6: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

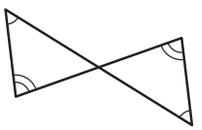
If **Angle** $\angle A \cong$ $\angle D$,
Angle $\angle C \cong$ $\angle F$, and
Side $\overline{BC} \cong$ $\overline{EF}$,
then $\triangle ABC \cong$ $\triangle DEF$.

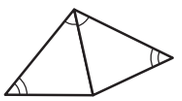


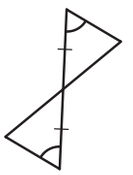
Your Notes

Example 1 ***Identify congruent triangles***

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. 

b. 

c. 

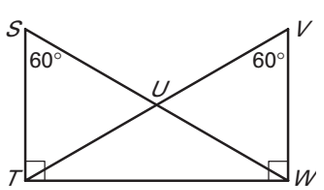
Solution

a. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

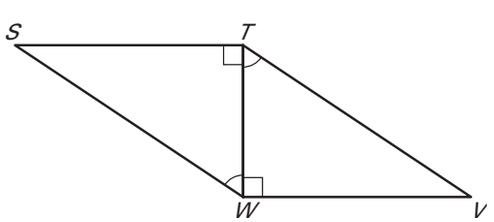
b. Two pairs of angles and a non-included pair of sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

c. The vertical angles are congruent, so two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

✔ ***Checkpoint*** **Can $\triangle STW$ and $\triangle VWT$ be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.**

1. 

Yes; AAS Congruence Theorem

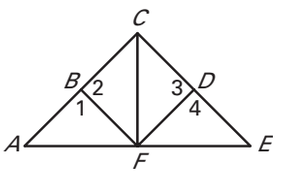
2. 

Yes; ASA Congruence Postulate

Your Notes

Example 2 ***Write a flow proof***

In the diagram, $\angle 1 \cong \angle 4$ and $\overline{CF}$ bisects $\angle ACE$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.



Solution

Given $\angle 1 \cong \angle 4$, $\overline{CF}$ bisects $\angle ACE$.

Prove $\triangle CBF \cong \triangle CDF$

$\angle 1 \cong \angle 4$ — Given

$\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 4$ are supplements.
— Def. of supplementary angles

$\overline{CF}$ bisects $\angle ACE$. — Given

$\angle 2 \cong \angle 3$ — Congruent Supps. Thm.

$\overline{CF} \cong \overline{CF}$ — Reflexive Prop.

$\angle ACF \cong \angle ECF$ — Def. of $\angle$ bisector

$\angle CBF \cong \angle CDF$ — AAS Congruence Theorem

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 2, suppose it is given that $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

$\overline{CF}$ bisects $\angle ACE$. — Given

$\overline{CF}$ bisects $\angle BFD$. — Given

$\angle ACF \cong \angle ECF$ — Def. of $\angle$ bisector

$\overline{CF} \cong \overline{CF}$ — Reflexive Prop.

$\angle BFC \cong \angle DFC$ — Def. of $\angle$ bisector

$\angle CBF \cong \angle CDF$ — ASA Congruence Postulate

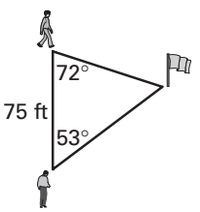
Your Notes

Example 3 ***Choose a postulate or theorem***

Games You and a friend are trying to find a flag hidden in the woods. Your friend is standing 75 feet away from you. When facing each other, the angle from you to the flag is 72° and the angle from your friend to the flag is 53°. Is there enough information to locate the flag?

Solution

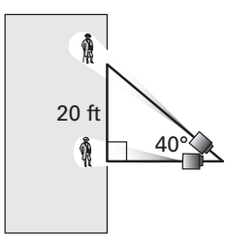
The locations of you, your friend, and the flag form a triangle. The measures of two angles and an included side of the triangle are known.



By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the flag location is given by the third vertex.

Checkpoint **Complete the following exercise.**

4. Theater You are working two spotlights for a play. Two actors are standing apart from each other on the end of the stage. The spotlights are located and pointed as shown in the diagram. Can one of the actors move without requiring the spotlight to move and without changing the distance between the other actor?



The measures of two angles and a nonincluded side of the triangle are known. By the AAS Congruence Theorem, all triangles with these measures are congruent. So, the triangle formed is unique, and one of the actors cannot move without requiring the spotlight to move and without changing the distance between the other actor.

Homework