8.5 Use Properties of Trapezoids and Kites

Goal • Use properties of trapezoids and kites.

Your Notes

VOCABULARY

Trapezoid A trapezoid is a quadrilateral with exactly one pair of parallel sides.

Bases of a trapezoid The parallel sides of a trapezoid are the bases.

Base angles of a trapezoid A trapezoid has two pairs of base angles. Each pair shares a base as a side.

Legs of a trapezoid The nonparallel sides of a trapezoid are the legs.

Isosceles trapezoid An isosceles trapezoid is a trapezoid in which the legs are congruent.

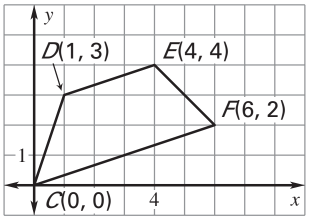
Midsegment of a trapezoid The midsegment of a trapezoid is the segment that connects the midpoints of its legs.

Kite A kite is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

Your Notes

Example 1 *Use a coordinate plane*

Show that *CDEF* is a trapezoid.



Solution

Compare the slopes of opposite sides.

Slope of $\overline{DE} = \frac{4-3}{4-1} = \frac{1}{3}$

Slope of $\overline{CF} = \frac{2-0}{6-0} = \frac{2}{6} = \frac{1}{3}$

The slopes of $\overline{DE}$ and $\overline{CF}$ are the same, so $\overline{DE} \parallel \overline{CF}$.

Slope of $\overline{EF} = \frac{2-4}{6-4} = \frac{-2}{2} = -1$

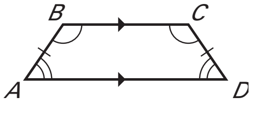
Slope of $\overline{CD} = \frac{3-0}{1-0} = \frac{3}{1} = 3$

The slopes of $\overline{EF}$ and $\overline{CD}$ are not the same, so $\overline{EF}$ is not parallel to $\overline{CD}$.

Because quadrilateral *CDEF* has exactly one pair of parallel sides, it is a trapezoid.

THEOREM 8.14

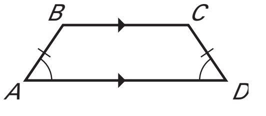
If a trapezoid is isosceles, then each pair of base angles is congruent.



If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

THEOREM 8.15

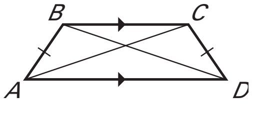
If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.



If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

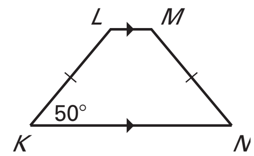


Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Your Notes

Example 2 ***Use properties of isosceles trapezoids***

Kitchen A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid. Find $m\angle N$, $m\angle L$, and $m\angle M$.



Solution

Step 1 **Find** $m\angle N$. *KLMN* is an isosceles trapezoid, so $\angle N$ and $\angle K$ are congruent base angles, and $m\angle N = m\angle K = 50°$.

Step 2 **Find** $m\angle L$. Because $\angle K$ and $\angle L$ are consecutive interior angles formed by $\overleftrightarrow{KL}$ intersecting two parallel lines, they are supplementary. So, $m\angle L = 180° - 50° = 130°$.

Step 3 **Find** $m\angle M$. Because $\angle M$ and $\angle L$ are a pair of base angles, they are congruent, and $m\angle M = m\angle L = 130°$.

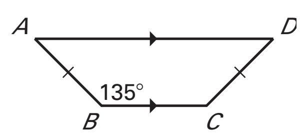
So, $m\angle N = 50°$, $m\angle L = 130°$, and $m\angle M = 130°$.

Checkpoint **Complete the following exercises.**

1. In Example 1, suppose the coordinates of point *E* are (7, 5). What type of quadrilateral is *CDEF*? *Explain.*

Parallelogram; opposite pairs of sides are parallel.

2. Find $m\angle C$, $m\angle A$, and $m\angle D$ in the trapezoid shown.

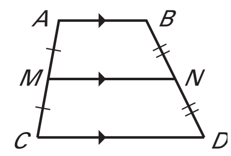


$m\angle C = 135°$, $m\angle A = 45°$, $m\angle D = 45°$

Your Notes

THEOREM 8.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

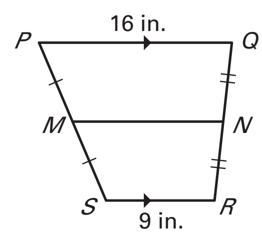


If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then

$\overline{MN} \parallel$ __$\overline{AB}$__, $\overline{MN} \parallel$ __$\overline{DC}$__, and $MN = \frac{1}{2}$(__AB__ + __CD__).

Example 3 ***Use the midsegment of a trapezoid***

In the diagram, $\overline{MN}$ is the midsegment of trapezoid *PQRS*. Find *MN*.



Solution

Use Theorem 8.17 to find *MN*.

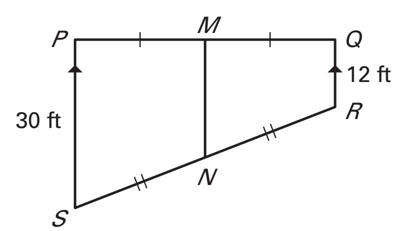
$MN = \frac{1}{2}$(__PQ__ + __SR__)		**Apply Theorem 8.17.**
$= \frac{1}{2}$(__16__ + __9__)		**Substitute __16__ for *PQ* and __9__ for *SR*.**
= __12.5__		**Simplify.**

The length *MN* is __12.5__ inches.

Checkpoint **Complete the following exercise.**

3. Find *MN* in the trapezoid at the right.

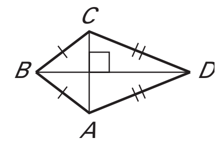
MN = 21 ft



Your Notes

THEOREM 8.18

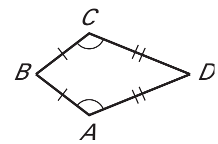
If a quadrilateral is a kite, then its diagonals are perpendicular.



If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.

THEOREM 8.19

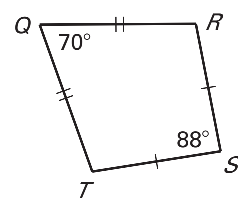
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.



If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.

Example 4 ***Apply Theorem 8.19***

Find $m\angle T$ in the kite shown at the right.



Solution

By Theorem 8.19, *QRST* has exactly one pair of congruent opposite angles. Because $\angle Q \not\cong \angle S$, $\angle R$ and $\angle T$ must be congruent. So, $m\angle R = m\angle T$. Write and solve an equation to find $m\angle T$.

$m\angle T + m\angle R + 70° + 88° = 360°$ **Corollary to Theorem 8.1**

$m\angle T + m\angle T + 70° + 88° = 360°$ **Substitute $m\angle T$ for $m\angle R$.**

$2(m\angle T) + 158° = 360°$ **Combine like terms.**

$m\angle T = 101°$ **Solve for $m\angle T$.**

Homework

Checkpoint **Complete the following exercise.**

4. Find $m\angle G$ in the kite shown at the right.

$m\angle G = 100°$