8.3 Show that a Quadrilateral is a Parallelogram
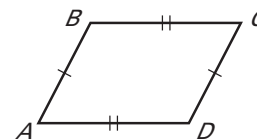
Goal • Use properties to identify parallelograms.

Your Notes

THEOREM 8.7

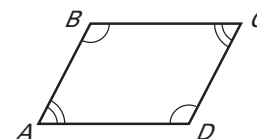If both pairs of opposite __sides__ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \underline{\overline{CD}}$ and $\overline{BC} \cong \underline{\overline{AD}}$, then *ABCD* is a parallelogram.

THEOREM 8.8

If both pairs of opposite __angles__ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \underline{\angle C}$ and $\angle B \cong \underline{\angle D}$, then *ABCD* is a parallelogram.


Example 1 *Solve a real-world problem*

Basketball In the diagram at the right, $\overline{AB}$ and $\overline{DC}$ represent adjustable supports of a basketball hoop. *Explain* why $\overline{AD}$ is always parallel to $\overline{BC}$.

A 18 in.
36 in.
D
36 in. 18 in.
B
C

Solution

The shape of quadrilateral *ABCD* changes as the adjustable supports move, but its __side lengths__ do not change. Both pairs of opposite __sides__ are congruent, so *ABCD* is a parallelogram by __Theorem 8.7__.

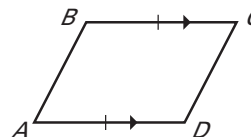By the definition of a parallelogram, $\overline{AD} \parallel \underline{\overline{BC}}$.
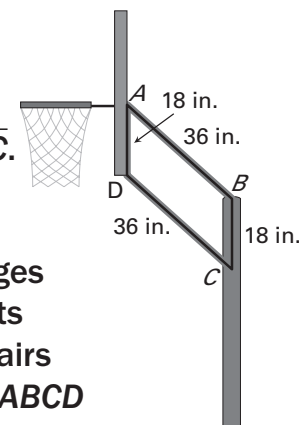
THEOREM 8.9

If one pair of opposite sides of a quadrilateral are __congruent__ and __parallel__ , then the quadrilateral is a parallelogram.
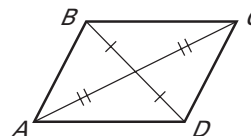
If $\overline{BC}$ __∥__ $\overline{AD}$ and $\overline{BC}$ __≅__ $\overline{AD}$, then ABCD is a parallelogram.
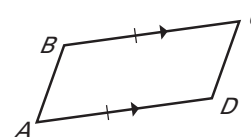
THEOREM 8.10

If the diagonals of a quadrilateral __bisect__ each other, then the quadrilateral is a parallelogram.

If $\overline{BD}$ and $\overline{AC}$ __bisect__ each other, then ABCD is a parallelogram.

Example 2 *Identify a parallelogram*

Lights The headlights of a car have the shape shown at the right. *Explain* how you know that $\angle B \cong \angle D$.

Solution

In the diagram, $\overline{BC}$∥ __$\overline{AD}$__ and $\overline{BC} \cong$ __$\overline{AD}$__ . By __Theorem 8.9__ , quadrilateral ABCD is a parallelogram. By __Theorem 8.4__ , you know that opposite angles of a parallelogram are congruent. So, $\angle B \cong$ __$\angle D$__ .
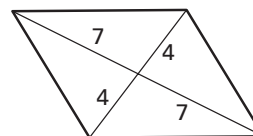
✔ *Checkpoint* **Complete the following exercises.**

1. In quadrilateral GHJK, $m\angle G = 55°$, $m\angle H = 125°$, and $m\angle J = 55°$. Find $m\angle K$. What theorem can you use to show that GHJK is a parallelogram?

   $m\angle K =$ 125°; Theorem 8.8

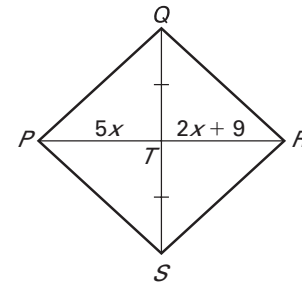2. What theorem can you use to show that the quadrilateral is a parallelogram?

