4.8 Perform Congruence Transformations

Goal • Create an image congruent to a given triangle.

Your Notes

VOCABULARY

Transformation A transformation is an operation that moves or changes a geometric figure in some way to produce a new figure.

Image The new figure produced by a transformation is the image.

Translation A translation moves every point of a figure the same distance in the same direction.

Reflection A reflection uses a *line of reflection* to create a mirror image of the original figure.

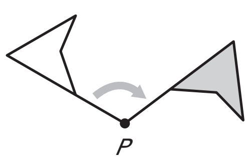
Rotation A rotation turns a figure about a fixed point, called the *center of rotation*.

Congruence Transformation A congruence transformation changes the position of a figure without changing its size or shape.

Example 1 ***Identify transformations***

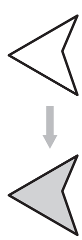
Name the type of transformation demonstrated in each picture.

a.



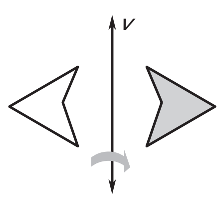
Rotation about a point

b.



Translation in a straight path

c.



Reflection in a vertical line

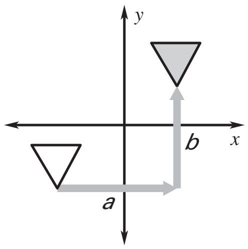
Your Notes

COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the unshaded figure is translated horizontally a units and vertically b units.



Example 2 *Translate a figure in the coordinate plane*

Figure $ABCD$ has the vertices $A(1, 2)$, $B(3, 3)$, $C(4, -1)$, and $D(1, -2)$. Sketch $ABCD$ and its image after the translation $(x, y) \rightarrow (x - 4, y + 2)$.

Solution

First draw $ABCD$. Find the translation of each vertex by subtracting 4 from its x-coordinate and adding 2 to its y-coordinate. Then draw $ABCD$ and its image.

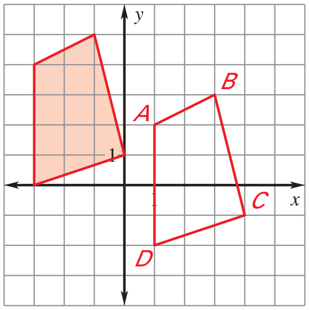
$(x, y) \rightarrow (x - 4, y + 2)$

$A(1, 2) \rightarrow (-3, 4)$

$B(3, 3) \rightarrow (-1, 5)$

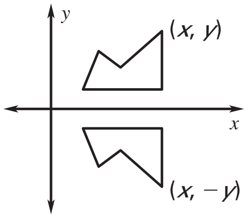
$C(4, -1) \rightarrow (0, 1)$

$D(1, -2) \rightarrow (-3, 0)$



COORDINATE NOTATION FOR A REFLECTION

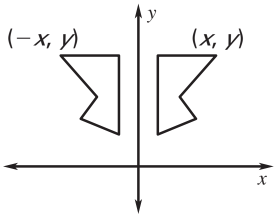
Reflection in the x-axis



Multiply y-coordinate by -1.

$(x, y) \rightarrow (x, -y)$

Reflection in the y-axis



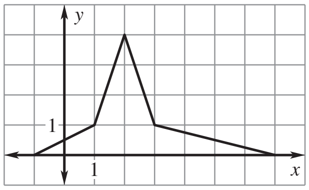
Multiply x-coordinate by -1.

$(x, y) \rightarrow (-x, y)$

Your Notes

Example 3 *Reflect a figure in the x-axis*

Shapes You are cutting figures out of paper. Use a reflection in the *x*-axis to draw the other half of the figure.



Solution

Multiply the <u>*y*-coordinate</u> of each vertex by −1 to find the corresponding vertex in the image. Then draw the image.

$(x, y) \rightarrow$ <u>$(x, -y)$</u>

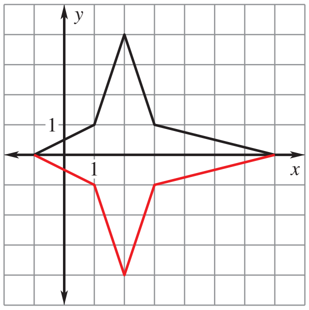
$(-1, 0) \rightarrow$ <u>$(-1, 0)$</u>

$(1, 1) \rightarrow$ <u>$(1, -1)$</u>

$(2, 4) \rightarrow$ <u>$(2, -4)$</u>

$(3, 1) \rightarrow$ <u>$(3, -1)$</u>

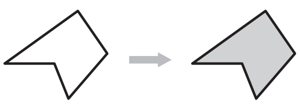
$(7, 0) \rightarrow$ <u>$(7, 0)$</u>



You can check your results by looking to see if each original point and its image are the same distance from the <u>*x*-axis</u>.

Checkpoint Complete the following exercises.

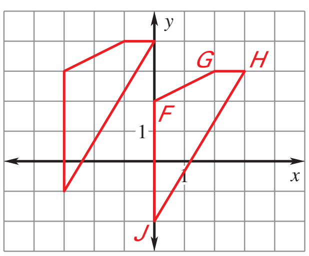
1. Name the type of transformation shown.



