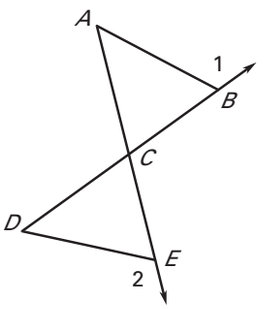
4.6 Use Congruent Triangles

Goal • Use congruent triangles to prove corresponding parts congruent.

Your Notes

Example 1 *Use congruent triangles*

Explain **how you can use the given information to prove that the triangles are congruent.**



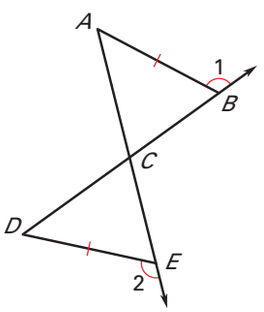
Given $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$

Prove $\overline{DC} \cong \overline{AC}$

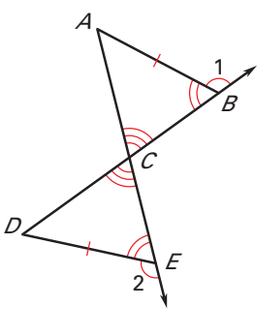
Solution

If you can show that $\triangle ABC \cong \triangle DEC$, you will know that $\overline{DC} \cong \overline{AC}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle ABC$ and $\angle DEC$ are supplementary to congruent angles, so $\angle ABC \cong \angle DEC$. Also, $\angle ACB \cong \angle DCE$.

Mark given information.



Add deduced information.



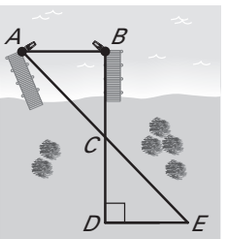
Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle ABC \cong \triangle DEC$. Because corresponding parts of congruent triangles are congruent, $\overline{DC} \cong \overline{AC}$.

Your Notes

When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

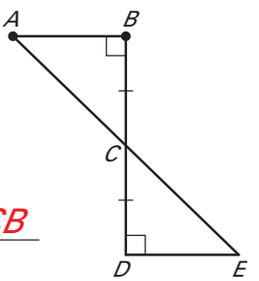
Example 2 *Use congruent triangles for measurement*

Boats Use the following method to find the distance between two docked boats, from point *A* to point *B*.



- Place a marker at *D* so that $\overline{AB} \perp \overline{BD}$.
- Find *C*, the midpoint of $\overline{BD}$.
- Locate the point *E* so that $\overline{BD} \perp \overline{DE}$ and *A*, *C*, and *E* are collinear.
- *Explain* how this plan allows you to find the distance.

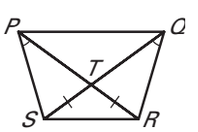
Solution



Because $\overline{AB} \perp \overline{BD}$ and $\overline{BD} \perp \overline{DE}$, $\angle B$ and $\angle D$ are congruent right angles. Because *C* is the midpoint of $\overline{BD}$, $\overline{BC} \cong \overline{DC}$. The vertical angles $\angle ACB$ and $\angle ECD$ are congruent. So, $\triangle CBA \cong \triangle CDE$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{BA} = \overline{DE}$. So, you can find the distance *AB* between the boats by measuring $\overline{DE}$.

Checkpoint **Complete the following exercises.**

1. *Explain* how you can prove that $\overline{PR} \cong \overline{QS}$.



Use the AAS Congruence Theorem to show $\triangle PTS \cong \triangle QTR$. Because corresponding pairs of congruent triangles are congruent, $\overline{PT} \cong \overline{QT}$. Then $\overline{PR} \cong \overline{QS}$ because $\overline{ST} \cong \overline{RT}$.

2. In Example 2, does it matter how far away from point *B* you place a marker at point *D*? *Explain*.

Point *D* should be placed far enough away from point *B* so that it is on land. This allows $\overline{DE}$ to be easily measured. However, the method will work regardless of how far *D* is from *B*.

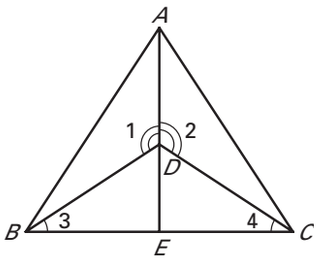
Your Notes

Example 3 *Plan a proof involving pairs of triangles*

Use the given information to write a plan for proof.

Given $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

Prove $\triangle ABD \cong \triangle ACD$



Solution

In $\triangle ABD$ and $\triangle ACD$, you know that $\angle 1 \cong$ ∠2 and $\overline{AD} \cong \overline{AD}$. If you can show that $\overline{BD} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

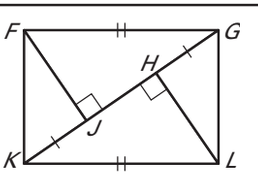
To prove that $\overline{BD} \cong \overline{CD}$, you can first prove that $\triangle BED \cong$ $\triangle CED$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property and $\angle BDE \cong$ $\angle CDE$ by the Congruent Supplements Theorem. You can use the AAS Congruence Theorem to prove that $\triangle BED \cong$ $\triangle CED$.

Plan for Proof Use the AAS Congruence Theorem to prove that $\triangle BED \cong$ $\triangle CED$. Then state that $\overline{BD} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle ABD \cong \triangle ACD$.

✔ ***Checkpoint*** **Use the given information to write a plan for proof.**

3. Given $\overline{GH} \cong \overline{KJ}, \overline{FG} \cong \overline{LK}$, $\angle FJG$ and $\angle LHK$ are rt. ∠s.

Prove $\triangle FJK \cong \triangle LHG$

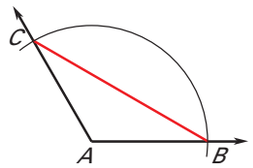


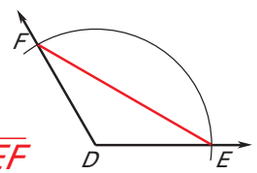
Plan for Proof: Use the HL Congruence Theorem to prove that $\triangle FJG \cong \triangle LHK$. Then state that $\overline{FJ} \cong \overline{LH}$. Then show that $\angle FJK \cong \angle LHG$ and use the SAS Congruence Postulate to prove that $\triangle FJK \cong \triangle LHG$.

Your Notes

Example 4 ***Prove a construction***

Write a proof to verify that the construction for copying an obtuse angle is valid.





Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$ are determined by the same compass setting, as are $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.

Given $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

Prove $\angle D \cong \angle A$

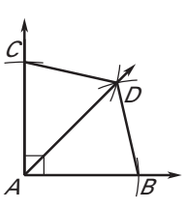
Plan for Proof Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle D$ and $\angle A$ are congruent.

Plan for Action

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$	**1.** Given
2. $\triangle CAB \cong \triangle FDE$	**2.** SSS Congruence Postulate
3. $\angle D \cong \angle A$	**3.** Corresp. parts of $\cong$ triangles are $\cong$.

✔ ***Checkpoint*** **Complete the following exercise.**

4. Write a paragraph proof to verify that the construction for bisecting a right angle is valid.



You know that $\overline{AC} \cong \overline{AB}$ and $\overline{BD} \cong \overline{CD}$ because they are determined by the same compass settings. Also, $\overline{AD} \cong \overline{AD}$ by the Reflexive Property. So, by the SSS Congruence Postulate, $\triangle CAD \cong \triangle BAD$. Thus, $\angle CAD \cong \angle BAD$ because corresponding parts of congruent triangles are congruent.

Homework