11.5 Areas of Circles and Sectors

Goal • Find the areas of circles and sectors.

Your Notes

VOCABULARY

Sector of a circle A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.

THEOREM 11.9: AREA OF A CIRCLE

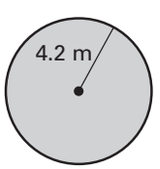
The area of a circle is π times the square of the radius.

$A = \pi r^2$

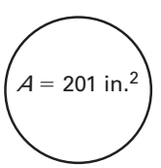
Example 1 *Use the formula for area of a circle*

Find the indicated measure.

a. Area



b. Diameter



Solution

a. $A = \pi r^2$ **Write formula for the area of a circle.**

$= \pi(\ 4.2\ )^2$ **Substitute** 4.2 **for** r.

$=$ 17.64 π **Simplify.**

$\approx$ 55.42 m^2 **Use a calculator.**

b. $A = \pi r^2$ **Write formula for the area of a circle.**

201 $= \pi r^2$ **Substitute** 201 **for** A.

$\frac{201}{\pi} = r^2$ **Divide each side by** π.

8 in. $\approx r$ **Find the positive square root of each side.**

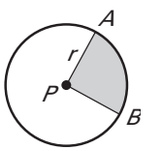
The radius is about 8 inches, so the diameter is about 16 inches.

Your Notes

THEOREM 11.10: AREA OF A SECTOR

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.

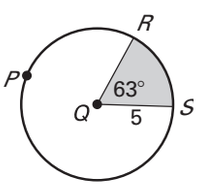
$$\frac{\text{Arc of sector } APB}{\boxed{\pi r^2}} = \frac{\boxed{\overset{\frown}{mAB}}}{360°}, \text{ or}$$



$$\text{Area of sector } APB = \frac{\boxed{\overset{\frown}{mAB}}}{360°} \cdot \underline{\pi r^2}$$

Example 2 ***Find areas of sectors***

Find the areas of the sectors formed by $\angle RQS$.



Solution

Step 1 **Find** the measures of the minor and major arcs.

Because $m\angle RQS =$ 63°, $m\overset{\frown}{RS} =$ 63° and $m\overset{\frown}{RPS} = 360° -$ 63° $=$ 297°.

Step 2 **Find** the areas of the small and large sectors.

$$\text{Area of small sector} = \frac{m\overset{\frown}{RS}}{360°} \cdot \pi r^2$$

$$= \frac{\boxed{63°}}{360°} \cdot \pi \cdot \underline{5}^2$$

$$\approx \underline{13.74}$$

$$\text{Area of large sector} = \frac{m\overset{\frown}{RPS}}{360°} \cdot \pi r^2$$

$$= \frac{\boxed{297°}}{360°} \cdot \pi \cdot \underline{5}^2$$

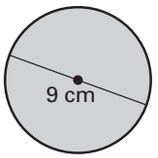
$$\approx \underline{64.80}$$

The areas of the small and large sectors are about 13.74 square units and 64.80 square units, respectively.

Your Notes

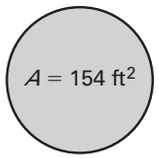
✔ ***Checkpoint*** **In Exercises 1 and 2, use the diagram to find the indicated measure.**

1. Area



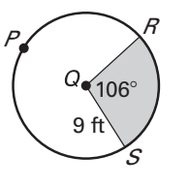
about 63.62 cm^2

2. Radius



about 7 ft

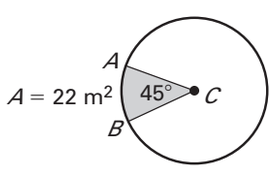
3. Find the areas of the sectors formed by $\angle RQS$.



small sector: about 74.93 ft^2;
large sector: about 179.54 ft^2

Example 3 ***Use the Area of a Sector Theorem***

Use the diagram to find the area of $\odot C$.



Solution

Area of sector $ACB = \frac{m\overset{\frown}{AB}}{360°} \cdot$ Area of $\odot C$

$\underline{22} = \frac{\boxed{45°}}{360°} \cdot$ Area of $\odot C$

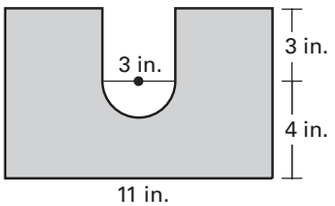
$\underline{176}$ = Area of $\odot C$

The area of $\odot C$ is $\underline{176}$ square meters.

Your Notes

Example 4 ***Find an area***

Construction A contractor needs to cut a section out of a rectangular piece of wood as shown. To the nearest square inch, what is the area of the remaining wood?



Solution

The area you need to find is the area of the rectangle minus the area of the cut-out section. The cut-out can be divided into a semicircle and a square.

Area of wood

= Area of rectangle − [Area of semicircle + Area of square]

= 11 (7) − [$\frac{180^\circ}{360^\circ}$ • ($\pi \cdot 1.5^2$) + 3^2]

= 77 − [1.125π + 9]

≈ 64.47

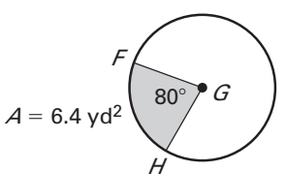
The area is about 64.5 square inches.

Use the radius (1.5 in.) not the diameter (3 in.) when you calculate the area of the semicircle.

Checkpoint **Complete the following exercises.**

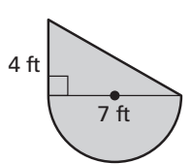
4. Find the area of $\odot G$.

28.8 yd^2



5. Find the area of the figure.

about 33.24 ft^2



Homework