5.2 Use Perpendicular Bisectors

Goal • Use perpendicular bisectors to solve problems.

Your Notes

VOCABULARY

Perpendicular bisector A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a perpendicular bisector.

Equidistant A point is equidistant from two figures if the point is the *same distance* from each figure.

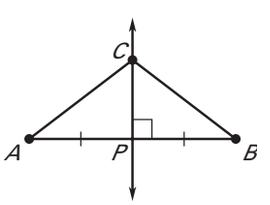
Concurrent When three or more lines, rays, or segments intersect in the same point, they are called concurrent lines, rays, or segments.

Point of concurrency The point of intersection of concurrent lines, rays, or segments is called the point of concurrency.

Circumcenter The point of concurrency of the three perpendicular bisectors of a triangle is called the circumcenter of the triangle.

THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM

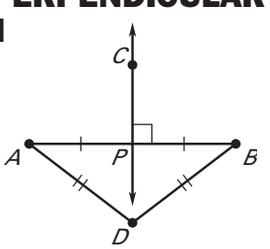
In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA =$ CB.

THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

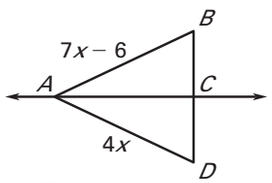


If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.

Your Notes

Example 1 *Use the Perpendicular Bisector Theorem*

$\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$. Find *AD*.



Solution

$AD =$ *AB* **Perpendicular Bisector Theorem**

4x $=$ *7x − 6* **Substitute.**

$x =$ *2* **Solve for *x*.**

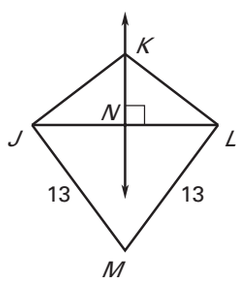
$AD =$ *4x* $=$ *4(2)* $=$ *8* .

Example 2 *Use perpendicular bisectors*

In the diagram, $\overleftrightarrow{KN}$ is the perpendicular bisector of $\overline{JL}$.

a. What segment lengths in the diagram are equal?

b. Is *M* on $\overleftrightarrow{KN}$?



Solution

a. $\overleftrightarrow{KN}$ bisects $\overline{JL}$, so *NJ* $=$ *NL* . Because *K* is on the perpendicular bisector of $\overline{JL}$, *KJ* $=$ *KL* by Theorem 5.2. The diagram shows that *MJ* $=$ *ML* $= 13$.

b. Because $MJ = ML$, *M* is *equidistant* from *J* and *L*. So, by the *Converse of the Perpendicular Bisector Theorem* , *M* is on the perpendicular bisector of $\overline{JL}$, which is $\overleftrightarrow{KN}$.

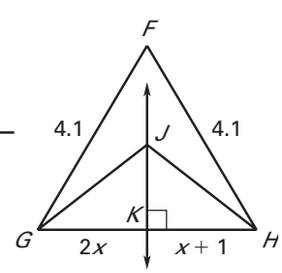
✔ ***Checkpoint*** **In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{GH}$.**

1. What segment lengths are equal?

KG = KH, JG = JH, FG = FH

2. Find *GH*.

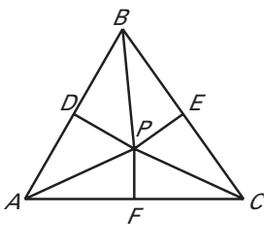
GH = 4



Your Notes

THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE

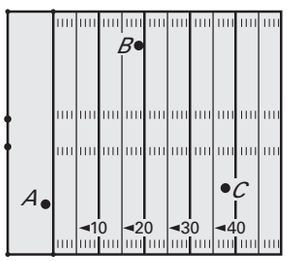
The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.



If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA =$ *PB* $=$ *PC*.

Example 3 ***Use the concurrency of perpendicular bisectors***

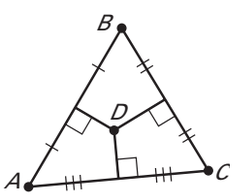
Football Three friends are playing catch. You want to join and position yourself so that you are the same distance from your friends. Find a location for you to stand.



Solution

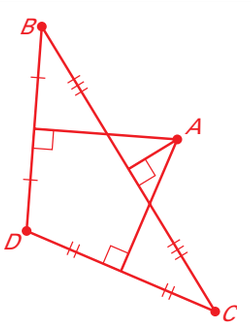
Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points *A*, *B*, and *C* and connect those points to draw $\triangle ABC$. Then use a ruler and a protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency *D* is a location for you to stand.



✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, your friend at location *A* wants to move to a location that is the same distance from everyone else. Find a new location for *A*.



Homework