5.3 Use Angle Bisectors of Triangles

Goal • Use angle bisectors to find distance relationships.

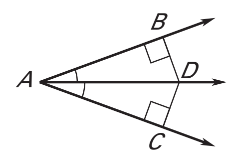
Your Notes

VOCABULARY

Incenter The point of concurrency of the three angle bisectors of a triangle is called the incenter of the triangle.

THEOREM 5.5: ANGLE BISECTOR THEOREM

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

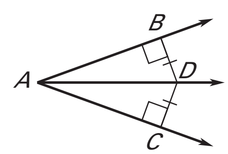


If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB =$ DC.

In Geometry, *distance* means the *shortest* length between two objects.

THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

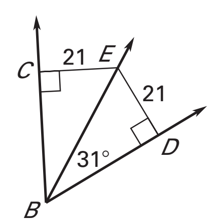


If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

Example 1 ***Use the Angle Bisector Theorems***

Find the measure of $\angle CBE$.

Solution

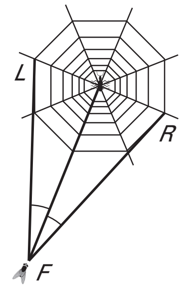


Because $\overline{EC} \perp$ $\overrightarrow{BC}$ and $\overline{ED} \perp$ $\overrightarrow{BD}$ and $EC = ED = 21$, $\overrightarrow{BE}$ bisects $\angle CBD$ by the Converse of the Angle Bisector Theorem. So, $m\angle CBE = m\angle$ DBE $=$ $31°$.

Your Notes

Example 2 ***Solve a real-world problem***

Web A spider's position on its web relative to an approaching fly and the opposite sides of the web forms congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?



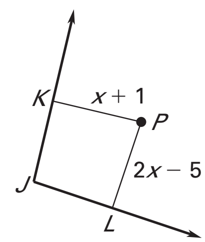
Solution

The congruent angles tell you that the spider is on the __bisector__ of $\angle LFR$. By the __Angle Bisector Theorem__, the spider is equidistant from $\overrightarrow{FL}$ and $\overrightarrow{FR}$.

So, the spider must move the __same distance__ to reach each edge.

Example 3 ***Use algebra to solve a problem***

For what value of x does P lie on the bisector of $\angle J$?



Solution

From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle J$ if P is equidistant from the sides of $\angle J$, so when __PK__ = __PL__.

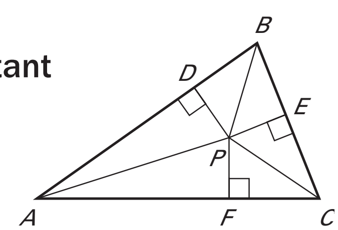
__PK__ = __PL__		**Set segment lengths equal.**
__$x + 1$__ = __$2x - 5$__		**Substitute expressions for segment lengths.**
__6__ = x		**Solve for x.**

Point P lies on the bisector of $\angle J$ when x = __6__.

THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

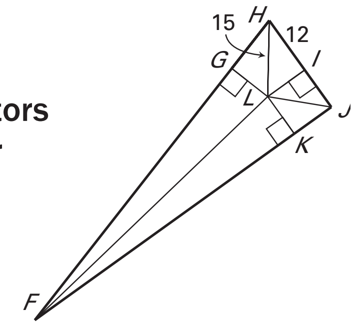
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then PD = __PE__ = __PF__.



Your Notes

Example 4 *Use the concurrency of angle bisectors*

In the diagram, L is the incenter of $\triangle FHJ$. Find LK.



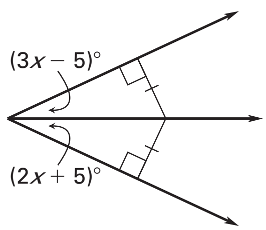
By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter L is equidistant from the sides of $\triangle FHJ$. So, to find LK, you can find LI in $\triangle LHI$. Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$		Pythagorean Theorem
$15^2 = LI^2 + 12^2$		Substitute known values.
$81 = LI^2$		Simplify.
$9 = LI$		Take the positive square root of each side.

Because $LI = LK$, $LK = 9$.

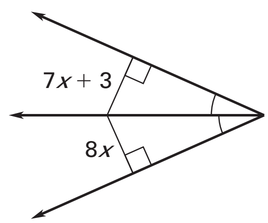
Checkpoint In Exercises 1 and 2, find the value of x.

1.



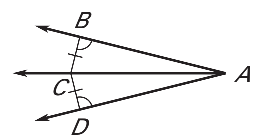
$x = 10$

2.



$x = 3$

3. Do you have enough information to conclude that $\overrightarrow{AC}$ bisects $\angle DAB$? *Explain.*



No, you must know that $m\angle ABC = m\angle ADC = 90°$ before you can conclude that $\overrightarrow{AC}$ bisects $\angle DAB$.

4. In Example 4, suppose you are not given HL or HI, but you are given that $JL = 25$ and $JI = 20$. Find LK.

$LK = 15$

Homework