

6.3

Use Similar Polygons

Goal • Use proportions to identify similar polygons.

Your Notes

VOCABULARY

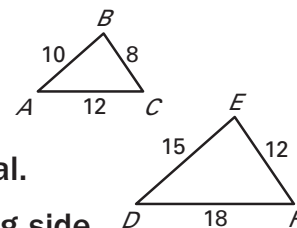
Similar polygons Two polygons are similar polygons if corresponding angles are congruent and corresponding side lengths are proportional.

Scale factor of two similar polygons If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the scale factor.

Example 1 Use similarity statements

In the diagram, $\triangle ABC \sim \triangle DEF$.

- List all pairs of congruent angles.
- Check that the ratios of corresponding side lengths are equal.
- Write the ratios of the corresponding side lengths in a statement of proportionality.



In a statement of proportionality, any pair of ratios forms a true proportion.

Solution

a. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

b. $\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$ $\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$
 $\frac{CA}{FD} = \frac{12}{18} = \frac{2}{3}$

c. The ratios in part (b) are equal, so

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Checkpoint Complete the following exercise.

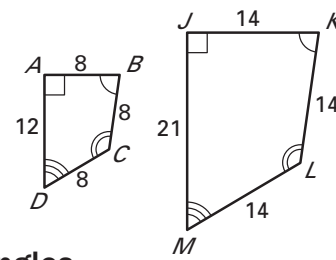
- Given $\triangle PQR \sim \triangle XYZ$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

$$\angle P \cong \angle X, \angle Q \cong \angle Y, \angle R \cong \angle Z, \frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX}$$

Your Notes

Example 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of $ABCD$ to $JKLM$.



Solution

Step 1 Identify pairs of congruent angles.

From the diagram, you can see that $\angle B \cong \angle K$, $\angle C \cong \angle L$, and $\angle D \cong \angle M$. Angles A and J are right angles, so $\angle A \cong \angle J$. So, the corresponding angles are congruent.

Step 2 Show that corresponding side lengths are proportional.

$$\frac{AB}{JK} = \frac{8}{14} = \frac{4}{7} \qquad \frac{BC}{KL} = \frac{8}{14} = \frac{4}{7}$$

$$\frac{CD}{LM} = \frac{8}{14} = \frac{4}{7} \qquad \frac{AD}{JM} = \frac{12}{21} = \frac{4}{7}$$

The ratios are equal, so the corresponding side lengths are proportional.

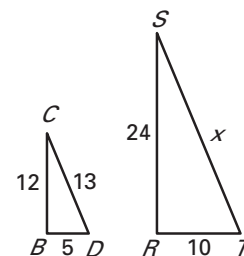
So $ABCD \sim JKLM$. The scale factor of $ABCD$ to $JKLM$ is $\frac{4}{7}$.

Example 3 Use similar polygons

In the diagram, $\triangle BCD \sim \triangle RST$. Find the value of x .

Solution

The triangles are similar, so the corresponding side lengths are proportional.



Write proportion.

$$\frac{BC}{RS} = \frac{CD}{ST}$$

Substitute.

$$\frac{12}{24} = \frac{13}{x}$$

Cross Products Property

$$12x = 312$$

Solve for x .

$$x = 26$$

There are several ways to write the proportion. For example, you could write $\frac{BD}{RT} = \frac{CD}{ST}$.

Your Notes

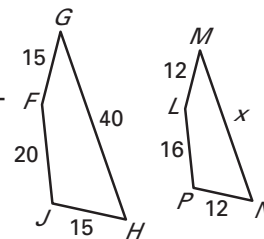
✔ **Checkpoint** In the diagram, $FGHJ \sim LMNP$.

2. What is the scale factor of $LMNP$ to $FGHJ$?

$$\frac{4}{5}$$

3. Find the value of x .

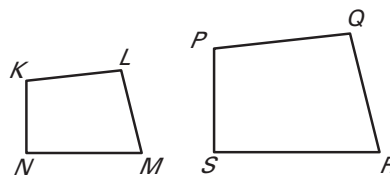
$$32$$



THEOREM 6.1: PERIMETERS OF SIMILAR POLYGONS

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then



$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$$

Example 4 Find perimeters of similar figures

Basketball A larger cement court is being poured for a basketball hoop in place of a smaller one. The court will be 20 feet wide and 25 feet long. The old court was similar in shape, but only 16 feet wide.

- Find the scale factor of the new court to the old court.
- Find the perimeters of the new court and the old court.

Solution

- Because the new court will be similar to the old court, the scale factor is the ratio of the widths, $\frac{20}{16} = \frac{5}{4}$.
- The new court's perimeter is $2(20) + 2(25) = 90$ feet. Use Theorem 6.1 to find the perimeter x of the old court.

$$\frac{90}{x} = \frac{5}{4} \quad \text{Use Theorem 6.1 to write a proportion.}$$

$$x = 72 \quad \text{Simplify.}$$

The perimeter of the old court was 72 feet.

Your Notes

CORRESPONDING LENGTHS IN SIMILAR POLYGONS

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

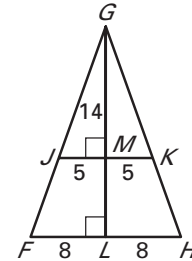
Example 5 Use a scale factor

In the diagram, $\triangle FGH \sim \triangle JGK$.
Find the length of the altitude GL .

Solution

First, find the scale factor of $\triangle FGH$ to $\triangle JGK$.

$$\frac{FH}{JK} = \frac{8 + 8}{5 + 5} = \frac{16}{10} = \frac{8}{5}$$



Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$$\frac{GL}{GM} = \frac{8}{5} \quad \text{Write proportion.}$$

$$\frac{GL}{14} = \frac{8}{5} \quad \text{Substitute } 14 \text{ for } GM.$$

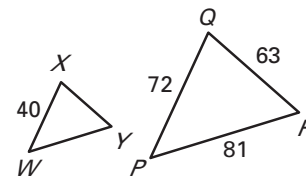
$$GL = 22.4 \quad \text{Multiply each side by } 14 \text{ and simplify.}$$

The length of altitude GL is 22.4.

✓ Checkpoint In the diagrams, $\triangle PQR \sim \triangle WXY$.

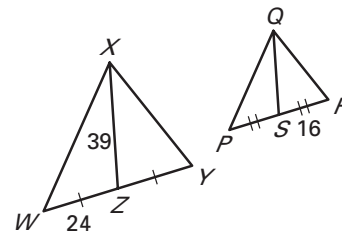
4. Find the perimeter of $\triangle WXY$.

The perimeter of $\triangle WXY$ is 120.



5. Find the length of median QS .

$$QS = 26$$



Homework