8.1 Find Angle Measures in Polygons

Goal • Find angle measures in polygons.

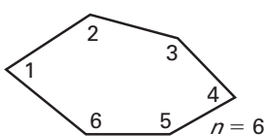
Your Notes

VOCABULARY

Diagonal A diagonal of a polygon is a segment that joins two *nonconsecutive vertices*.

THEOREM 8.1: POLYGON INTERIOR ANGLES THEOREM

The sum of the measures of the interior angles of a convex n-gon is $(n - \underline{2}) \cdot \underline{180^\circ}$.



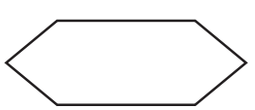
$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - \underline{2}) \cdot \underline{180^\circ}$

COROLLARY TO THEOREM 8.1: INTERIOR ANGLES OF A QUADRILATERAL

The sum of the measures of the interior angles of a quadrilateral is $\underline{360^\circ}$.

Example 1 ***Find the sum of angle measures in a polygon***

Find the sum of the measures of the interior angles of a convex hexagon.



Solution

A hexagon has $\underline{6}$ sides. Use the Polygon Interior Angles Theorem.

$(n - \underline{2}) \cdot \underline{180^\circ} = (\underline{6} - \underline{2}) \cdot \underline{180^\circ}$ **Substitute $\underline{6}$ for n.**

$= \underline{4} \cdot \underline{180^\circ}$ **Subtract.**

$= \underline{720^\circ}$ **Multiply.**

The sum of the measures of the interior angles of a hexagon is $\underline{720^\circ}$.

Your Notes

Example 2 ***Find the number of sides of a polygon***

The sum of the measures of the interior angles of a convex polygon is 1260°. Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n. Then solve the equation to find the number of sides.

$(n - \underline{2}) \cdot \underline{180^\circ} = \underline{1260^\circ}$ **Polygon Interior Angles Theorem**

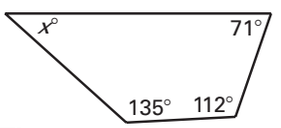
$n - \underline{2} = \underline{7}$ **Divide each side by** $\underline{180^\circ}$.

$n = \underline{9}$ **Add** $\underline{2}$ **to each side.**

The polygon has $\underline{9}$ sides. It is a $\underline{\text{nonagon}}$.

Example 3 ***Find an unknown interior angle measure***

Find the value of x in the diagram shown.



Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x. Then solve the equation.

$x^\circ + \underline{135^\circ} + \underline{112^\circ} + \underline{71^\circ} = \underline{360^\circ}$ **Corollary to Theorem 8.1**

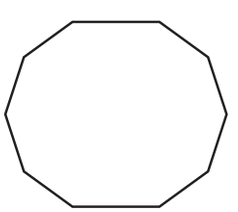
$x + \underline{318} = \underline{360}$ **Combine like terms.**

$x = \underline{42}$ **Subtract** $\underline{318}$ **from each side.**

✔ ***Checkpoint*** **Complete the following exercise.**

1. Find the sum of the measures of the interior angles of the convex decagon.

1440°



Your Notes

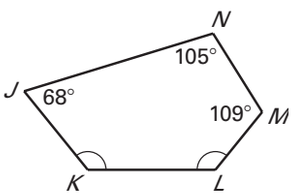
✔ ***Checkpoint*** **Complete the following exercises.**

2. The sum of the measures of the interior angles of a convex polygon is 1620°. Classify the polygon by the number of sides.

11-gon

3. Use the diagram at the right. Find $m\angle K$ and $m\angle L$.

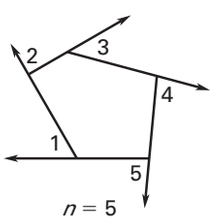
$m\angle K = m\angle L = 129°$



THEOREM 8.2: POLYGON EXTERIOR ANGLES THEOREM

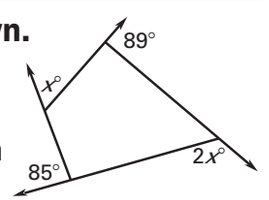
The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360° .

$m\angle 1 + m\angle 2 + \cdots + m\angle n =$ 360°



Example 4 ***Find unknown exterior angle measures***

Find the value of *x* in the diagram shown.



Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° +$ 2x° $+$ 85° $+$ 89° $=$ 360° **Polygon Exterior Angles Theorem.**

3 $x +$ 174 $=$ 360 **Combine like terms.**

$x =$ 62 **Solve for *x*.**

Your Notes

Example 5 ***Find angle measures in regular polygons***

Lamps The base of a lamp is in the shape of a regular 15-gon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

Solution

a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n - \underline{2}) \cdot \underline{180^\circ} = (\underline{15} - \underline{2}) \cdot \underline{180^\circ}$

$= \underline{2340^\circ}$

Then find the measure of one interior angle. A regular 15-gon has $\underline{15}$ congruent interior angles. Divide $\underline{2340^\circ}$ by $\underline{15}$: $\underline{2340^\circ} \div \underline{15} = \underline{156^\circ}$.

The measure of each interior angle in the 15-gon is $\underline{156^\circ}$.

b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is $\underline{360^\circ}$. Divide $\underline{360^\circ}$ by $\underline{15}$: $\underline{360^\circ} \div \underline{15} = \underline{24^\circ}$.

The measure of each exterior angle in the 15-gon is $\underline{24^\circ}$.

✔ ***Checkpoint*** **Complete the following exercises.**

4. A convex pentagon has exterior angles with measures 66°, 77°, 82°, and 62°. What is the measure of an exterior angle at the fifth vertex?

73°

5. Find the measure of (a) each interior angle and (b) each exterior angle of a regular nonagon.

a. 140°

b. 40°

Homework