11.3 Perimeter and Area of Similar Figures

Goal • Use ratios to find areas of similar figures.

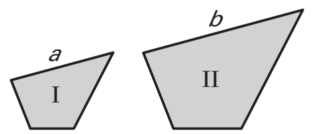
Your Notes

THEOREM 11.7: AREAS OF SIMILAR POLYGONS

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is a^2 : b^2.

$$\frac{\text{Side length of Polygon I}}{\text{Side length of Polygon II}} = \frac{a}{b}$$

$$\frac{\text{Area of Polygon I}}{\text{Area of Polygon II}} = \frac{a^2}{b^2}$$



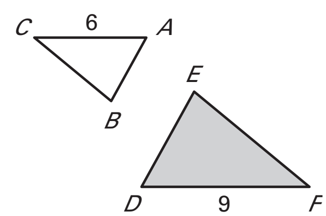
Polygon I ~ Polygon II

Example 1 ***Find ratios of similar polygons***

In the diagram, △*ABC* ~ △*DEF*. Find the indicated ratio.

a. Ratio (shaded to unshaded) of the perimeters

b. Ratio (shaded to unshaded) of the areas



Solution

The ratio of the lengths of corresponding sides is $\frac{9}{6} = \frac{3}{2}$, or 3 : 2.

a. By Theorem 6.1, the ratio of the perimeters is 3 : 2.

b. By Theorem 11.7 above, the ratio of the areas is 3^2 : 2^2, or 9 : 4.

You can also compare the measures with fractions. The perimeter of △*DEF* is three halves the perimeter of △*ABC*. The area of △*DEF* is nine fourths the area of △*ABC*.

Your Notes

Example 2 *Solve a real-world problem*

Windows You buy two rectangular pieces of aluminum window screening. One is 15 feet long and costs \$135. The other is similar in shape and is 20 feet long. The screen is sold by the square foot. What is the cost of the longer roll?

Solution

The ratio of the length of the longer roll to the shorter roll is 20 : 15, or 4 : 3. So, the ratio of the areas is 4^2 : 3^2, or 16 : 9. This ratio is also the ratio of the screen costs. Let x be the cost of the longer roll.

$$\frac{16}{9} = \frac{x}{135}$$

← **Cost of longer roll** (x)

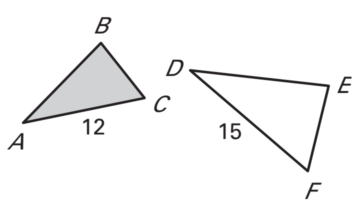
← **Cost of shorter roll** (135)

$x = 240$ **Solve for x.**

It costs \$240 for the longer roll.

✔ ***Checkpoint*** **Complete the following exercises.**

1. Given $\triangle ABC \sim \triangle DEF$, find the ratio (shaded to unshaded) of the perimeters and areas.



4 : 5; 16 : 25

2. In Example 2, suppose you decide to buy a different kind of screening but in the same dimensions. The longer roll costs \$200. What is the cost of the shorter roll?

\$112.50

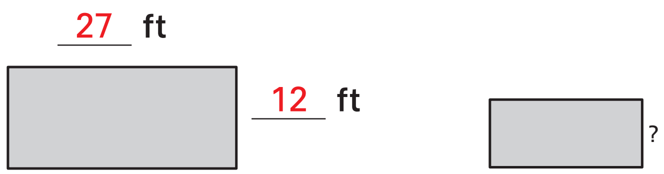
Your Notes

Example 3 *Use a ratio of areas*

Billboards A large rectangular billboard is 12 feet high and 27 feet long. A smaller billboard is similar to the large billboard. The area of the smaller billboard is 144 square feet. Find the height of the smaller billboard.

Solution

First draw a diagram to represent the problem. Label dimensions and areas.



$A = 27(12) = 324\ \text{ft}^2$ $\qquad$ $A = 144\ \text{ft}^2$

Then use Theorem 11.7. If the area ratio is $a^2 : b^2$, then the length ratio is $a : b$.

$$\frac{\text{area of smaller billboard}}{\text{area of large billboard}} = \frac{144}{324} = \frac{4}{9}$$

$$\frac{\text{length of smaller billboard}}{\text{length of large billboard}} = \frac{2}{3}$$

Any length in the smaller billboard is $\frac{2}{3}$ of the corresponding length in the large billboard. So, the height of the smaller billboard is $\frac{2}{3}$(12 feet) = 8 feet.

Checkpoint **Complete the following exercise.**

3. In Example 3, suppose the area of the smaller billboard is 225 square feet. Find the height of the smaller billboard.

10 ft

Your Notes

Example 4 ***Solve a multi-step problem***

Stop sign A stop sign rug is a regular octagon. Each side is 2 feet and the area is about 19.3 square feet. You make a stop sign mat with a perimeter of 72 inches. Find the area of the mat to the nearest tenth of a square inch.

Solution

All regular octagons are similar, so the rug and mat are similar.

Step 1 **Find** the ratio of the lengths of the rug and mat by finding the ratio of the perimeters. Use the same units for both lengths in the ratio.

$$\frac{\text{Perimeter of rug}}{\text{Perimeter of mat}} = \frac{\boxed{8(2\text{ ft})}}{72\text{ in.}} = \frac{\boxed{16}\text{ ft}}{\boxed{6}\text{ ft}} = \frac{8}{3}$$

So, the ratio of corresponding lengths (rug to mat) is 8 : 3.

Step 2 **Calculate** the area of the mat. Let x be this area.

$$\frac{(\text{length in rug})^2}{(\text{length in mat})^2} = \frac{\text{area of rug}}{\text{area of mat}}$$

$$\frac{8^2}{3^2} = \frac{\boxed{19.3\text{ ft}^2}}{x\text{ ft}^2}$$

$$64x = 173.7$$

$$x \approx 2.714\text{ ft}^2$$

Step 3 **Convert** the area to square inches.

$$2.714\text{ ft}^2 \cdot \frac{\boxed{144}\text{ in.}^2}{1\text{ ft}^2} \approx 390.8\text{ in.}^2$$

The area of the mat is about 390.8 square inches.

Checkpoint **Complete the following exercise.**

Homework

4. Rectangles I and II are similar. The perimeter of Rectangle I is 48 inches. Rectangle II is 30 inches by 18 inches. Find the area of Rectangle I.

135 in.^{2}