

6.6

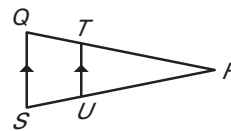
Use Proportionality Theorems

Goal • Use proportions with a triangle or parallel lines.

Your Notes

THEOREM 6.4: TRIANGLE PROPORTIONALITY THEOREM

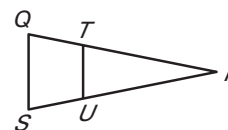
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

THEOREM 6.5: CONVERSE OF THE TRIANGLE PROPORTIONALITY THEOREM

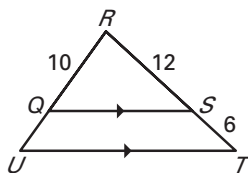
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Example 1 Find the length of a segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RQ = 10$, $RS = 12$, and $ST = 6$. What is the length of \overline{QU} ?



Solution

$$\frac{RQ}{QU} = \frac{RS}{ST}$$

Triangle Proportionality Theorem

$$\frac{10}{QU} = \frac{12}{6}$$

Substitute.

$$60 = 12 \cdot QU$$

Cross Products Property

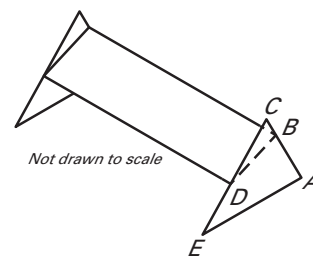
$$5 = QU$$

Divide each side by 12.

Your Notes

Example 2 Solve a real-world problem

Aerodynamics A spoiler for a remote controlled car is shown where $AB = 31$ mm, $BC = 19$ mm, $CD = 27$ mm, and $DE = 23$ mm. Explain why \overline{BD} is not parallel to \overline{AE} .



Solution

Find and simplify the ratios of lengths determined by \overline{BD} .

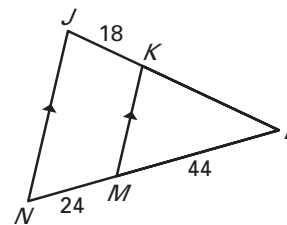
$$\frac{CD}{DE} = \frac{27}{23} \quad \frac{CB}{BA} = \frac{31}{19}$$

Because $\frac{27}{23} \neq \frac{31}{19}$, \overline{BD} is not parallel to \overline{AE} .

✓ **Checkpoint** Complete the following exercises.

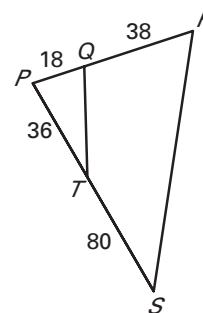
1. Find the length of \overline{KL} .

$$KL = 33$$



2. Determine whether $\overline{QT} \parallel \overline{RS}$.

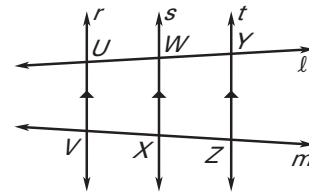
No; \overline{QT} is not parallel to \overline{RS} .



Your Notes

THEOREM 6.6

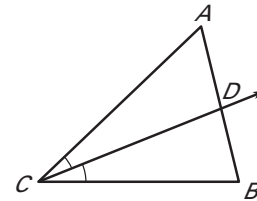
If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$$\frac{UW}{WY} = \frac{VX}{XZ}$$

THEOREM 6.7

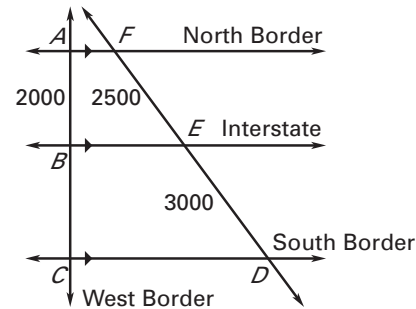
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



$$\frac{AD}{DB} = \frac{CA}{CB}$$

Example 3 Use Theorem 6.6

Farming A farmer's land is divided by a newly constructed interstate. The distances shown are in meters. Find the distance CA between the north border and the south border of the farmer's land.



Use Theorem 6.6.

$$\frac{CB}{BA} = \frac{DE}{EF}$$

$$\frac{CB + BA}{BA} = \frac{DE + EF}{EF}$$

$$\frac{CA}{2000} = \frac{3000 + 2500}{2500}$$

$$\frac{CA}{2000} = \frac{5500}{2500}$$

$$CA = 4400$$

Parallel lines divide transversals proportionally.

Property of proportions (Property 4)

Substitute.

Simplify.

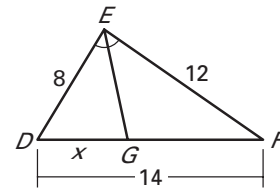
Multiply each side by 2000 and simplify.

The distance between the north border and the south border is 4400 meters.

Your Notes

Example 4 Use Theorem 6.7

In the diagram, $\angle DEG \cong \angle GEF$.
Use the given side lengths to
find the length of \overline{DG} .



Solution

Because \overrightarrow{EG} is an angle bisector of $\angle DEF$, you can apply Theorem 6.7. Let $GD = x$. Then $GF = 14 - x$.

$$\frac{GF}{GD} = \frac{EF}{ED}$$

Angle bisector divides opposite side proportionally.

$$\frac{14 - x}{x} = \frac{12}{8}$$

Substitute.

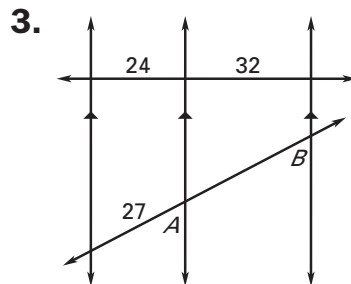
$$12x = 112 - 8x$$

Cross Products Property

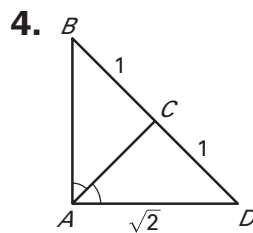
$$x = 5.6$$

Solve for x .

✓ **Checkpoint** Find the length of \overline{AB} .



$$AB = 36$$



$$AB = \sqrt{2}$$

Homework