5.1 Midsegment Theorem and Coordinate Proof

Goal • Use properties of midsegments and write coordinate proofs.

Your Notes

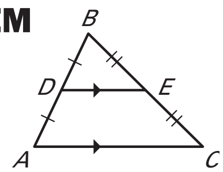
VOCABULARY

Midsegment of a triangle A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

Coordinate proof A coordinate proof involves placing geometric figures in a coordinate plane.

THEOREM 5.1: MIDSEGMENT THEOREM

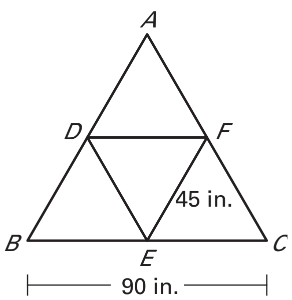
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.



$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

Example 1 ***Use the Midsegment Theorem to find lengths***

Windows A large triangular window is segmented as shown. In the diagram, $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$. Find DF and AB.



In the diagram for Example 1, midsegment $\overline{DF}$ can be called "the midsegment opposite $\overline{BC}$."

Solution

$DF = \frac{1}{2} \cdot BC = \frac{1}{2}(90 \text{ in.}) = 45 \text{ in.}$

$AB = 2 \cdot FE = 2(45 \text{ in.}) = 90 \text{ in.}$

Checkpoint **Complete the following exercise.**

1. In Example 1, consider $\triangle ADF$. What is the length of the midsegment opposite $\overline{DF}$?

22.5 in.

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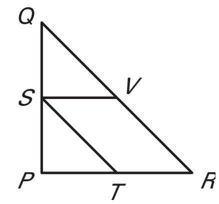
Example 2 *Use the Midsegment Theorem*

In the diagram at the right, $QS = SP$ and $PT = TR$. Show that $\overline{QR} \parallel \overline{ST}$.



Solution

Because $QS = SP$ and $PT = TR$, S is the midpoint of $\overline{QP}$ and T is the midpoint of $\overline{PR}$ by definition. Then $\overline{ST}$ is a midsegment of $\triangle PQR$ by definition and $\overline{QR} \parallel \overline{ST}$ by the Midsegment Theorem.

Checkpoint **Complete the following exercise.**

2. In Example 2, if V is the midpoint of $\overline{QR}$, what do you know about $\overline{SV}$?

$\overline{SV}$ is a midsegment of $\triangle PQR$ and $\overline{SV} \parallel \overline{PR}$.

Example 3 *Place a figure in a coordinate plane*

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

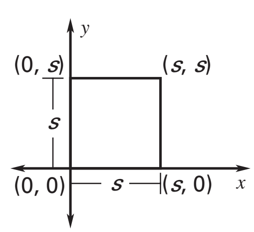
a. a square

b. an acute triangle

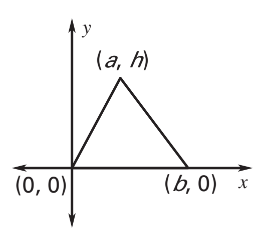
Solution

It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

a. Let s represent the side length.



b. You need to use three different variables.



The square represents a general square because the coordinates are based only on the definition of a square. If you use this square to prove a result, the result will be true for all squares.

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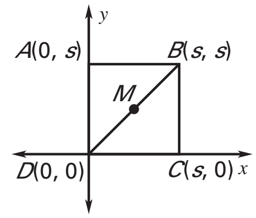
Example 4 *Apply variable coordinates*

In Example 3 part (a), find the length and midpoint of a diagonal of the square.

Solution



Draw a diagonal and its midpoint. Assign letters to the points.

Use the distance formula to find BD.

$BD = \sqrt{(s-0)^2 + (s-0)^2}$

$= \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2}$

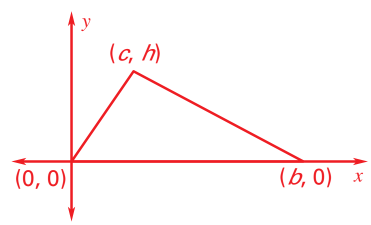
Use the midpoint formula to find the midpoint M.

$M\left(\frac{s+0}{2}, \frac{s+0}{2}\right) = M\left(\frac{s}{2}, \frac{s}{2}\right)$

Checkpoint **Complete the following exercises.**

3. Place an obtuse scalene triangle in a coordinate plane that is convenient for finding side lengths. Assign coordinates to each vertex.

Sample answer:



4. In Example 4, find the length and midpoint of diagonal $\overline{AC}$. What do you notice? *Explain* why this is true for all squares.

length: $s\sqrt{2}$; midpoint: $M\left(\frac{s}{2}, \frac{s}{2}\right)$; The lengths of the diagonals are the same, and the midpoints of the diagonals are the same. This is true for all squares because the coordinates are based only on the definition of a square.

Homework

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