5.4 Use Medians and Altitudes

Goal • Use medians and altitudes of triangles.

Your Notes

VOCABULARY

Median of a triangle The median of a triangle is a segment from a vertex to the midpoint of the opposite side.

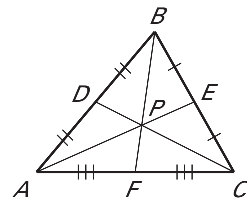
Centroid The point of concurrency of the three medians of a triangle is the centroid.

Altitude of a triangle An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

Orthocenter The point at which the lines containing the three altitudes of a triangle intersect is called the orthocenter of the triangle.

THEOREM 5.8: CONCURRENCY OF MEDIANS OF A TRIANGLE

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

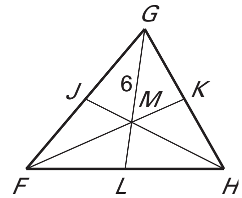


The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}$ AE, $BP = \frac{2}{3}$ BF, and $CP = \frac{2}{3}$ CD.

Your Notes

Example 1 ***Use the centroid of a triangle***

In $\triangle FGH$, M is the centroid and $GM = 6$. Find ML and GL.



$GM = \frac{2}{3}GL$ **Concurrency of Medians of a Triangle Theorem**

$6 = \frac{2}{3}GL$ **Substitute 6 for GM.**

$9 = GL$ **Multiply each side by the reciprocal, $\frac{3}{2}$.**

Then $ML = GL - GM = 9 - 6 = 3$.

So, $ML = 3$ and $GL = 9$.

Checkpoint **Complete the following exercise.**

1. In Example 1, suppose $FM = 10$. Find MK and FK.

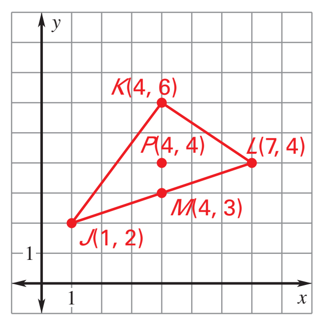
$MK = 5$, $FK = 15$

Example 2 ***Find the centroid of a triangle***

The vertices of $\triangle JKL$ are $J(1, 2)$, $K(4, 6)$, and $L(7, 4)$. Find the coordinates of the centroid P of $\triangle JKL$.

Sketch $\triangle JKL$. Then use the Midpoint Formula to find the midpoint M of $\overline{JL}$ and sketch median $\overline{KM}$.

$$M\left(\frac{1+7}{2}, \frac{2+4}{2}\right) = M(4, 3)$$



The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex K to point M is $6 - 3 = 3$ units. So, the centroid is $\frac{2}{3}(3) = 2$ units down from K on $\overline{KM}$.

The coordinates of the centroid P are $(4, 6 - 2)$, or $(4, 4)$.

Median $\overline{KM}$ was used in Example 2 because it is easy to find distances on a vertical segment. You can check by finding the centroid using a different median.

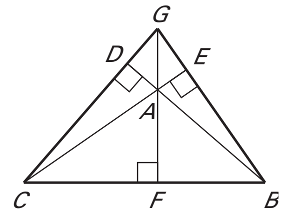
Your Notes

> Notice that in a right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE

The lines containing the altitudes of a triangle are <u>concurrent</u>.

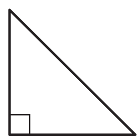
The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.



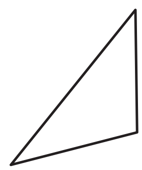
Example 3 ***Find the orthocenter***

Find the orthocenter *P* in the triangle.

a.

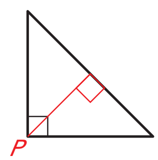


b.



Solution

a.



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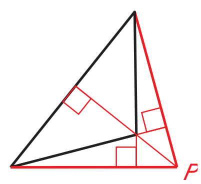
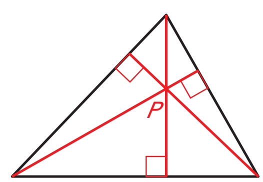


✔ ***Checkpoint*** **Complete the following exercises.**

2. In Example 2, where do you need to move point K so that the centroid is $P(4, 5)$?

Point K should be moved to $(4, 9)$.

3. Find the orthocenter P in the triangle.

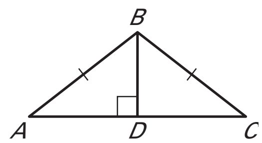


Your Notes

Example 4 *Prove a property of isosceles triangles*

Prove that the altitude to the base of an isosceles triangle is a median.

Solution



Given $\triangle ABC$ is isosceles, with base $\overline{AC}$. $\overline{BD}$ is the altitude to base $\overline{AC}$.

Prove $\overline{BD}$ is a median of $\triangle ABC$.

Proof Legs $\overline{AB}$ and $\overline{CD}$ of $\triangle ABC$ are congruent. $\angle ADB$ and $\angle CDB$ are congruent right angles because $\overline{BD}$ is the altitude to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ADB \cong \triangle CDB$ by the HL Congruence Theorem.

$\overline{AD} \cong \overline{CD}$ because corresponding parts of congruent triangles are congruent. So, D is the midpoint of $\overline{AC}$ by definition. Therefore, $\overline{BD}$ intersects $\overline{AC}$ at its midpoint, and $\overline{BD}$ is a median of $\triangle ABC$.

Checkpoint Complete the following exercise.

4. Prove that the altitude $\overline{BD}$ in Example 4 is also an angle bisector.

Proof Legs $\overline{AB}$ and $\overline{CB}$ of $\triangle ABC$ are congruent. $\angle ADB$ and $\angle CDB$ are congruent right angles because $\overline{BD}$ is the altitude to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ADB \cong \triangle CDB$ by the HL Congruence Theorem. $\angle ABD \cong \angle CBD$ because corresponding parts of congruent triangles are congruent. Therefore, $\overline{BD}$ bisects $\angle ABC$, and $\overline{BD}$ is an angle bisector.

Homework