11.6 Areas of Regular Polygons

Goal • Find areas of regular polygons inscribed in circles.

Your Notes

VOCABULARY

Center of a polygon The center of a polygon is the center of its circumscribed circle.

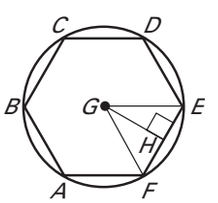
Radius of a polygon The radius of a polygon is the radius of its circumscribed circle.

Apothem of a polygon The distance from the center to any side of the polygon is the apothem.

Central angle of a regular polygon A central angle of a regular polygon is an angle formed by two radii drawn to consecutive vertices of the polygon.

Example 1 ***Find angle measures in a regular polygon***

In the diagram, *ABCDEF* is a regular hexagon inscribed in ⊙*G*. Find each angle measure.



a. $m\angle EGF$

b. $m\angle EGH$

c. $m\angle HEG$

Solution

a. $\angle EGF$ is a central angle, so $m\angle EGF =$ $\frac{360^\circ}{6}$, or 60°.

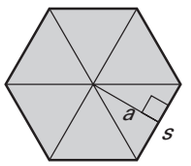
b. $\overline{GH}$ is an apothem, which makes it an altitude of isosceles $\triangle EGF$. So, $\overline{GH}$ bisects $\angle EGF$ and $m\angle EGH = \frac{1}{2}\ m\angle EGF = 30^\circ$.

c. The sum of the measures of right $\triangle HEG$ is 180°. So, $90^\circ + 30^\circ + m\angle HEG = 180^\circ$, and $m\angle HEG = 60^\circ$.

Your Notes

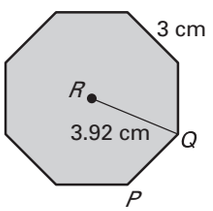
THEOREM 11.11: AREA OF A REGULAR POLYGON

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}$ *aP*, or $A = \frac{1}{2}$ *a* • *ns*.



Example 2 ***Find the area of a regular polygon***

Coaster A wooden coaster is a regular octagon with 3 centimeter sides and a radius of about 3.92 centimeters. What is the area of the coaster?



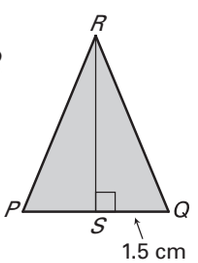
Solution

Step 1 **Find** the perimeter P of the coaster. An octagon has 8 sides, so $P =$ 8 (3) = 24 centimeters.

Step 2 **Find** the apothem a. The apothem is height *RS* of $\triangle PQR$. Because $\triangle PQR$ is isosceles, altitude $\overline{RS}$ bisects $\overline{QP}$.

So, $QS = \frac{1}{2}(QP) = \frac{1}{2}$ (3)

$= 1.5$ cm.



To find RS, use the Pythagorean Theorem for $\triangle RQS$.

$a = RS$

$\approx \sqrt{3.92^2 - 1.5^2} = \sqrt{13.1164} \approx 3.622$

Step 3 **Find** the area A of the coaster.

$A = \frac{1}{2}aP$ **Formula for area of regular polygon**

$\approx \frac{1}{2}$(3.622)(24) **Substitute.**

$\approx$ 43.5 **Simplify.**

The area of the coaster is about 43.5 square centimeters.

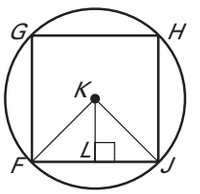
In general, your answer will be more accurate if you avoid rounding until the last step. Round your final answers to the nearest tenth unless you are told otherwise.

Your Notes

Checkpoint **Complete the following exercises.**

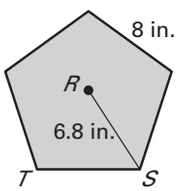
1. In the diagram, *FGHJ* is a square inscribed in $\odot K$. Find $m\angle FKJ$ and $m\angle KJF$.

$m\angle FKJ = 90°$; $m\angle KJF = 45°$



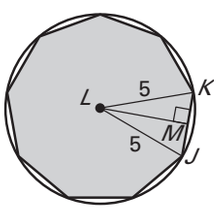
2. The radius of the regular pentagon is about 6.8 inches. Find the area to the nearest square inch.

about 110 $in.^2$



Example 3 ***Find the perimeter and area of a regular polygon***

A regular nonagon is inscribed in a circle with radius 5 units. Find the perimeter *P* and area *A* of the nonagon.

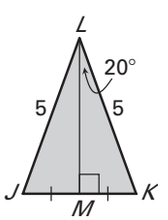


The measure of central $\angle JLK$ is $\frac{360°}{9}$, or $40°$.

Apothem $\overline{LM}$ bisects the central angle, so $m\angle KLM$ is $20°$. To find the lengths of the legs, use trigonometric ratios for right $\triangle KLM$.

$\sin 20° = \frac{MK}{5}$ $\qquad$ $\cos 20° = \frac{LM}{5}$

$5 \sin 20° = MK$ $\qquad$ $5 \cos 20° = LM$



The regular nonagon has side lengths $s = 2MK = 2(5 \sin 20°) = 10 \sin 20°$ and apothem $a = LM = 5 \cos 20°$.

So,

$P = 9s = 9(10 \sin 20°) = 90 \sin 20° \approx 30.8$ units,

and

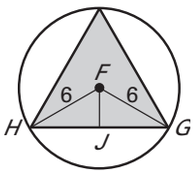
$A = \frac{1}{2}aP$

$= \frac{1}{2}(5 \cos 20°)(90 \sin 20°) \approx 72.3$ square units.

Your Notes

✔ ***Checkpoint*** **Complete the following exercise.**

3. Find the perimeter and area of the equilateral triangle inscribed in $\odot F$.



Perimeter: about 31.2 units;
Area: about 46.8 square units

FINDING LENGTHS IN A REGULAR *N*-GON

To find the area of a regular *n*-gon with radius *r*, you may need to first find the apothem *a* or the side length *s*.

You can use . . .	. . . when you know *n* and . . .	. . . as in . . .
Pythagorean Theorem: $\left(\frac{1}{2}s\right)^2 + a^2 = r^2$	Two measures: *r* and *a*, or *r* and *s*	Example 2 and Checkpoint Ex. 2
Special Right Triangles	Any one measure: *r* or *a* or *s* **And** the value of *n* is 3, 4, or 6	Checkpoint Ex. 3
Trigonometry	Any one measure: *r* or *a* or *s*	Example 3 and Checkpoint Ex. 3

Homework