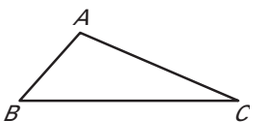
6.5 Prove Triangles Similar by SSS and SAS

Goal • Use the SSS and SAS Similarity Theorems.

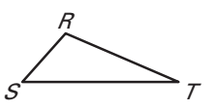
Your Notes

THEOREM 6.2: SIDE-SIDE-SIDE (SSS) SIMILARITY THEOREM

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

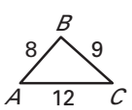


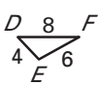
If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

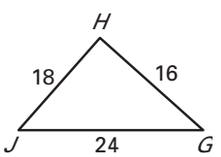


Example 1 *Use the SSS Similarity Theorem*

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?







Solution

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Shortest sides	Longest sides	Remaining sides
$\frac{AB}{DE} = \frac{8}{4} = 2$	$\frac{CA}{FD} = \frac{12}{8} = \frac{3}{2}$	$\frac{BC}{EF} = \frac{9}{6} = \frac{3}{2}$

All the ratios are not equal, so $\triangle ABC$ and $\triangle DEF$ are not similar.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

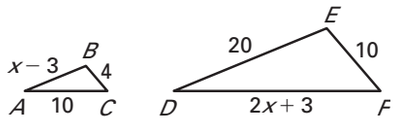
Shortest sides	Longest sides	Remaining sides
$\frac{AB}{GH} = \frac{8}{16} = \frac{1}{2}$	$\frac{CA}{JG} = \frac{12}{24} = \frac{1}{2}$	$\frac{BC}{HJ} = \frac{9}{18} = \frac{1}{2}$

All the ratios are equal, so $\triangle ABC \sim \triangle GHJ$.

Your Notes

Example 2 *Use the SSS Similarity Theorem*

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



Solution

Step 1 **Find** the value of x that makes corresponding side lengths proportional.

$\frac{4}{10} = \frac{x-3}{20}$ **Write proportion.**

$4 \cdot 20 = 10(x-3)$ **Cross Products Property**

$80 = 10x - 30$ **Simplify.**

$11 = x$ **Solve for x.**

Step 2 **Check** that the side lengths are proportional when $x =$ 11 .

$AB = x - 3 =$ 8 $DF = 2x + 3 =$ 25

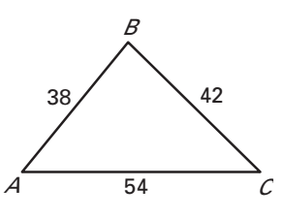
$\frac{BC}{EF} \stackrel{?}{=} \frac{AB}{DE}$ $\frac{4}{10} = \frac{8}{20}$ ✓ $\frac{BC}{EF} \stackrel{?}{=} \frac{AC}{DF}$ $\frac{4}{10} = \frac{10}{25}$ ✓

When $x =$ 11 , the triangles are similar by the SSS Similarity Theorem .

Checkpoint **Complete the following exercises.**

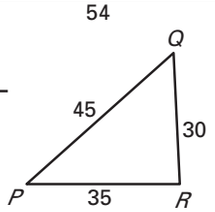
1. Which of the three triangles are similar?

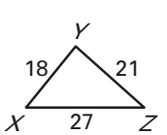
$\triangle PQR \sim \triangle ZXY$



2. Suppose AB is not given in $\triangle ABC$. What length for AB would make $\triangle ABC$ similar to $\triangle QRP$?

$AB = 36$

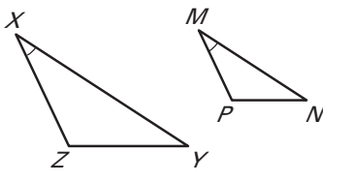




Your Notes

THEOREM 6.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM

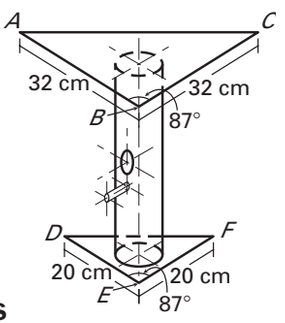
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



If $\angle X \cong \angle M$, and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Example 3 ***Use the SAS Similarity Theorem***

Birdfeeder You are drawing a design for a birdfeeder. Can you construct the top so it is similar to the bottom using the angle measure and lengths shown?



Solution

Both $m\angle$ B and $m\angle$ E equal 87°, so $\angle$ B $\cong \angle$ E. Next, compare the ratios of the lengths of the sides that include $\angle B$ and $\angle E$.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{32}{20} = \frac{8}{5}$$

The lengths of the sides that include $\angle B$ and $\angle E$ are proportional.

So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle DEF$.
Yes, you can make the top similar to the bottom.

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose you use equilateral triangles on the top and bottom. Are the top and bottom similar? *Explain*.

Yes, the top and bottom are similar. If the side length of the top is a and the side length of the bottom is b, the ratios of the side lengths are $\frac{a}{b}$ and the angles are all 60°. The triangles are similar by SAS or SSS.

Your Notes

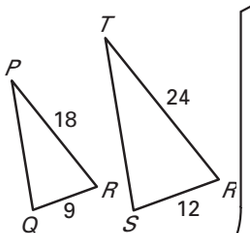
TRIANGLE SIMILARITY POSTULATE AND THEOREMS

AA Similarity Postulate If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

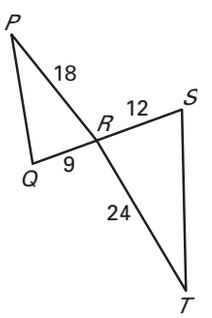
SAS Similarity Theorem If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Example 4 *Choose a method*

Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{QR}{RS} = \frac{9}{12} = \frac{3}{4}$

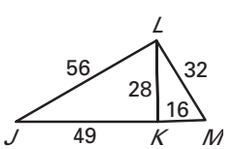
Longer sides $\frac{PR}{RT} = \frac{18}{24} = \frac{3}{4}$

The corresponding side lengths are proportional. The included angles $\angle PRQ$ and $\angle TRS$ are congruent because they are vertical angles. So, $\triangle PQR \sim \triangle TSR$ by the SAS Similarity Theorem.

✔ ***Checkpoint*** **Complete the following exercise.**

4. *Explain* how to show $\triangle JKL \sim \triangle LKM$.

Show that the corresponding side lengths are proportional, then use the SSS Similarity Theorem to show $\triangle JKL \sim \triangle LKM$.



Homework