   Theorem 8.10

Example 3 *Use algebra with parallelograms*

For what value of *x* is quadrilateral *PQRS* a parallelogram?

Solution

By Theorem 8.10, if the diagonals of *PQRS* __bisect__ each other, then it is a parallelogram. You are given that $\overline{QT} \cong \underline{\ \overline{ST}\ }$. Find *x* so that $\overline{PT} \cong \underline{\ \overline{RT}\ }$.

$PT = \underline{\ RT\ }$ Set the segment lengths equal.

$5x = \underline{\ 2x+9\ }$ Substitute 5*x* for *PT* and $\underline{\ 2x+9\ }$ for $\underline{\ RT\ }$.

$\underline{\ 3\ }x = \underline{\ 9\ }$ Subtract $\underline{\ 2x\ }$ from each side.

$x = \underline{\ 3\ }$ Divide each side by $\underline{\ 3\ }$.

When $x = \underline{\ 3\ }$, $PT = 5(\underline{\ 3\ }) = \underline{\ 15\ }$ and $RT = 2(\underline{\ 3\ }) + 9 = \underline{\ 15\ }$.

Quadrilateral *PQRS* is a parallelogram when $x = \underline{\ 3\ }$.

CONCEPT SUMMARY: WAYS TO PROVE A QUADRILATERAL IS A PARALLELOGRAM

1. Show both pairs of opposite sides are parallel. (**Definition**)

2. Show both pairs of opposite sides are congruent. (**Theorem 8.7**)
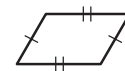
3. Show both pairs of opposite angles are congruent. (**Theorem 8.8**)

4. Show one pair of opposite sides are congruent and parallel. (**Theorem 8.9**)

5. Show the diagonals bisect each other. (**Theorem 8.10**)

Example 4   *Use coordinate geometry*

Show that quadrilateral *KLMN* is a parallelogram.

$L(4, 4)$
$K(2, 2)$
$M(6, 0)$
$N(4, -2)$

Solution

One way is to show that a pair of sides are congruent and parallel. Then apply **Theorem 8.9** .

First use the Distance Formula to show that $\overline{KL}$ and $\overline{MN}$ are **congruent** .

$KL = \sqrt{(4 - 2)^2 + (4 - 2)^2} = \sqrt{8}$

$MN = \sqrt{(6 - 4)^2 + [0 - (-2)]^2} = \sqrt{8}$

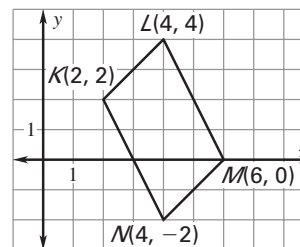Because $KL = MN = \sqrt{8}$, $\overline{KL} \cong \overline{MN}$.

Then use the slope formula to show that $\overline{KL} \parallel \overline{MN}$.

Slope of $\overline{KL} = \dfrac{4 - 2}{4 - 2} = 1$

Slope of $\overline{MN} = \dfrac{0 - (-2)}{6 - 4} = 1$

$\overline{KL}$ and $\overline{MN}$ have the same slope, so they are **parallel** .

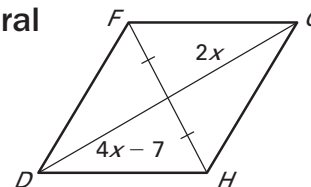$\overline{KL}$ and $\overline{MN}$ are congruent and parallel. So, *KLMN* is a parallelogram by **Theorem 8.9** .

✔ *Checkpoint* **Complete the following exercises.**

3. For what value of x is quadrilateral *DFGH* a parallelogram?

x = 3.5

F G $2x$ $4x - 7$ D H

Homework

4. *Explain* another method that can be used to show that quadrilateral *KLMN* in Example 4 is a parallelogram.

Sample Answer: Draw the diagonals and find the point of intersection. Show the diagonals bisect each other and apply Theorem 8.10.