

# 5.5

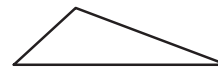
## Use Inequalities in a Triangle

**Goal** • Find possible side lengths of a triangle.

### Your Notes

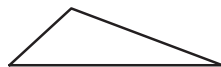
#### Example 1 *Relate side length and angle measure*

Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown at the right. What do you notice?



#### Solution

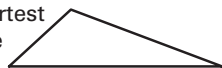
largest angle



longest side

The longest side and largest angle are opposite each other.

shortest side

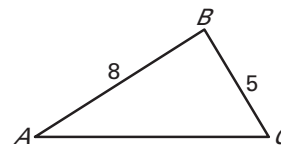


smallest angle

The shortest side and smallest angle are opposite each other.

#### THEOREM 5.10

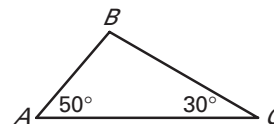
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$AB > BC$ , so  
 $m\angle C > m\angle A$ .

#### THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



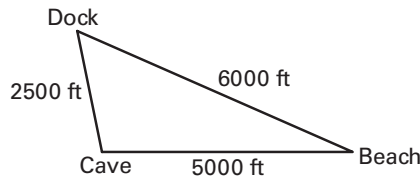
$m\angle A > m\angle C$ ,  
so  $BC > AB$ .

Be careful not to confuse the symbol  $\angle$  meaning *angle* with the symbol  $<$  meaning *is less than*. Notice that the bottom edge of the angle symbol is horizontal.

## Your Notes

### Example 2 Find angle measures

**Boating** A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown below. One of the angles in the path is about  $55^\circ$  and one is about  $24^\circ$ . What is the angle measure of the path made at the cave?



### Solution

The cave is opposite the longest side so, by Theorem 5.10, the cave angle is the largest angle.

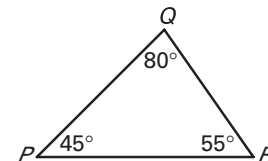
The angle measures sum to  $180^\circ$ , so the third angle measure is  $180^\circ - (55^\circ + 24^\circ) = 101^\circ$ .

The angle measure made at the cave is  $101^\circ$ .

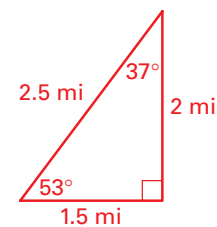
✓ **Checkpoint** Complete the following exercises.

1. List the sides of  $\triangle PQR$  in order from shortest to longest.

$\overline{QR}, \overline{PQ}, \overline{PR}$



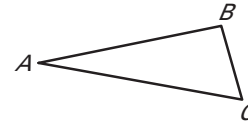
2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of  $90^\circ$ , about  $53^\circ$ , and about  $37^\circ$ . Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.



## Your Notes

### THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$$\underline{AB} + \underline{BC} > AC$$

$$AC + \underline{BC} > \underline{AB}$$

$$\underline{AB} + AC > \underline{BC}$$

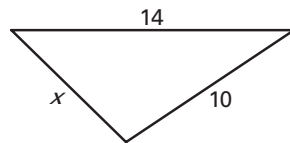
### Example 3 Find possible side lengths

A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

#### Solution

Let  $x$  represent the length of the third side. Draw diagrams to help visualize the small and large values of  $x$ . Then use the Triangle Inequality Theorem to write and solve inequalities.

#### Small values of $x$



$$x + \underline{10} > \underline{14}$$

$$x > \underline{4}$$

#### Large values of $x$



$$\underline{10} + \underline{14} > x$$

$$\underline{24} > x, \text{ or } x < \underline{24}$$

The length of the third side must be greater than 4 and less than 24.

### ✓ Checkpoint Complete the following exercise.

#### Homework

3. A triangle has one side of 23 meters and another of 17 meters. Describe the possible lengths of the third side.

The length of the third side must be greater than 6 meters and less than 40 meters.