

6.1

Ratios, Proportions, and the Geometric Mean

Goal • Solve problems by writing and solving proportions.

Your Notes

VOCABULARY

Ratio of a to b If a and b are two numbers or quantities and $b \neq 0$, then the ratio of a to b is $\frac{a}{b}$.

Proportion An equation that states that two ratios are equal is a proportion.

Means, extremes In the proportion $\frac{a}{b} = \frac{c}{d}$, b and c are the means, and a and d are the extremes.

Geometric mean The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.

Example 1 Simplify ratios

Simplify the ratio. (See Table of Measures, p. 921)

a. 76 cm : 8 cm

b. $\frac{4 \text{ ft}}{24 \text{ in.}}$

Solution

a. Write 76 cm : 8 cm as $\frac{76 \text{ cm}}{8 \text{ cm}}$. Then divide out the units and simplify.

$$\frac{76 \cancel{\text{ cm}}}{8 \cancel{\text{ cm}}} = \frac{19}{2} = 19 : 2$$

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{4 \text{ ft}}{24 \text{ in.}} = \frac{4 \cancel{\text{ ft}}}{24 \cancel{\text{ in.}}} \cdot \frac{12 \cancel{\text{ in.}}}{1 \cancel{\text{ ft}}} = \frac{48}{24} = \frac{2}{1}$$

For help with conversion factors, see p. 886.

Your Notes

Example 2 Use a ratio to find a dimension

Painting You are painting barn doors. You know that the perimeter of the doors is 64 feet and that the ratio of the length to the height is 3:5. Find the area of the doors.

Solution

Step 1 Write expressions for the length and height.

Because the ratio of the length to height is 3:5, you can represent the length by 3 x and the height by 5 x .

Step 2 Solve an equation to find x .

$$2l + 2w = P \quad \text{Formula for perimeter}$$

$$2(\underline{3} x) + 2(\underline{5} x) = \underline{64} \quad \text{Substitute.}$$

$$\underline{16} x = \underline{64} \quad \text{Multiply and combine like terms.}$$

$$x = \underline{4} \quad \text{Divide each side by } \underline{16} .$$

Step 3 Evaluate the expressions for the length and height. Substitute the value of x into each expression.

$$\text{Length: } \underline{3} x = \underline{3} (\underline{4}) = \underline{12}$$

$$\text{Height: } \underline{5} x = \underline{5} (\underline{4}) = \underline{20}$$

The doors are 12 feet long and 20 feet high, so the area is 12 \cdot 20 = 240 ft².

✔ **Checkpoint** In Exercises 1 and 2, simplify the ratio.

1. 4 meters to 18 meters

2 to 9

2. 33 yd:9 ft

11:1

3. The perimeter of a rectangular table is 21 feet and the ratio of its length to its width is 5:2. Find the length and width of the table.

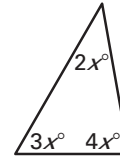
length: 7.5 feet, width: 3 feet

Example 3 Use extended ratios

The measures of the angles in $\triangle BCD$ are in the *extended ratio* of 2:3:4. Find the measures of the angles.

Solution

Begin by sketching the triangle. Then use the extended ratio of 2:3:4 to label the measures as $\underline{2}x^\circ$, $\underline{3}x^\circ$, and $\underline{4}x^\circ$.



$$\underline{2}x^\circ + \underline{3}x^\circ + \underline{4}x^\circ = 180^\circ \quad \text{Triangle Sum Theorem}$$

$$\underline{9}x = 180 \quad \text{Combine like terms.}$$

$$x = \underline{20} \quad \text{Divide each side by } \underline{9}.$$

The angle measures are $2(\underline{20^\circ}) = \underline{40^\circ}$,
 $3(\underline{20^\circ}) = \underline{60^\circ}$, and $4(\underline{20^\circ}) = \underline{80^\circ}$.

Checkpoint Complete the following exercise.

4. A triangle's angle measures are in the extended ratio of 1:4:5. Find the measures of the angles.

$18^\circ, 72^\circ, 90^\circ$

A PROPERTY OF PROPORTIONS

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $\underline{ad} = \underline{bc}$.

$$\frac{2}{3} = \frac{4}{6} \quad \begin{array}{l} \curvearrowright 3 \cdot \underline{4} = \underline{12} \\ \curvearrowleft 2 \cdot \underline{6} = \underline{12} \end{array}$$

Your Notes

In part (a), you could multiply each side by the denominator, 16.

Then

$$16 \cdot \frac{3}{4} = 16 \cdot \frac{x}{16}$$

$$\text{so } \underline{12} = x.$$

Example 4 Solve proportions

Solve the proportion.

a. $\frac{3}{4} = \frac{x}{16}$

Original proportion

$$3 \cdot \underline{16} = \underline{4} \cdot x$$

Cross Products Property

$$\underline{48} = \underline{4}x$$

Multiply.

$$\underline{12} = x$$

Divide each side by 4.

b. $\frac{3}{x+1} = \frac{2}{x}$

Original proportion

$$\underline{3} \cdot x = \underline{2}(x+1)$$

Cross Products Property

$$\underline{3}x = \underline{2}x + \underline{2}$$

Distributive Property

$$x = \underline{2}$$

Subtract 2x from each side.

Example 5 Solve a real-world problem

Bowling You want to find the total number of rows of boards that make up 24 lanes at a bowling alley. You know that there are 117 rows in 3 lanes. Find the total number of rows of boards that make up the 24 lanes.

Solution

Write and solve a proportion involving two ratios that compare the number of rows with the number of lanes.

$$\frac{\underline{117}}{\underline{3}} = \frac{\underline{n}}{\underline{24}}$$

← number of rows

← number of lanes

Write proportion.

$$\underline{117} \cdot \underline{24} = \underline{3} \cdot \underline{n}$$

Cross Products Property

$$\underline{936} = n$$

Simplify.

There are 936 rows of boards that make up the 24 lanes.

GEOMETRIC MEAN

The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.

$$\text{So, } x^2 = \underline{ab} \text{ and } x = \sqrt{\underline{ab}}.$$

Your Notes

Example 6 Find a geometric mean

Find the geometric mean of 16 and 48.

Solution

$$\begin{aligned}x &= \sqrt{ab} && \text{Definition of geometric mean} \\&= \sqrt{16 \cdot 48} && \text{Substitute } 16 \text{ for } a \text{ and } 48 \text{ for } b. \\&= \sqrt{16 \cdot 16 \cdot 3} && \text{Factor.} \\&= 16\sqrt{3} && \text{Simplify.}\end{aligned}$$

The geometric mean of 16 and 48 is $16\sqrt{3} \approx 27.7$.

✓ **Checkpoint** Complete the following exercises.

5. Solve $\frac{8}{y} = \frac{2}{5}$.

$$y = 20$$

6. Solve $\frac{x-3}{3} = \frac{2x}{9}$.

$$x = 9$$

7. A small gymnasium contains 10 sets of bleachers. You count 192 spectators in 3 sets of bleachers and the spectators seem to be evenly distributed. Estimate the total number of spectators.

about 640 spectators

8. Find the geometric mean of 14 and 16.

$$4\sqrt{14} \approx 15.0$$

Homework