8.4 Properties of Rhombuses, Rectangles, and Squares

Goal • Use properties of rhombuses, rectangles, and squares.

Your Notes

VOCABULARY

Rhombus A rhombus is a parallelogram with four congruent sides.

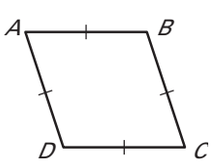
Rectangle A rectangle is a parallelogram with four right angles.

Square A square is a parallelogram with four congruent sides and four right angles.

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

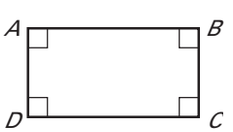
ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

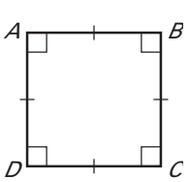
ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



SQUARE COROLLARY

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Your Notes

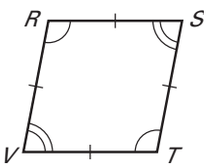
Example 1 ***Use properties of special quadrilaterals***

For any rhombus *RSTV*, decide whether the statement is always or sometimes true. Draw a sketch and explain your reasoning.

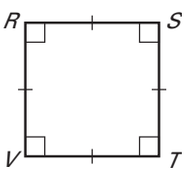
a. $\angle S \cong \angle V$ **b.** $\angle T \cong \angle V$

Solution

a. By definition, a rhombus is a parallelogram with four congruent sides. By Theorem 8.4, opposite angles of a parallelogram are congruent. So, $\angle S \cong \angle V$. The statement is always true.

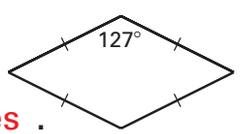


b. If rhombus *RSTV* is a square, then all four angles are congruent right angles. So $\angle T \cong \angle V$ if *RSTV* is a square. Because not all rhombuses are also squares, the statement is sometimes true.



Example 2 ***Classify special quadrilaterals***

Classify the special quadrilateral. *Explain* your reasoning.



The quadrilateral has four congruent sides. One of the angles is not a right angle, so the rhombus is not also a square. By the Rhombus Corollary, the quadrilateral is a rhombus.

✔ ***Checkpoint*** **Complete the following exercises.**

1. For any square *CDEF*, is it *always* or *sometimes* true that $\overline{CD} \cong \overline{DE}$? *Explain* your reasoning.

Always; a square has four congruent sides.

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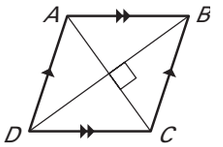
2. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

square

Your Notes

THEOREM 8.11

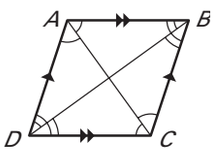
A parallelogram is a rhombus if and only if its diagonals are perpendicular.



▱*ABCD* is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

THEOREM 8.12

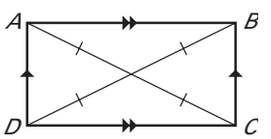
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.



▱*ABCD* is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$ and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.



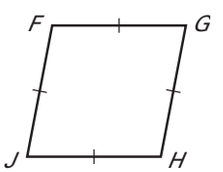
▱*ABCD* is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Example 3 ***List properties of special parallelograms***

Sketch rhombus *FGHJ*. List everything you know about it.

Solution

By definition, you need to draw a figure with the following properties:



- The figure is a parallelogram.
- The figure has four congruent sides.

Because *FGHJ* is a parallelogram, it has these properties:

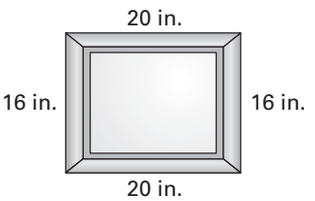
- Opposite sides are parallel and congruent.
- Opposite angles are congruent. Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.11, the diagonals of *FGHJ* are perpendicular. By Theorem 8.12, each diagonal bisects a pair of opposite angles.

Your Notes

Example 4 ***Solve a real-world problem***

Framing You are building a frame for a painting. The measurements of the frame are shown at the right.



a. The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain*.

b. You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

Solution

a. No, you cannot. The boards on opposite sides are the same length, so they form a parallelogram. But you do not know whether the angles are right angles.

b. By Theorem 8.13, the diagonals of a rectangle are congruent. The diagonals of the frame are congruent, so the frame forms a rectangle.

✔ ***Checkpoint*** **Complete the following exercises.**

3. Sketch rectangle *WXYZ*. List everything that you know about it.

WXYZ is a parallelogram with four right angles. Opposite sides are parallel and congruent. Opposite angles are congruent and consecutive angles are supplementary. The diagonals are congruent and bisect each other.

4. Suppose the diagonals of the frame in Example 4 are not congruent.

Could the frame still be a rectangle? *Explain*.

No; by Theorem 8.13, a rectangle must have congruent diagonals.

Homework