

6.7

Perform Similarity Transformations

Goal • Perform dilations.

Your Notes

VOCABULARY

Dilation A dilation is a transformation that stretches or shrinks a figure to create a similar figure.

Center of dilation In a dilation, a figure is enlarged or reduced with respect to a fixed point called the center of dilation.

Scale factor of a dilation The scale factor k of a dilation is the ratio of a side length of the image to the corresponding side length of the original figure.

Reduction A dilation where $0 < k < 1$ is a reduction.

Enlargement A dilation where $k > 1$ is an enlargement.

COORDINATE NOTATION FOR A DILATION

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a reduction. If $k > 1$, the dilation is an enlargement.

Your Notes

All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Example 1 Draw a dilation with a scale factor greater than 1

Draw a dilation of quadrilateral $ABCD$ with vertices $A(2, 0)$, $B(6, -4)$, $C(8, 2)$, and $D(6, 4)$. Use a scale factor of $\frac{1}{2}$.

First draw $ABCD$. Find the dilation of each vertex by multiplying its coordinates by $\frac{1}{2}$. Then draw the dilation.

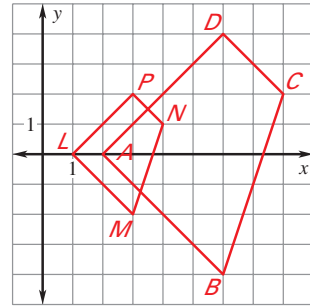
$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y \right)$$

$$A(2, 0) \rightarrow L(1, 0)$$

$$B(6, -4) \rightarrow M(3, -2)$$

$$C(8, 2) \rightarrow N(4, 1)$$

$$D(6, 4) \rightarrow P(3, 2)$$



Example 2 Verify that a figure is similar to its dilation

A triangle has the vertices $A(2, -1)$, $B(4, -1)$, and $C(4, 2)$. The image of $\triangle ABC$ after a dilation with a scale factor of 2 is $\triangle DEF$.

- Sketch $\triangle ABC$ and $\triangle DEF$.
- Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

Solution

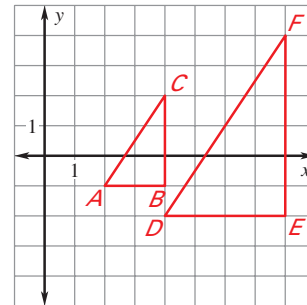
- The scale factor is greater than 1, so the dilation is an **enlargement**.

$$(x, y) \rightarrow (2x, 2y)$$

$$A(2, -1) \rightarrow D(4, -2)$$

$$B(4, -1) \rightarrow E(8, -2)$$

$$C(4, 2) \rightarrow F(8, 4)$$



- Because $\angle B$ and $\angle E$ are both right angles, $\angle B \cong \angle E$. Show that the lengths of the sides that include $\angle B$ and $\angle E$ are proportional.

$$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \quad \frac{2}{4} = \frac{3}{6} \checkmark$$

The lengths are proportional. So, $\triangle ABC \sim \triangle DEF$ by the **SAS Similarity Theorem**.

Example 3 Find a scale factor

Magnets You are making your own photo magnets. Your photo is 8 inches by 10 inches. The image on the magnet is 2.8 inches by 3.5 inches. What is the scale factor of the reduction?

Solution

The scale factor is the ratio of a side length of the magnet image to a side length of the

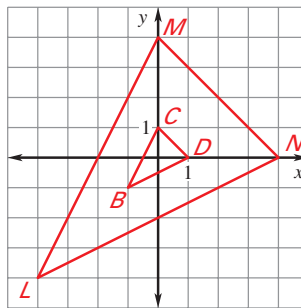
original photo, or $\frac{2.8 \text{ in.}}{8 \text{ in.}}$. In simplest form,

the scale factor is $\frac{7}{20}$.

✔ **Checkpoint** Complete the following exercises.

1. A triangle has the vertices $B(-1, -1)$, $C(0, 1)$, and $D(1, 0)$. Find the coordinates of L , M , and N so that $\triangle LMN$ is a dilation of $\triangle BCD$ with a scale factor of 4. Sketch $\triangle BCD$ and $\triangle LMN$.

$L(-4, -4)$,
 $M(0, 4)$,
 $N(4, 0)$



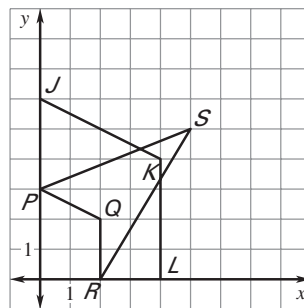
2. In Example 3, what is the scale factor of the reduction if your photo is 4 inches by 5 inches?

$\frac{7}{10}$

Your Notes

Example 4 Find missing coordinates

You want to create a quadrilateral $JKLM$ that is similar to quadrilateral $PQRS$. What are the coordinates of M ?



Solution

Determine if $JKLM$ is a dilation of $PQRS$ by checking whether the same scale factor can be used to obtain J , K , and L from P , Q , and R .

$$(x, y) \rightarrow (kx, ky)$$

$$P(0, 3) \rightarrow J(0, 6) \quad k = 2$$

$$Q(2, 2) \rightarrow K(4, 4) \quad k = 2$$

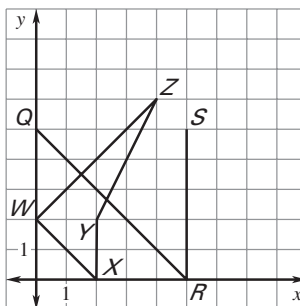
$$R(2, 0) \rightarrow L(4, 0) \quad k = 2$$

Because k is the same in each case, the image is a **dilation** with a scale factor of **2**. So, you can use the scale factor to find the image M of point S .

$$S(5, 5) \rightarrow M(2 \cdot 5, 2 \cdot 5) = M(10, 10)$$

✔ **Checkpoint** Complete the following exercise.

3. You want to create a quadrilateral $QRST$ that is similar to quadrilateral $WXYZ$. What are the coordinates of T ?



$$T(10, 15)$$

Homework