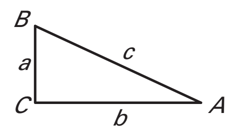
7.2 Use the Converse of the Pythagorean Theorem

Goal • Use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

Your Notes

THEOREM 7.2: CONVERSE OF THE PYTHAGOREAN THEOREM

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

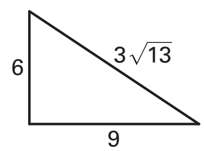


If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

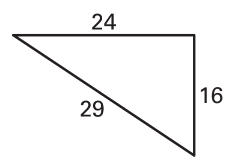
Example 1 ***Verify right triangles***

Tell whether the given triangle is a right triangle.

a.



b.



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{13})^2 \stackrel{?}{=} 6^2 + 9^2$

$9 \cdot 13 \stackrel{?}{=} 36 + 81$

$117 = 117$ ✓

The triangle is a right triangle.

b. $29^2 \stackrel{?}{=} 24^2 + 16^2$

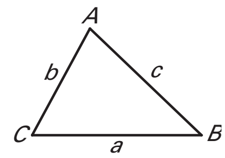
$841 \stackrel{?}{=} 576 + 256$

$841 \neq 832$

The triangle is not a right triangle.

Your Notes

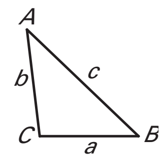
THEOREM 7.3



If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an acute triangle.

If $c^2 < a^2 + b^2$, then the triangle *ABC* is acute.

THEOREM 7.4



If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an obtuse triangle.

If $c^2 > a^2 + b^2$, then the triangle *ABC* is obtuse.

Example 2 ***Classify triangles***

Can segments with lengths of 2.8 feet, 3.2 feet, and 4.2 feet form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

Solution

The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Step 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$2.8 + 3.2 = 6$	$2.8 + 4.2 = 7$	$3.2 + 4.2 = 7.4$
$6 > 4.2$	$7 > 3.2$	$7.4 > 2.8$

Step 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \ ? \ a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$4.2^2 \ ? \ 2.8^2 + 3.2^2$	**Substitute.**
$17.64 \ ? \ 7.84 + 10.24$	**Simplify.**
$17.64 < 18.08$	**c^2 is less than $a^2 + b^2$.**

The side lengths 2.8 feet, 3.2 feet, and 4.2 feet form an acute triangle.

Your Notes

Example 3 *Use the Converse of the Pythagorean Theorem*

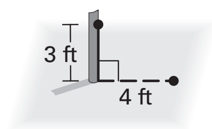
Lights You are helping install a light pole in a parking lot. When the pole is positioned properly, it is perpendicular to the pavement. How can you check that the pole is perpendicular using a tape measure?

Solution

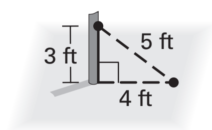
To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

Think of the pole as a line and the pavement as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the pole is perpendicular to different lines on the pavement.

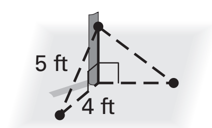
First mark 3 feet up the pole and mark on the pavement 4 feet from the pole.



Use the tape measure to check that the distance between the two marks is 5 feet. The pole makes a right angle with the line on the pavement.

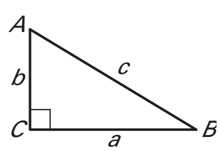


Finally, repeat the procedure to show that the pole is perpendicular to another line on the pavement.



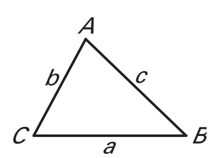
METHODS FOR CLASSIFYING A TRIANGLE BY ANGLES USING ITS SIDE LENGTHS

Theorem 7.2



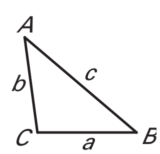
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

Theorem 7.4

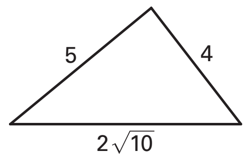


If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Your Notes

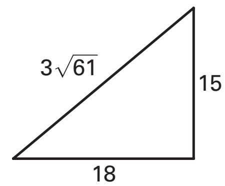
✔ ***Checkpoint*** **In Exercises 1 and 2, tell whether the triangle is a right triangle.**

1.



not a right triangle

2.



right triangle

3. Can segments with lengths of 6.1 inches, 9.4 inches, and 11.3 inches form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

Yes; obtuse

4. In Example 3, could you use triangles with side lengths 50 inches, 120 inches, and 130 inches to verify that you have perpendicular lines? *Explain*.

Yes; A triangle with side lengths 50 inches, 120 inches, and 130 inches is a right triangle. The right triangle shows that you have perpendicular lines.

Homework