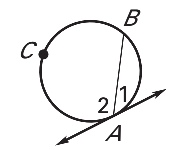
10.5 Apply Other Angle Relationships in Circles

Goal • Find the measures of angles inside or outside a circle.

Your Notes

THEOREM 10.11

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



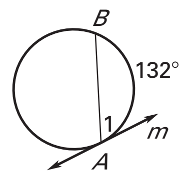
$m\angle 1 = \frac{1}{2}$ $m\overset{\frown}{AB}$

$m\angle 2 = \frac{1}{2}$ $m\overset{\frown}{BCA}$

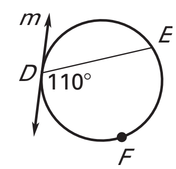
Example 1 ***Find angle and arc measures***

Line *m* is tangent to the circle. Find the indicated measure.

a. $m\angle 1$



b. $m\overset{\frown}{EFD}$



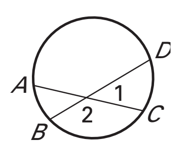
Solution

a. $m\angle 1 = \frac{1}{2}(132°) = 66°$

b. $m\overset{\frown}{EFD} = 2(110°) = 220°$

THEOREM 10.12: ANGLES INSIDE THE CIRCLE THEOREM

If two chords intersect *inside* a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



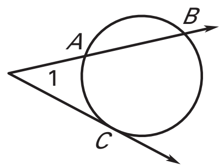
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

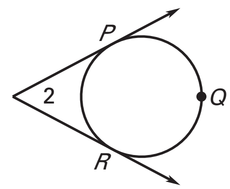
Your Notes

THEOREM 10.13: ANGLES OUTSIDE THE CIRCLE THEOREM

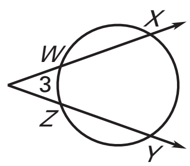
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



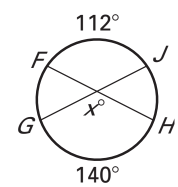
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Example 2 ***Find an angle measure inside a circle***

Find the value of *x*.



The chords $\overline{FH}$ and $\overline{GJ}$ intersect inside the circle.

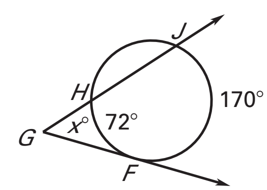
$x° = \frac{1}{2}(m\overset{\frown}{FJ} + m\overset{\frown}{HG})$ **Use Theorem 10.12.**

$x° = \frac{1}{2}(112° + 140°)$ **Substitute.**

$x = 126$ **Simplify.**

Example 3 ***Find an angle measure outside a circle***

Find the value of *x*.



The tangent $\overrightarrow{GF}$ and the secant $\overrightarrow{GJ}$ intersect outside the circle.

$m\angle FGH = \frac{1}{2}(m\overset{\frown}{FJ} - m\overset{\frown}{FH})$ **Use Theorem 10.13.**

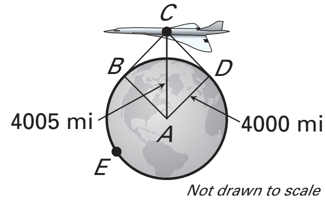
$x° = \frac{1}{2}(170° - 72°)$ **Substitute.**

$x = 49$ **Simplify.**

Your Notes

Example 4 *Solve a real-world problem*

Airplane You are flying in an airplane about 5 miles above the ground. What is the measure of arc BD, the part of Earth that you can see? (Earth's radius is about 4000 miles.)



Solution

Because $\overline{CB}$ and $\overline{CD}$ are tangents, $\overline{CB} \perp$ $\overline{AB}$ and $\overline{CD} \perp$ $\overline{AD}$. Also, $\overline{BC} \cong$ $\overline{DC}$. So, $\triangle ABC \cong$ $\triangle ADC$ by the HL Congruence Theorem, and $\angle BCA \cong$ $\angle DCA$. Solve right triangle CBA to find that $m\angle BCA \approx$ 87.1°. So, $m\angle BCD \approx 2($ 87.1° $) =$ 174.2°. Let $m\overset{\frown}{BD} = x°$.

Because the value for $m\angle BCD$ is an approximation, use the symbol $\approx$ instead of $=$.

$m\angle BCD = \frac{1}{2}(m\overset{\frown}{DEB} - m\overset{\frown}{BD})$ **Use Theorem 10.13.**

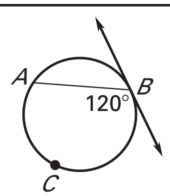
$174.2° \approx \frac{1}{2}[(360° - x°) - x°]$ **Substitute.**

$x \approx 5.8$ **Solve for x.**

From the airplane, you can see an arc of about 5.8°.

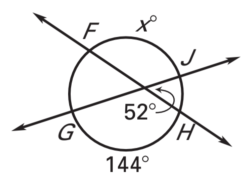
Checkpoint Find the indicated measure or the value of the variable.

1. $m\overset{\frown}{ACB}$



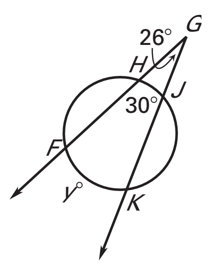
$m\overset{\frown}{ACB} = 240°$

2.



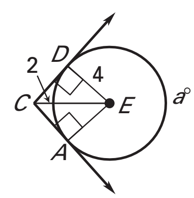
$x = 112$

3.



$y = 82$

4.



$a \approx 263.6$

Homework