11.4 Circumference and Arc Length

Goal • Find arc lengths and other measures.

Your Notes

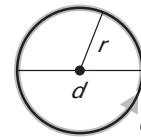**VOCABULARY**

Circumference The circumference of a circle is the distance around the circle.

Arc length An arc length is a portion of the circumference of a circle.

THEOREM 11.8: CIRCUMFERENCE OF A CIRCLE

The circumference C of a circle is
$C = \underline{\pi d}$ or $C = \underline{2\pi r}$, where d
is the diameter of the circle and
r is the radius of the circle.

$C = \underline{\pi d} = \underline{2\pi r}$

Example 1 *Use the formula for circumference*

Find the indicated measure.

a. Circumference of a circle with radius 11 meters

b. Radius of a circle with circumference 18 yards

Solution

a. $C = 2\pi r$

$= 2 \cdot \pi \cdot \underline{11}$

$= \underline{22}\ \pi$

$\approx \underline{69.12}\ $ m

b. $C = 2\pi r$

$\underline{18} = 2\pi r$

$\dfrac{18}{2\pi} = r$

$\underline{2.86}\ $ yd $\approx r$

✔ *Checkpoint* **Complete the following exercise.**

1. Find the circumference of a circle with diameter 23 inches.

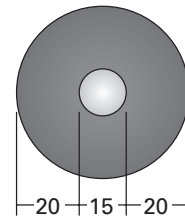about 72.26 inches

---

Example 2 *Use circumference to find distance traveled*

Skateboarding The dimensions of the skateboard wheel shown at the right are in millimeters. To the nearest meter, how far does the wheel travel when it makes 35 revolutions?

—20—15—20—

Solution

Step 1 Find the diameter of the wheel.

$$d = \underline{15} + 2(\underline{20}) = \underline{55} \text{ mm}$$

Step 2 Find the circumference of the wheel.

$$C = \pi d = \pi(\underline{55}) \approx \underline{172.8} \text{ mm}$$

Step 3 Find the distance the wheel travels in 35 revolutions. In one revolution, the wheel travels a distance equal to its __circumference__. In 35 revolutions, the wheel travels a distance equal to __35__ times its circumference.

$$\frac{\text{Distance}}{\text{traveled}} = \frac{\text{Number of}}{\text{revolutions}} \cdot \text{Circumference}$$

$$\approx \underline{35} \cdot \underline{172.8} \text{ mm}$$

$$= \underline{6048} \text{ mm}$$

Step 4 Use unit analysis. Change __6048__ millimeters to meters.

$$\underline{6048} \text{ mm} \cdot \frac{1 \text{ m}}{1000 \text{ mm}} = \underline{6.048} \text{ m}$$

The wheel travels about __6__ meters.

✔ *Checkpoint* **Complete the following exercise.**

2. A skateboard wheel has a diameter of 56 millimeters. How many revolutions does the wheel make when traveling 3 meters?
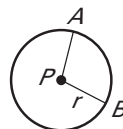
about 17 revolutions

ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or}$$

$$\text{Arc length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$$

Example 3 *Find and use arc lengths*

Find the indicated measure.
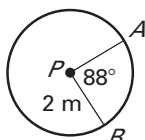
a. Arc length of $\overset{\frown}{AB}$ **b.** $m\overset{\frown}{RS}$
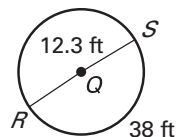
a. Arc length of $\overset{\frown}{AB} = \dfrac{88°}{360°} \cdot 2\pi(\underline{\;2\;}) \approx \underline{\;3.07\;}$ meters

b.
$$\frac{\text{Arc length of } \overset{\frown}{RS}}{2\pi r} = \frac{m\overset{\frown}{RS}}{360°} \qquad \textbf{Write equation.}$$

$$\frac{\boxed{38}}{2\pi\left(\boxed{12.3}\right)} = \frac{m\overset{\frown}{RS}}{360°} \qquad \textbf{Substitute.}$$

$$\underline{360°} \cdot \frac{\boxed{38}}{2\pi\left(\boxed{12.3}\right)} = m\overset{\frown}{RS} \qquad \begin{array}{l}\textbf{Multiply each side}\\\textbf{by } \underline{360°}.\end{array}$$

$$\underline{177°} \approx m\overset{\frown}{RS} \qquad \textbf{Use a calculator.}$$

✔ *Checkpoint* **Find the indicated measure.**
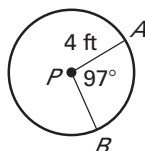
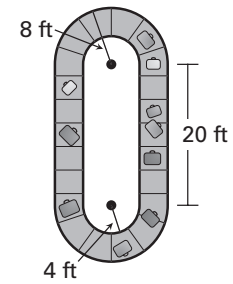| **3.** Arc length of $\overset{\frown}{AB}$	**4.** Circumference of $\odot Z$
about 6.77 ft	20.64 cm
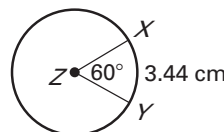
Example 4   *Use arc length to find distances*

Luggage A conveyor belt for luggage at an airport is shown at the right. The outer part of the belt forms a 180° arc at each end. For each arc, the radius is 8 feet. Approximate the distance around the belt for a coin on the outer portion. Round to the nearest foot.

8 ft

20 ft

4 ft

Solution

The outer portion is made of two straight sections and two semicircles. To find the distance around the outer portion, find the sum of the lengths of each part.

$$\text{Distance} = 2 \cdot \begin{matrix}\text{Length of each}\\ \text{straight section}\end{matrix} + 2 \cdot \begin{matrix}\text{Length of each}\\ \text{semicircle}\end{matrix}$$

$$= 2(\underline{\ 20\ }) + 2 \cdot \left(\frac{1}{2} \cdot 2\pi \cdot 8 \right)$$

$$\approx \underline{\ 90.27\ } \text{ feet}$$

The distance around the outer portion is about __90__ feet.

✔ *Checkpoint* **Complete the following exercise.**

5. In Example 4, the inner portion of the belt also has 180° arcs on each end. The radius of each arc is 4 feet. Find the distance around the belt for a coin on the inner portion. Round to the nearest foot.

about 65 ft

Homework