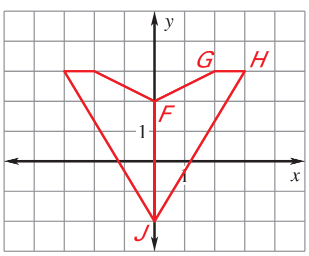
Translation

2. Figure *FGHJ* has the vertices *F*(0, 2), *G*(2, 3), *H*(3, 3), and *J*(0, −2). Sketch *FGHJ* and its image after (a) the translation $(x, y) \rightarrow (x - 3, y + 1)$ and (b) a reflection in the *y*-axis.

a.



b.



Your Notes

Example 4 *Identify a rotation*

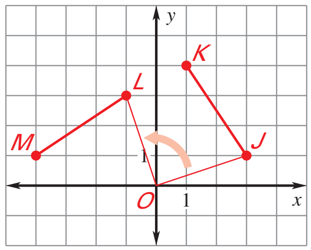
Graph $\overline{JK}$ and $\overline{LM}$. Tell whether $\overline{LM}$ is a rotation of $\overline{JK}$ about the origin. If so, give the angle and direction of rotation.

a. $J(3, 1)$, $K(1, 4)$, $L(-1, 3)$, $M(-4, 1)$

b. $J(-2, 1)$, $K(-1, 5)$, $L(1, 1)$, $M(2, 5)$

Solution

a.

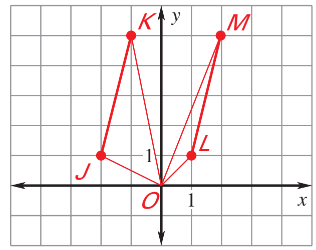


$m\angle JOL$ $\underline{\ =\ }$ $m\angle KOM$

$\underline{\ =\ }$ $\underline{\ 90°\ }$

$\underline{\text{90° counterclockwise rotation}}$

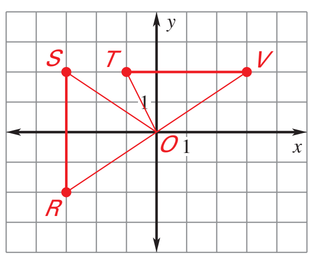
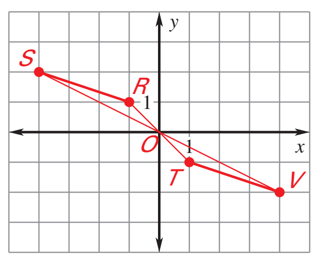
b.



$m\angle JOL$ $\underline{\ >\ }$ $m\angle KOM$

$\underline{\text{not a rotation}}$

Checkpoint **Graph $\overline{RS}$ and $\overline{TV}$. Tell whether $\overline{TV}$ is a rotation of $\overline{RS}$ about the origin. If so, give the angle of rotation.**

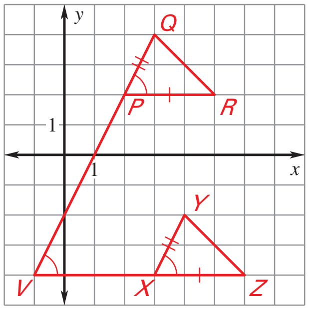
3. $R(-3, -2)$, $S(-3, 2)$, $T(-1, 2)$, $V(3, 2)$  $m\angle ROT < m\angle SOV$ not a rotation	**4.** $R(-1, 1)$, $S(-4, 2)$, $T(1, -1)$, $V(4, -2)$  $m\angle ROT = m\angle SOV$ $= 180°$ 180° rotation

Your Notes

Example 5 *Verify congruence*

The vertices of $\triangle PQR$ are $P(2, 2)$, $Q(3, 4)$, and $R(5, 2)$. The notation $(x, y) \rightarrow (x + 1, y - 6)$ describes the translation of $\triangle PQR$ to $\triangle XYZ$. Show that $\triangle PQR \cong \triangle XYZ$ to verify that the translation is a congruence transformation.

Solution



S You can see that PR = XZ = 3, so $\overline{PR} \cong \overline{XZ}$.

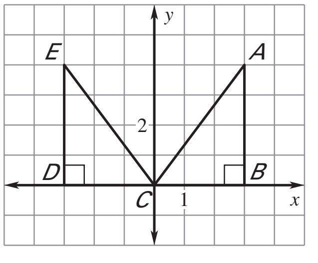
A Using the slopes, $\overline{PQ} \parallel \overline{XY}$ and $\overline{QR} \parallel \overline{YZ}$. If you extend $\overline{PQ}$ and $\overline{XZ}$ to form $\angle V$, the Corresponding Angles Postulate gives you $\angle QPR \cong \angle V$ and $\angle V \cong \angle YXZ$. Then, $\angle QPR \cong \angle YXZ$ by the Transitive Property of Congruence.

S Using the distance formula, PQ = XY = $\sqrt{5}$ so $\overline{PQ} \cong \overline{XY}$. So, $\triangle PQR \cong \triangle XYZ$ by the SAS Congruence Postulate.

Because $\triangle PQR \cong \triangle XYZ$, the translation is a congruence transformation.

✔ ***Checkpoint*** **Complete the following exercise.**

5. Show that $\triangle ABC \cong \triangle EDC$ to verify that the transformation is a congruence transformation.



You can see that $AB = ED = 4$ and $BC = DC = 3$. Also, $\angle B$ and $\angle D$ are congruent right angles. So, $\triangle ABC \cong \triangle EDC$ by the SAS Congruence Postulate.

Homework